2026 chapter ICMC SUMMER MEETING

02-04 FEBRUARY

SÃO CARLOS - SP BRAZIL

LIST OF SESSIONS

Conservation Laws and Transport Equations
Dispersive Equations
Domain perturbation for PDEs and applications
Dynamical Systems via Ordinary Differential Equations
Elliptic Equations
Free Boundaries problems and related topics
Harmonic Analysis and Related Topics
Integral and Functional Differential Equations
Linear Partial Differential Equations
Nonlinear Dynamical Systems

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Plenary Lectures | **Special Sessions** | **Poster Sessions**

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Welcome

It is a pleasure to welcome you to the *ICMC Summer Meeting on Differential Equations - 2026 Chapter* and to São Carlos. We wish you a pleasant stay and that you enjoy the meetings.

Scientific committee

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Session Organizers

Jean Silva (UFMG/Brazil) & Gerardo Jonatan Huaroto Cardenas (UFAL/Brazil): Conservation Laws and Transport Equations.

Mahendra Panthee (UNICAMP/Brazil) & Argenis Jose Mendez Garcia (UNICAMP/Brazil): Dispersive Equations.

Alessandra Verri (UFSCar/Brazil) & Marcone C. Pereira (USP/Brazil): Domain Perturbation for PDEs and Applications.

Ederson Moreira dos Santos (USP/Brazil) & Gustavo Ferron Madeira (UFSCar/Brazil): Elliptic Equations.

João Vitor da Silva (UNICAMP/Brazil), Disson dos Prazeres (UFS/Brazil) & Ginaldo de Santana Sá (CMM/Chile): Recent Trends in Nonlinear PDEs and Free boundary Problems.

Guilherme da Silva (USP/Brasil), Tiago Picon (USP/Brazil) & Charles Ferreira dos Santos (USP/Brazil): Harmonic Analysis and Related Topics.

Everaldo de Mello Bonotto (USP/Brazil), Jaqueline G. Mesquita (UnB/Brazil) & Pierluigi Benevieri (USP/Brazil): Integral and Functional Differential Equations.

Nicholas Braun Rodrigues (UFSCar/Brasil) & Vinícius Novelli da Silva (USP/Brazil): Linear Partial Differential Equations.

Juliana Fernandes da Silva Pimentel (UFRJ/Brazil), Maykel Boldrin Belluzi (UFSCar/Brazil) & Phillipo Lappicy (Universidad Complutense de Madrid/Spain): Nonlinear Dynamical Systems.

Alex Carlucci Rezende (UFSCar/Brazil) & Márcio Ricardo Alves Gouveia (IBILCE UNESP/Brazil): Dynamical Systems via Ordinary Differential Equations.

Address

Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo
Avenida Trabalhador São-carlense, 400
CEP: 13566-590 - São Carlos - SP
FAX: +55 (16) 3371-2238
e-mail: summer@icmc.usp.br

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Maps

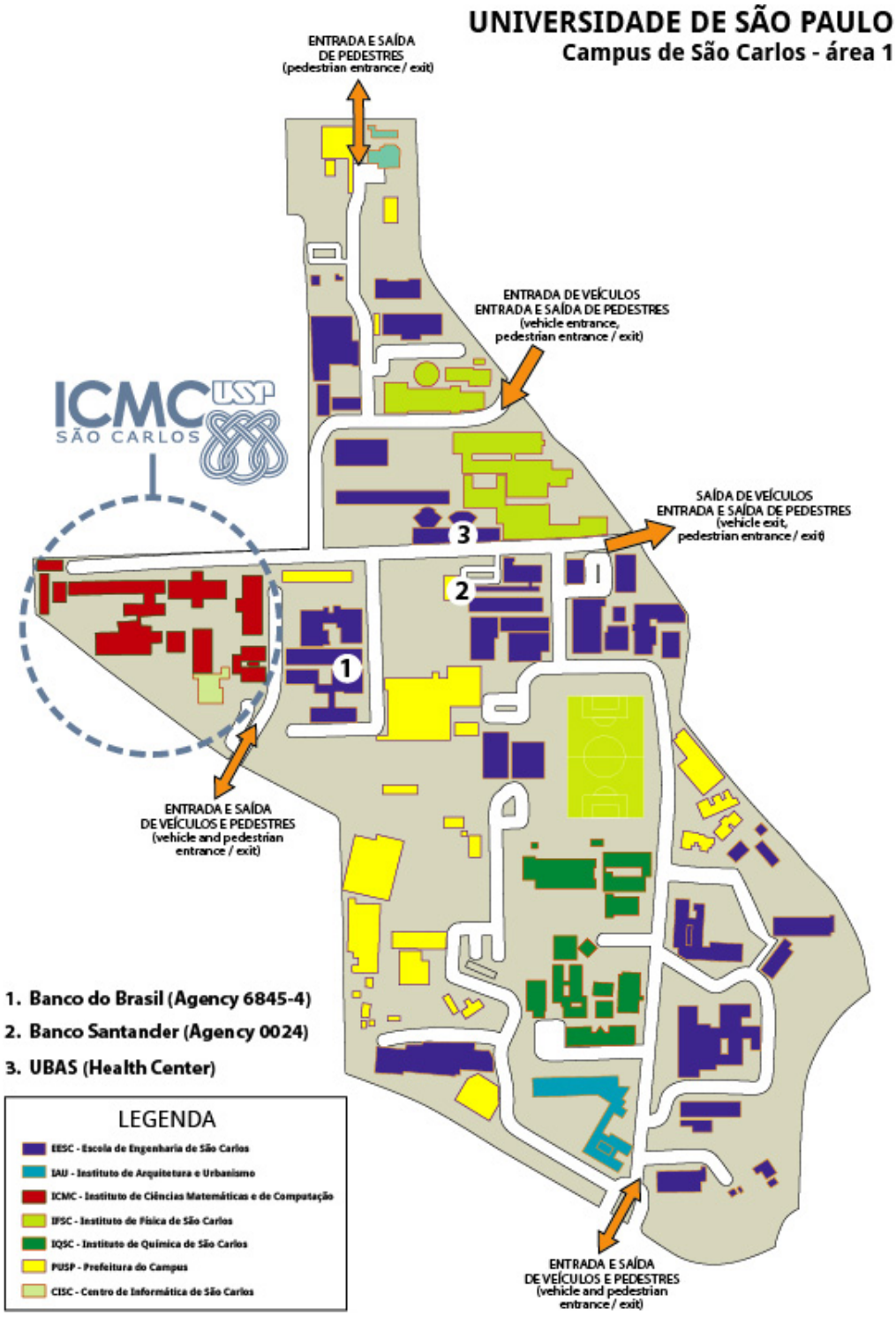

Figure 1: Campus map

Figure 2: ICMC map

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

General Information

Conference site

The meeting will take place at Buildings 4, 5 and 6. These buildings are indicated in the map on page 6.

Plenary talks and Session on Nonlinear Dynamical Systems will take place at the Auditorium (Building 6).

Session "Dynamical Systems via Ordinary Differential Equations" will take place in room 4001 (Building 4). All other sessions will take place in Building 5.

Poster session will take place at the Ground floor of the Library.

The Coffee Room is on the ground floor of the Library (1, map on page 6).

Registration

The registrations will be made in the following schedule:

Sunday, February 01^{st}: From 15:30 to 18:00 in the entrance of the ICMC Auditorium (Building 6).

Monday, January 02^{nd}: From 08:00 to 08:40 in the entrance of the ICMC Auditorium (Building 6).

Those who cannot register on Sunday or Monday can also do it during the week at any time.

We will provide you with a badge at registration. Please wear your badge at the event to access the event rooms.

The Events Office will set up a help desk at the entrance of the Auditorium and will be at your disposal for any questions and information, also the Events Office (3, map on page 6) will be at your disposal.

Registration Fees

Student Fee: R$ 100,00 (reais).

Regular Fee: R$ 250,00 (reais).

Financial support

The financial support from the local organizing committee will be available on Monday, February 02, from 11:30 to 13:30 and on Tuesday, February 03, from 11:30 to 12:30, at the help desk at the entrance of the Auditorium (Building 6, map on page 6). In order to receive your support, it is mandatory to completely fill out the on-line registration form available at `summer.icmc.usp.br/user_summer/` .

Meals and refreshments

There are some restaurants near the campus (walk distance).

Social events

Tuesday, February 03: Photo of the meeting at 11:30 at ICMC.

Tuesday, February 03: Conference Dinner at 20:00 at Barone Restaurant.

Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

Money exchanges

In case you need to exchange your money, we recommend:

• Confidence Câmbio at Shopping Center Iguatemi. The working hours are from 10:00 to 20:00 (Mon-Fri) and from 10:00 to 18:00 (Sat).

• JIS Câmbio at 1931, São Sebastião Street. The working hours are from 09:30 to 17:30 (Mon-Fri).

Smoking

Smoking is prohibited inside any of the ICMC buildings also in the canteen and on the ground floor of the library.

Computer and wireless LAN use

The University provides access to wireless internet connection via **eduroam**. If you do not possess an eduroam account you can access another of our wireless connection through the following steps:

1. Enable wireless on your device.
2. Join the ICMC-GUEST wireless network.
3. Open a browser and try to visit any website.
4. You will be redirected to a login page. Enter the login and password as follows:

   User Name: summer2026

   Password: 2026summer
5. You may freely browse the internet after logging in. You may occasionally need to re-authenticate using the above procedure.

Contents

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Plenary Lectures

Plenary Lectures

A Cauchy Problem in Hypocomplex Structures

Abdelhamid Meziani

Florida International University, USA

Initial value problems in a class of hypocomplex structures over an open set $\mathcal{O} \in \boldsymbol{R}^N$ are investigated. Existence and uniqueness of solution are established for the linear Cauchy problem $w_t = \sum A_j M_j w + Bw + C$, $w(0,x) = \phi(x)$, when all data are annihilated by the local sections $L_1, \cdots, L_r$ that define the structure. Here the M_j's are vector fields so that $L_1, \cdots, L_r, M_1, \cdots, M_n$ form a basis of $\mathbb{C}T\mathcal{O}$. A quasilinear problem is considered and an application to an initial value problem in CR structures of hypersurface type is given.

Symmetries of first order linear PDEs

Bernhard Lamel

University of Vienna, Austria

We will discuss properties of symmetries of first order linear PDEs. This talk is going to contrast properties known from the more classical case of CR geometry with properties of more general involutive systems, outline some recent results, and discuss applications and open problems.

On a Class of Free Boundary Problems Related to Thermal Insulation

Cristina Trombetti

Università degli Studi di Napoli Federico II, Italy

Free boundary problems in partial differential equations (PDEs) constitute a class of mathematical models in which both the solution and the domain where it is defined are unknown and must be determined simultaneously. Such problems naturally arise in a wide range of physical and engineering applications, including fluid dynamics, solid mechanics, and heat conduction. In this talk, we focus on a class of free boundary problems related to thermal insulation. In these models, the free boundary may represent either an interface whose location is not known a priori or the optimal configuration of an insulating material. The aim is to characterize the resulting free boundaries and to analyze how their geometry influences heat loss and energy efficiency.

Recent developments in shape optimization theory and overdetermined problems

Filomena Pacella

Sapienza Università di Roma, Italy

Starting from classical results we present recent developments for shape optimization and overdetermined problems in a relative setting, i.e. when the differential problems are posed in domains constrained to be inside a fixed region. In particular we will discuss the case when the unbounded region is an infinite cylinder, showing that the best shape to optimize some Dirichlet energy is not always the most expected one.

Understanding Decay Rates for Nonlinear Dynamics in Critical Spaces

Gabriela Del Valle Planas

Universidade Estadual de Campinas, Brazil

Understanding how quickly solutions decay as time approaches infinity is essential for capturing how systems stabilise, how rapidly perturbations vanish, and whether the solutions efficiently reach equilibrium. This understanding provides a link between transient dynamics and the system's long-term behaviour.

In this talk, I will explore the decay rates of solutions in critical Sobolev spaces for a range of nonlinear dissipative systems. I will present recent results concerning the Navier-Stokes equations, the Navier-Stokes–Coriolis system, the energy-critical nonlinear heat equation, and the Hardy–Sobolev parabolic equation.

The decay estimates are expressed in terms of the decay character of the initial data, yielding algebraic decay rates and showing in detail the roles played by the linear and nonlinear parts. The proof is carried on purely in the critical space. This is the first instance in which such a method is used for obtaining decay bounds in a critical space for nonlinear equations.

In collaboration with M. Ikeda (Japan), L. Kosloff (Brazil), and C.J. Niche (Brazil).

On solutions to Interaction equations for short and long dispersive waves

Jose Felipe Linares Ramirez

IMPA, Brazil

In this lecture we will describe recent results concerning asymptotic behavior of solutions to a system of nonlinear dispersive equations called Schrödinger-Korteweg-de Vries $(S-KdV)$ system. This system is a model depicting the interaction of long and short waves and has been studied in several fields of physics and fluid dynamics.

On a comb-like domain with non-cylindrical teeth

José Maria Arrieta, Joaquín Domínguez de Tena

Universidad Complutense de Madrid, Spain

We consider the Laplace operator with homogeneous Neumann boundary conditions defined in a comb-like (or brush-like) domain where the teeth are not necessarily cylindrical. We obtain the limit problem which, as a difference from previous results in the literature, may be interpreted as an equation in a graph.

Generic behaviours and stability in dynamical systems

Maria Joana Torres

Universidade do Minho, Portugal

Dynamical systems is the study of the long-term behaviour of systems that evolve with time. The foundations were laid by Henri Poincaré's extraordinary work *Les Méthodes Nouvelles de la Mécanique Céleste* $(1892-1899)$, with fundamental questions concerning the stability and evolution of the solar system. Attempts to address these questions exposed the incapacity to solve *exactly* mathematical questions arising from physical systems. It became evident that understanding *typical* systems, or systems *in general*, was mathematically more fruitful.

The aim of the modern theory of dynamical systems is therefore to describe the behaviour of *typical* (or *generic*) trajectories under *typical* evolution laws. Moreover, when considering real-world systems, neither the initial data nor the evolution law is known exactly. For this reason, the focus is on properties that are *stable*, i.e., that persist under small perturbations of the evolution law.

In this talk, we present an overview and recent results on generic behaviours and stability in dynamical systems.

Analysis of Liquid Crystal Flows and Poroelastic Media

Matthias Hieber

Technische Universität Darmstadt, Germany

In this talk we consider first two models describing nematic liquid crystal flows: the de Gennes model and the Ericksen-Leslie model. Of special interest in the Ericksen-Leslie model is a fully nonlinear boundary condition which is necessary to guarantee that the system fulfills physical principles. We also consider the interaction between a fluid and a porous media described by the Biot system and the Beavers-Joseph interface conditions. This is joint work with T. Binz, A. Hussein, J. Li, A. Roy, M. Wilke and M. Wrona.

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Thematic Sessions

Conservation Laws and Transport Equations

Organizer: Jean Silva (UFMG/Brazil) & Gerardo Jonatan Huaroto Cardenas (UFAL/Brazil)

Pathwise quantitative particle approximation of nonlinear stochastic Fokker-Planck equations via relative entropy

Alexandre Batista de Souza, Christian Olivera

Universidade Estadual de Campinas, Brazil

We derive the nonlinear stochastic Fokker-Planck equation from stochastic systems particles with individual and environmental noises via relative entropy method, with pathwise quantitative bounds. Moreover, we prove the existence of a unique strong solution to the associated Fokker-Planck equation. Our proof is based on tools from PDE analysis, stochastic analysis, functional inequalities, such as the log-Sobolev inequality, and also we use the dissipation of entropy which provides some bound on the Fisher information of the particle system. The approach applies to repulsive and attractive kernels.

A Lagrangian-Eulerian approach for a class of nonlinear balance laws: numerical analysis via a weak asymptotic method

Eduardo Cardoso de Abreu, E. Pandini, W. Lambert

Universidade Estadual de Campinas, Brazil

In this work, we discuss a new class of Lagrangian–Eulerian schemes, recently introduced in [2], to solve initial value problems for $u(x,t)\colon \mathbb{R}\times\mathbb{R}_+\to\Omega\subset\mathbb{R}$,

$$\frac{\partial}{\partial t}A(u(x,t))+\frac{\partial}{\partial x}F(u(x,t))=G(u(x,t)),\quad x\in\mathbb{R},t\in\mathbb{R}^+,t>0,\ u(x,0)=u_0(x),\quad x\in\mathbb{R},\qquad(1)$$

where $u_0\in L^\infty(\mathbb{R})\cap BV(\mathbb{R})$ and A, F are assumed to be twice continuously differentiable functions; A is also invertible, and the function G is uniformly bounded. The motivation for this proposal arises from the study of transport phenomena and continuum fluid mechanics in the context of balance laws. The Cauchy problem has been studied in great detail for the case of conservation laws, i.e. $G(u)$=0; see [7]. The dynamics of nonlinear hyperbolic balance laws (1), typically under effects of stiff source term, is of uppermost importance in the several disciplines of fluid mechanics [1] and reaction transport problems in porous media [8, 4]. We developed both fully discrete and semi-discrete formulations, and extended the concept of *no-flow curves* [5, 3] to this general class of nonlinear balance laws. We established a rigorous mathematical numerical entropic-convergence study using weak asymptotic analysis [6], which involved investigating the existence, uniqueness, and regularity of entropy-weak solutions computed with our scheme (for details see [2]). The proposed method is Riemann-solver-free we subject to a novel weak CFL stability condition [3, 4]. To evaluate the shock capturing capabilities of the enhanced Lagrangian-Eulerian numerical scheme, we carried out numerical experiments that demonstrate its ability to accurately resolve the key features of balance laws [2, 4] and hyperbolic problems [5, 3]. *E. Abreu thanks the financial support of CNPq grant 307641/2023-6.*

References

[1] Lukáčová-Medvid'ová, Mária, and Christian Rohde. Mathematical challenges for the theory of hyperbolic balance laws in fluid mechanics: complexity, scales, randomness. Jahresbericht der Deutschen Mathematiker-Vereinigung 126, no. 4 (2024): 283-311.

[2] E. A, W. Lambert, E. P. Barros. An enhanced Lagrangian-Eulerian Method for a class of balance laws: numerical analysis via a weak asymptotic method with applications, Numerical Methods for Partial Differential Equations, Volume 41, Issue 1 (January 2025) e23163.

[3] E. A, J. Agudelo, W. Lambert, and J. Pérez. A Lagrangian-Eulerian Method on Regular Triangular Grids for Hyperbolic Problems: Error Estimates for the Scalar Case and a Positive Principle for Multidimensional Systems, Journal of Dynamics and Differential Equations, (Published: 26 June 2023) Volume 37, pages 749-814, (2025).

[4] E. Abreu, V. Matos, J. Perez and P. Rodriguez-Bermudez. Riemann problem solutions for a balance law under Dirac-Delta source with a discontinuous flux, Journal of Hyperbolic Differential EquationsVol. 21, No. 01, pp. 1-32 (2024).

[5] E. A and J. Pérez, A fast, robust, and simple Lagrangian-Eulerian solver for balance laws and applications. Computers and Mathematics with Applications, 77(9) (2019) 2310-2336.

[6] E. A, M. Colombeau, E. Panov, Weak asymptotic methods for scalar equations and systems, J. Math. Anal. Appl. 444 (2016) 1203-1232.

[7] C. M. Dafermos, Hyperbolic Conservation Laws in Continuum Physics. Grundlehren der Mathema- tischen Wissenschaften, vol. 325. Springer, Berlin (2016).

[8] E. Abreu, A. Bustos, W. Lambert), Non-monotonic traveling wave and computational solutions for gas dynamics Euler equations with stiff relaxation source terms. Computers & Mathematics with Applications, v.70, p.2155 - 2176 (2015).

Uniqueness of Entropy solution for Doubly nonlinear degenerate fractional parabolic

Gerardo Huaroto

Universidade Federal de Alagoas, Brazil

In this work we investigate the uniqueness of solutions for the double nonlinear isotropic degenerate fractional parabolic problem within bounded domains, subject to homogeneous Dirichlet boundary conditions. Our study employs the mathematical analysis techniques, doubling variable techniques by kružkov.

Lagrangian flows associated to singular vector fields

Henrique Borrin de Souza

Universidade de São Paulo, Brazil

In this lecture, we present an extension on the well-posedness of lagrangian flows associated to vector fields whose derivative be written as a convolution of a singular kernel of order different than the dimension and a Besov function. The proof follows mainly a generalization of the technique developed by Bouchut-Crippa (JHDE, 2013), where we heavily use the structure of grand maximal functions.

Modeling of Single-Phase Flows in Porous Media and Passive Contaminant Transport

Henrique Casellato Vitorio Rodrigues da Costa, Nikolai V. Chemetov
Universidade de São Paulo, Brazil

A porous medium is a material characterized by its ability to store fluids and is found in a variety of contexts, such as sedimentary rocks for oil extraction, underground aquifers, biological tissues, and fuel cells. In general, computational simulations are essential tools for understanding chemical, physical, and flow processes, complementing field observations, laboratory experiments, and analytical models. This work aims to introduce concepts of Numerical Methods for Partial Differential Equations and their applications in modeling flow in porous media. Specifically, numerical simulation methods for elliptic and hyperbolic equations are investigated.

A Kruzhkov-Type Uniqueness Theorem for a System of Conservation Laws Describing Chemical Flooding

Yulia Petrova, Nikita Rastegaev
Universidade de São Paulo, Brazil

A system of two hyperbolic conservation laws describing a two-phase, three-component displacement (usually water and oil phases, as well as a component of a chemical agent dissolved in water) is considered. This system is neither truly nonlinear nor strictly hyperbolic, which limits the applicability of general results pertaining to strictly hyperbolic, truly nonlinear systems.

Solutions to some initial-boundary value problems (e.g., the Riemann problem or the problem of injecting a chemical agent slug) for this system have been previously investigated using the transition to Lagrangian coordinates, in which the equations are separated. The solutions are constructed using the method of characteristics, but the uniqueness of the resulting solutions has not been investigated. We used the proposed coordinate change to prove a Kruzhkov-type uniqueness theorem for solving the Cauchy problem under certain constraints on the initial data and the class of admissible weak solutions. A local small parameter criterion (vanishing viscosity) was used to determine the admissibility of discontinuities. The uniqueness result is applied to the analysis of the Riemann problem solutions for an S-shaped flow function changing monotonicity exactly once.

This talk is based on joint work with N. Rastegaev, https://arxiv.org/abs/2512.07639

An Extension of the Subadditive Ergodic Theorem

Jean Silva
Universidade Federal de Minas Gerais, Brazil

In this talk, it will be presented an extension of the Kingman's subadditive ergodic theorem. Here, we will consider subadditive quantities composed with stochastic deformations, in which the stationarity assumption is missed, and we will show that the same conclusions hold as in the classical case. The proof involves a rather simple reduction to the additive case, where Birkhoff's ergodic theorem can be applied.

Regular variation, maximal functions, and DiPerna-Lions flows

João Fernando Nariyoshi, Henrique Borrin
Universidade de São Paulo, Brazil

Using maximal functions with weights of regular variation, we investigate the regularity of flows generated by Sobolev velocity fields. This is an ongoing joint work with Henrique Borrin (FFCLRP-USP), and it is

supported in part by the São Paulo Research Foundation (FAPESP, Process 2024/21041-1) and by the Pró-Reitoria de Pesquisa e Inovação of the University of São Paulo (Programa de Apoio a Novos Docentes, Grant 22.1.09345.01.2).

Wave-Curve Construction of Riemann Solutions in Three-Phase Flow Models

Luis Fernando Lozano, Frederico Furtado, Aparecido de Souza, Dan Marchesin

Universidade Federal de Juiz de Fora, Brazil

We investigate the Riemann problem for a 2×2 system of conservation laws that exhibits non-strict hyperbolicity due to the coincidence of characteristic speeds at an interior state. This system arises in models featuring large viscosity contrasts and serves as a representative example of the non-strictly hyperbolic dynamics observed in several multiphase transport processes.

Our work provides a detailed construction and classification of Riemann solutions for left states constrained to one boundary of the state domain and right states spanning almost the entire admissible region. We apply the wave-curve method, utilizing the viscous-profile criterion for discontinuities, combined with numerical techniques, to determine the bifurcation loci that separate regions in the space of right states with distinct Riemann solution structures.

We also analyze the sensitivity of these solutions to perturbations in the initial data and present numerical evidence supporting their L^1_{loc}-stability and overall robustness. The results highlight how interior degeneracies influence wave interactions and solution organization in non-strictly hyperbolic systems.

Finally, we validate the semi-analytical constructions through fully numerical simulations, confirming the consistency and accuracy of the proposed classification across the considered parameter regimes.

Concentration Effects of H-Measures in Differents Ergodic Settings

Luís Fernando Salvino, Jean Carlos da Silva

Universidade Federal de Minas Gerais, Brazil

The H-measure was introduced independently by L. Tartar, P. Gerárd and improved by E. Panov. It is a default measure which quantifies, in the phase (i.e. the physical space times the Fourier space of propagation directions), the lack of compactness of weakly converging sequences. In this paper, we address the task to reckon the H-measure generated by functions in known ergodic algebras, that is, in the almost periodic and in the weakly almost periodic algebras. We shall see that the H-measure generated by a function in these ergodic algebras has the same structure of one which is periodic: The direction of the Fourier variable ξ is a purely atomic measure. But this atomic behavior seems to be very restricted to the algebras of the "periodics families". Precisely, we shall computer the H-measure generated by a ergodic function which is beyond the weak almost periodic setting and its corresponding H-measure has a structure completely different: the direction of the Fourier variable ξ is a measure absolutely continuous with respect to Lebesgue measure.

Existence and large-time behavior of solutions to an incompressible inhomogeneous kinetic–fluid flocking system

Robson Moura, Gabriela Planas

Universidade Estadual de Campinas, Brazil

In this work, we prove the global existence of weak solutions and establish an a priori exponential time-asymptotic behavior for a kinetic–fluid flocking model. The kinetic component is the standard Cucker–Smale

system supplemented with a local alignment term and a noise term. This kinetic equation is coupled, via a drag force, to the incompressible inhomogeneous Navier-Stokes equations on the three-dimensional periodic domain.

The existence of weak solutions is obtained by passing to the limit in solutions of a regularized system. This limiting procedure relies on the averaging lemma and compactness results. Solutions to the regularized system are obtained via a fixed-point argument, in which the drag force is regularized to belong to L^2 and a cut-off is introduced in the kinetic equation. The remaining steps of the regularization follow usual methods for inhomogeneous incompressible fluids.

The large-time behavior is deduced from the boundedness of the derivative of a suitable energy–fluctuation functional.

The Lagrangian-Eulerian Method for Nonlocal Conservation Laws in Several Dimensions

Wanderson José Lambert, Eduardo Abreu, Juan Juajibioy, Richard de La Cruz
Universidade Federal de Alfenas, Brazil

This talk addresses the numerical approximation of multidimensional scalar conservation laws with nonlocal flux terms. We introduce a semi-discrete Lagrangian–Eulerian scheme for the following general model:

$$\partial_t \rho(t,\mathbf{x}) + \sum_{i=1}^{d} \partial_{x_i} \Big(V^i\big(W[\rho,\omega](t,\mathbf{x})\big)\, F^i\big(\rho(t,\mathbf{x})\big) \Big) = 0, \qquad (t,\mathbf{x}) \in (0,T)\times\mathbb{R}^d,$$

where $W[\rho,\omega]$ represents a nonlocal spatial averaging operator. Equations of this type appear in various contexts, including vehicular traffic, flow in porous media, and biological population dynamics, where long-range interactions play a fundamental role. The presence of the nonlocal coupling complicates both the mathematical analysis and the design of stable numerical methods.

The convergence study of our scheme relies on the weak asymptotic framework established in [1], which we adapt to the present multidimensional, nonlocal setting. Within this framework, we build a parametrized family of approximations possessing uniform bounds in $L^1(\mathbb{R}^d)\cap L^\infty(\mathbb{R}^d)$ and satisfying appropriate continuity estimates in time and space. These properties are sufficient to guarantee strong compactness in L^1, allowing us to pass to the limit and identify it as a weak entropy solution of the original nonlocal conservation law.

Finally, we provide a series of computational experiments that validate the theoretical results and demonstrate the practical effectiveness of the proposed approach. This is joint work with E. Abreu, R. de la Cruz, and J. Juajibioy, and the details can be found in [2].

References

[1] E. Abreu, M. Colombeau, E. Panov, *Weak asymptotic methods for scalar equations and systems*, J. Math. Anal. Appl. **444** (2016), 1203–1232.

[2] E. Abreu, R. De la Cruz, J. Juajibioy, W. Lambert, *Semi-discrete Lagrangian–Eulerian approach based on the weak asymptotic method for nonlocal conservation laws in several dimensions*, to appear in Journal of Computational and Applied Mathematics.

Sideris' affine flows for inviscid compressible gases: The mono-atomic case

Wladimir Neves, Denis Serre
Universidade Federal do Rio de Janeiro, Brazil

The special, but illuminating, class of affine flows was introduced by T. Sideris in the context of inviscid gases. In the compressible case, such motions are governed by an ODE in the matrix group $GL_d^+(\mathbb{R})$. When the gas is mono-atomic, this equation falls into a special class, of Calogero–Moser type, admitting therefore a larger invariance group. This allows us to derive finer results about the time asymptotics and the scattering operator.

Domain Perturbation for PDEs and Applications

Organizer: Alessandra Verri (UFSCar/Brazil) & Marcone C. Pereira (USP/Brazil)

Spectral Analysis in Cut Strips

Alessandra Verri, Rafael Amorim

Universidade Federal de São Carlos, Brazil

We study the Dirichlet Laplacian in cut strips, regions obtained by translating a line segment along a curve in R^3 in an appropriate way. We characterize the essential spectrum of the operator and establish conditions under which discrete eigenvalues appear below the spectral threshold, showing how the asymptotic behavior of the angle between the curve and the fixed translation vector determines the existence and number of such bound states.

Spectral properties of Laplace operator on waveguide surfaces

Diana Carolina Suarez Bello

Universidade Federal de São Carlos, Brazil

In this talk, we analyze the Laplace operator $-\Delta_{\mathcal{S}}$ on waveguide shaped surfaces. These surfaces $\mathcal{S} \subset \mathbb{R}^3$ are constructed by translating a closed curve along an unbounded spatial curve in a constant direction. Under the condition that the tangent vector of the spatial reference curve has a finite limit at infinity, we characterize the essential spectrum of $-\Delta_{\mathcal{S}}$ and discuss the conditions that lead to the emergence of discrete eigenvalues. Furthermore, we extend the analysis to the case of a broken sheared waveguide shaped surface. The results presented here are contained in [1].

Referências

[1] Spectrum of the Laplacian in waveguide shaped surfaces. Anal.Math.Phys. 15, 140 (2025). https://doi.org/10.1007/s13324-025-01137-8

Asymptotic profiles for solutions of a generalised shallow water model

Igor Leite Freire

Universidade Federal de São Carlos, Brazil

In this talk we discuss persistence properties of an equation enclosing shallow water models such as the Camassa-Holm equation. We show that for any further and fixed time, the solution is asymptotically dominated by the initial condition and a perturbation behaving like a peakon.

Asymptotic modelling of the current flow system described by the $p(x)$-Laplacian

Jean Carlos Nakasato

Universidade Federal do ABC, Brazil

Starting from the $p(x)$-Laplace thermistor model, in this paper we rigorously derive the effective model describing the current flow in a thin domain. We assume that the exponent $p(x)$ is constant by parts and exhibiting the oscillating interface. Depending on the parameter describing these oscillations, we investigate the asymptotic behavior of the solution by using the adaptation of the periodic unfolding method. Our work is motivated by the industrial applications consisting of thin organic light-emitting diodes. This is a Joint work with Igor Pazanin.

Initial-boundary value problem for a Manakov system

Priscila Leal da Silva, Dionyssis Mantzavinos

Universidade Federal de São Carlos, Brazil

In this talk we consider a generalised Manakov system in the half-line and, by making use of the unified transform method, we show that the initial-boundary value problem is well-posed in Sobolev spaces.

Existence, regularization and upper-semicontinuity of uniform attractors for a nonautonomous semilinear evolution equation of second order

Vinicius Tavares Azevedo

Universidade Federal de São Carlos, Brazil

We investigate the forward dynamics of a nonautonomous semilinear wave-type evolution problem, which models propagation phenomena in nonlinear elastic rods and ion-acoustic waves. We establish global well-posedness and prove the existence of a family of uniform attractors under appropriate growth and dissipativity conditions. Additionally, we demonstrate upper semicontinuity in a suitable space and derive regularity results in a more refined space. Finally, we characterize the uniform attractor through kernel sections for the problem under consideration.

Metric and domain perturbation theory for the degenerate Grushin operator

Wanessa Ferreira Tavares, Marcus Antonio Mendonça Marrocos

Universidade Federal do Amazonas, Brazil

We consider the eigenvalue problem with Dirichlet and Neumann boundary conditions for the elliptic operator in divergence form $\mathcal{L}_{T,g,\eta}f = \mathrm{div}\eta(T\nabla f) = \mathrm{div}g(T\nabla f) - g(\nabla\eta, T\nabla f)$ on a compact Riemannian manifold M with boundary. We prove that the symmetric functions of the eigenvalues depend real analytically upon metric perturbations and we prove an Hadamard-type formula for their shape differential. The applications are dedicated to analyzing the critical points of the symmetric eigenvalue functions restricted to the space of metrics with constant volume, the characterization of the critical points of the symmetric eigenvalue functions of the operator $\mathcal{L}_{T,g,\eta}$ parameterized by bounded domains in a Riemannian manifold and addressing the geometric optimization problem of the first non-zero eigenvalue μ_2 for the operator $\mathcal{L}_{T,g,\eta}$ with Neumann boundary conditions on bounded subdomains Ω of a Riemannian manifold (M, g).

Gradient Einstein-type warped products: Rigidity, existence and nonexistence results via a nonlinear PDE

Willian Isao Tokura

Universidade Federal da Grande Dourados, Brazil

In this talk, we establish the necessary and sufficient conditions for constructing gradient Einstein-type warped metrics. One of these conditions leads us to a general Lichnerowicz equation with analytic and geometric coefficients for this class of metrics on the space of warping functions. In this way, we prove gradient estimates for positive solutions of a nonlinear elliptic differential equation on a complete Riemannian manifold with associated Bakry–Émery Ricci tensor bounded from below. As an application, we provide nonexistence and rigidity results for a large class of gradient Einstein-type warped metrics.

Recent Trends in Nonlinear PDEs and Free boundary Problems

Organizer: João Vitor da Silva (UNICAMP/Brazil), Disson dos Prazeres (UFS/Brazil), & Ginaldo de Santana Sá (CMM/Chile)

On a Harnack inequality to the normalized $\infty-$laplacian with nonlinear gradient terms

Ahmed Mohammed

Ball State University, USA

Harnack inequality is a fundamental tool in analysis, providing uniform con- trol over the oscillation of non-negative solutions to elliptic and parabolic partial differential equations on small sets. Such control is often indispensable for es- tablishing higher regularity, compactness, and qualitative properties of solutions. Beyond its central role in analysis, Harnack inequality also has deep implications in geometry and related fields. A striking and well-known example is its use in Grigori Perelman's proof of the Poincar´e Conjecture, where it served as a crucial ingredient in controlling the evolution of curvature under the Ricci flow. In this talk, I will discuss the validity of Harnack inequality for the normalized infinity Laplacian in the presence of nonlinear lower-order gradient terms. This operator is both highly degenerate and singular, and the addition of nonlinear gradient interactions introduces further complexity. Despite these challenges, the method employed relies on tools that are simpler than those traditionally used in the analysis of highly degenerate elliptic equations, yet sufficiently robust to produce new Harnack estimates.

On a class of free transmission problems for degenerate fully nonlinear equations

Giane Casari Rampasso

Universidade Federal de Itajubá, Brazil

In this talk we discuss some results for viscosity solutions to a fully nonlinear degenerate free transmission problem. Our findings include existence of a viscosity solution to the associated Dirichlet problem and regularity in $\mathcal{C}^{1,\alpha}$ spaces, with the appropriate estimates. In particular, we give an explicit characterization of α.

Sharp and improved regularity estimates for weighted quasilinear elliptic equations of $p-$Laplacian type and applications

Ginaldo de Santana Sá, João Vitor da Silva, Disson dos Prazeres, Gleydson Ricarte

Universidad de Chile, Chile

In this work, we obtain sharp and improved regularity estimates for weak solutions of weighted quasilinear elliptic models of Hardy-Hénon-type, featuring an explicit regularity exponent depending only on universal parameters. Our approach is based on geometric tangential methods and uses a refined oscillation mechanism, compactness, and scaling techniques. In some specific scenarios, we establish higher regularity estimates and non-degeneracy properties, providing further geometric insights into such solutions. Our regularity estimates both enhance and, to some extent, extend the results arising from the $C^{p'}$ conjecture for the p-Laplacian with a bounded source term. As applications of our results, we address some Liouville-type results for our class of equations.

Lipschitz regularity of weakly coupled vectorial almost-minimizers for Alt-Caffarelli functionals with non-standard growth

João Vitor da Silva, Pedro Fellype Pontes, Minbo Yang

Universidade Estadual de Campinas, Brazil

For a fixed constant $\lambda > 0$ and a bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ with $n \geq 2$, we establish that almost-minimizers (functions satisfying a sort of variational inequality) of the Alt-Caffarelli type functional

$$\mathcal{J}_G(\mathbf{v};\Omega) := \int_\Omega \left(\sum_{i=1}^m G\big(|\nabla v_i(x)|\big) + \lambda \chi_{\{|\mathbf{v}|>0\}}(x) \right) dx,$$

where $\mathbf{v} = (v_1, \ldots, v_m)$ and $m \in \mathbb{N}$, exhibit optimal Lipschitz continuity on compact subsets of Ω, where G is a Young function satisfying specific growth conditions. Furthermore, we obtain universal gradient estimates for non-negative almost-minimizers in the interior of non-coincidence sets.

Our work extends the recent regularity results for weakly coupled vectorial almost-minimizers for the p-Laplacian addressed in Bayrami *et al* [1], and even the scalar case treated in da Silva *et al* [2], Dipierro *et al* [3], and Pellegrino-Teixeira [4], thereby providing new insights and approaches applicable to a variety of non-linear one or two-phase free boundary problems with non-standard growth.

This is a joint work with Pedro Fellype Pontes and Minbo Yang (Zhejiang Normal University, People's Republic of China).

References

[1] M. Bayrami, M. Fotouhi, and H. Shahgholian, *Lipschitz regularity of a weakly coupled vectorial almost-minimizers for the p-Laplacian*, J. Differential Equations, 412 (2024), 447-473.

[2] J.V. da Silva, A. Silva and H. Vivas, *Lipschitz regularity of almost minimizers in a Bernoulli problem with non-standard growth*. Discrete Contin. Dyn. Syst. 44 (2024), no. 6, 1555-1586.

[3] S. Dipierro, F. Ferrari, N. Forcillo, and E. Valdinoci, *Lipschitz regularity of almost minimizers in one-phase problems driven by the p-Laplace operator*. Indiana Univ. Math. J. 73 (2024), no. 3, 813-854.

[4] D. M. Pellegrino and E.V. Teixeira, *Regularity for quasi-minima of the Alt-Caffarelli functional*. Calc. Var. Partial Differential Equations 63 (2024), no. 6, Paper No. 149, 12 pp.

Removable Sets for Hölder Continuous Solutions of Quasilinear Elliptic Equations with Variable Exponent

Juan Pablo Alcon Apaza

Universidade Federal de São Carlos, Brazil

This talk explores the existence and continuity of solutions to obstacle problems with $p(x)$-type nonstandard growth, as well as the removability of closed sets for Hölder continuous solutions of elliptic equations in the framework of variable exponent Sobolev spaces. We address the case where a closed set E is removable for $C^{0,\alpha}$ Hölder continuous $p(x)$-harmonic functions in a bounded open domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, provided that, for each compact subset K of E, the $\big(n - p_K + \alpha(p_K - 1)\big)$-Hausdorff measure of K is zero, where $p_K = \max_{x \in K} p(x)$.

Regularity to Thin Obstacle Problem in Orlicz spaces

Junior da Silva Bessa, Paulo Henryque da Costa Silva, Alan Pio Sousa
Universidade Estadual de Campinas, Brazil

In this talk, we present regularity results for minimizers of the energy functional associated with the thin obstacle problem in the framework of Orlicz spaces. More precisely, we prove Lipschitz continuity and Hölder continuity of the gradient of minimizers. Our analysis combines techniques from De Giorgi's classical regularity theory with variational methods adapted to the Orlicz setting.

A Geometric Tangential Method for Schauder Estimates in Fully Nonlinear Elliptic PDEs with Dini Data

Laura A. Ospina, Junior da Silva Bessa, João Vitor da Silva
Universidade Estadual de Campinas, Brazil

In this talk, we establish local Schauder estimates for flat viscosity solutions, that is, solutions with sufficiently small norms, to a class of fully nonlinear elliptic partial differential equations of the form

$$F(D^2u, x) + \langle \mathfrak{B}(x), Du \rangle = f(x) \quad \text{in} \quad \mathrm{B}_1 \subset \mathbb{R}^n,$$

where the operator F is differentiable, though not necessarily convex or concave. In addition, we impose suitable Dini-type continuity assumptions on the data. Our methodology is based on geometric tangential techniques, combined with compactness and perturbative arguments. This approach is strongly motivated by recent advances in the theory of nonlinear elliptic equations and free boundary problems. Our results can be viewed as an extension of the work by dos Prazeres and Teixeira [2, Theorem 2.2], now within the framework of linear drift terms and Dini continuity assumptions.

References

[1] Caffarelli, L.A. and Cabré, X. *Fully nonlinear elliptic equations*. Amer. Math. Soc. Colloq. Publ., 43 American Mathematical Society, Providence, RI, 1995. vi+104 pp. ISBN:0-8218-0437-5.

[2] dos Prazeres, D. and Teixeira, E.V. *Asymptotics and regularity of flat solutions to fully nonlinear elliptic problems*. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 15 (2016), 485–500.

[3] Kovats, J. *Fully nonlinear elliptic equations and the Dini condition*. Comm. PDE, 22, 1911–1927 (1997).

[4] Nadirashvili, N. and Vlăduţ, S. *Nonclassical solutions of fully nonlinear elliptic equations*. Geom. Funct. Anal. 17 (2007), no. 4, 1283-1296.

Regularity Estimates for a Nonhomogeneous Free Boundary Interface Problem

Leandro da Silva Tavares, Mayra Soares, Romildo Lima
Universidade Federal do ABC, Brazil

In this talk, we present regularity results for the minimizers of a new nonhomogeneous interface problem including a free boundary in Orlicz-Sobolev spaces. The nonlinearities satisfy structural conditions of Lieberman's type with subcritical growth. The main challenges arise from the nonho- mogeneous setting and the absence of representation formulas. Our results complement the recent literature and exhibit novelties even for the

Laplacian operator. This work was done in collaboration with Mayra Soares (UnB/Brazil) and Romildo Lima (UFCG/Brazil).

Regularizing Effect for a Class of Elliptic Systems

Luís Henrique De Miranda, Ayana de Castro Santana
Universidade de Brasília, Brazil

The main goal of this talk is to discuss some aspects of recent developments on regularity of solutions to Maxwell-Schrodinger systems

$$\begin{cases} -\operatorname{div}(\mathcal{M}(x)\nabla u) + A\phi|u|^{r-2}u = f \text{ em } \Omega, \\ -\operatorname{div}(\mathcal{M}(x)\nabla \phi) = |u|^s \text{ em } \Omega. \\ u = \phi = 0 \text{ sobre } \partial\Omega, \end{cases} \tag{S}$$

As it turns out, solutions to (8) have better regularity properties than what is guaranteed by the standard regularity results, see [3] even when the data is poor. We are going to recall a couple of results due to L. Boccardo et. ally, see [1], and later on introduce some recent contributions and also new trends on the subject, see [2].

References

[1] L. Boccardo e L. Orsina , *Regularizing effect for a system of Schrödinger–Maxwell equations*, Adv. Calc. Var., 11.1 (2016), 75–87

[2] A. Santana L.H. de Miranda, *Regularizing effect for a class of Maxwell-Schrödinger Systems*, Nonlinear Analysis 248(9–10):113625 DOI: 10.1016/j.na.2024.113625 (2024).

[3] G. Stampacchia, *Some limit cases of L^p-estimates for solutions of second order elliptic equations*, Comm. Pure Appl. Math. 16 1963 505–510.

Interior and Boundary Regularity of Mixed Local–Nonlocal Problem with Singular Data and Its Applications

Ritabrata Jana, R. Dhanya, Jacques Giacomoni
Universidade Estadual de Campinas, Brazil

In this talk, we address the Hölder regularity of solutions to equations involving a nonhomogeneous mixed local–nonlocal operator $-\Delta_p + (-\Delta)^q_s$, with singular data, under the minimal assumption that $p > sq$. The regularity result is twofold: we establish interior gradient Hölder regularity for locally bounded data and boundary regularity for singular data. In particular, we present both boundary Hölder and boundary gradient Hölder regularity results, depending on the degree of singularity. We conclude the talk by discussing several applications of these regularity results to mixed local–nonlocal problems.

Sharp regularity for the obstacle problem for quasilinear elliptic equations of $p-$Laplacian type

Romário Tomilhero Frias, João Vitor da Silva, Elzon Cézar Bezerra Júnior
Instituto Federal de São Paulo, Brazil

In this talk, we establish regularity properties of weak solutions for the obstacle problem for a quasilinear elliptic equation of p-Laplacian type with varying coefficients and possibly unbounded source terms. Specifically, we obtain sharp gradient estimates, namely $C^{1,\alpha}_{loc}(B_1)$ estimates for the solution, with an explicit regularity exponent depending only on universal parameters.

Regularity theory for Lipschitz viscosity solutions to the false mean curvature equation

Sergio H. Monari Soares, Diego Moreira, Jefferson Abrantes
Universidade de São Paulo, Brazil

We establish an equivalence between the distributional and viscosity notions of the inhomogeneous false mean curvature equation $Q_\beta(u) = 2\Delta_\infty u + \beta\Delta u = f$ in the realm of locally Lipschitz solutions, subsolutions, and supersolutions. This equivalence enables a truncation scheme that yields uniform ellipticity, leading first to $C^{1,\alpha}$ regularity via Lieberman's theory and subsequently to the full spectrum of classical estimates, including $W^{2,p}$, $W^{2,n}$ and $C^{2,\alpha}$. Within this unified framework, we derive a quantitative version of the Hopf–Oleinik lemma, a Krylov-type boundary regularity result for differential inequalities, and a flipping one-sided regularity theorem for supersolutions, in the spirit of Caffarelli, Kohn, Nirenberg and Spruck estimates proven in the 80s. We also classify locally Lipschitz viscosity solutions of the homogeneous false mean curvature equation $Q_\beta(u) = 0$ in half-spaces that vanish continuously on the flat boundary and exhibit linear growth at infinity, thereby improving a Liouville-type theorem we had established earlier for $C^1(\mathbb{R}^n_+)$ solutions.

Integral and Functional Differential Equations

Organizer: Everaldo de Mello Bonotto (USP/Brazil), Jaqueline G. Mesquita (Unicamp/Brazil) & Pierluigi Benevieri (USP/Brazil)

Two positive solutions for a nonlinear mixed boundary value problem with sign-changing weight

Adriano Peixoto, Pierluigi Benevieri

Universidade de São Paulo, Brazil

In this work, we investigate a second-order nonlinear boundary value problem with mixed boundary conditions, where the weight function is positive near the origin and negative near the endpoint. Under new assumptions on the nonlinear term, we establish the existence of at least two positive solutions. Our approach is based on Mawhin's coincidence degree. These results extend previous works on mixed boundary problems with sign-changing weights and provide new insights into the multiplicity of positive solutions.

Bifurcation for a nonlinear boundary value problem

Aldo Pereira

Universidad de La Serena, Chile

In this talk, we consider a parameterized boundary value problem defined on a discrete set, involving Dirichlet conditions. The main result is to prove the existence of a continuous branch of nontrivial solutions arising from a trivial solution of the problem. The approach to get such main result is based on the concept of characteristic value of operators defined on Banach spaces. In addition, we obtain a result on existence and uniqueness of a solution for such boundary value problems, by using the theory of time scales.

Global Stability and Periodicity in a Differential Delay Model

Anatoli Ivanov

Pennsylvania State University, USA

Scalar differential delay equation of the form

$$x'(t) = f(x(t))g(x(t-\tau)) - h(x(t))$$

is considered, which is a generalization of a physiological model of platelet production in human body. Sufficient conditions for the global asymptotic stability of the positive equilibrium are derived. They are given in terms of one-dimensional maps, one set being delay independent, the other one depends explicitly on the size of delay τ. Existence of periodic solutions in the model is proved under the assumption of the instability of the linearized equation about the equilibrium and certain general boundedness assumptions on the nonlinearities f, g, and h involved.

A study of evolution equations through operator theory

Andréa Cristina Prokopczyk Arita

Universidade Estadual Paulista, Brazil

In this work we study the well-posedness of some classes of differential equations by means of operator theory. In particular, recalling the classical ideas of semigroup theory applied for the first-order Cauchy problem, and of the sine and cosine families of operators for the second-order Cauchy problem, we consider two types of abstract equations: the first is a third-order equation that includes in its formulation the generalized Blackstock-Crighton-Westervelt equation, and the second is an equation with a fractional-order derivative and a damping term, also of fractional order, which includes in its formulation some important equations in the literature, for example, the strongly damped Klein-Gordon linear equation, or even the strongly damped wave equation, the linear part of the viscous Cahn-Hilliard equation, the linear part of the Kuznetsov equation and the fractional-order telegraph equation.

References

[1] J. Aquino, C. Lizama, A. Prokopczyk, Well-posedness for strongly damped abstract Cauchy problems of fractional order, *Proceedings of the Royal Society of Edinburgh: Section A Mathematics* (2025), 1-27.

[2] L. Gambera, C. Lizama, A. Prokopczyk, Well-posedness for the abstract Blackstock–Crighton–Westervelt equation, *Journal of Evolution Equations*, 21, (2021), 313-337.

Numerical Persistence and Shadowing in Integrodifference Equations

Christian Pötzsche

University of Klagenfurt, Austria

Integrodifference equations are versatile and popular models in the life sciences which simultaneously describe the temporal evolution and spatial dispersal of populations or infective diseases in discrete generations. As infinite-dimensional dynamical systems their behavior is often illustrated by simulations and a rigorous foundation of related numerical methods is due.

In this talk, we establish persistence (of global attractivity) and shadowing results for integrodifference equations. They guarantee that global attraction persists under discretization and that true solutions are shadowed by approximate solutions (also known as pseudo-solutions) and conversely. These results are illustrated by means of spatial discretizations using projection methods.

On attractors for functional differential equations with state-dependent delay

Denis Fernandes da Silva

Universidade de São Paulo, Brazil

Differential equations with nonlinear terms and state-dependent delays present significant challenges in mathematical analysis, particularly because they are generally not well-posed when initial conditions are taken in the space of continuous functions.

In this talk, I will discuss recent results related to the existence of a finite fractal-dimensional global attractor for the multivalued semigroup generated by a class of abstract differential equations with state-dependent

delays. The focus will be on understanding the dynamics in the space of continuous functions. The class under consideration takes the form

$$\begin{aligned} u'(t) &= Au(t) + F(u(t), u(t-\sigma(t,u_t))),\ t \geq 0, \\ u_0 &= \varphi \in C([-p,0];X), \end{aligned}$$

in which $A : D(A) \subset X \to X$ is the generator of an analytic C_0-semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ acting on a Banach space $(X, \|\cdot\|)$. The analysis addresses both subsets of and the entire space $C([-p,0];X)$ as possible phase spaces for initial conditions.

References

[1] Coti Zelati, M. On the theory of global attractors and Lyapunov functionals. *Set-Valued Var. Anal.* **21**, (2013), 127-149.

[2] Efendiev, M. Finite and infinite dimensional attractors for evolution equations of mathematical physics. *Gakuto International Series. Mathematical Sciences and Applications*, **33**, Gakkotosho Co. Ltd., Tokyo, 2010.

[3] Hernandez, E., Fernandes, D., Wu, J. Existence and uniqueness of solutions, well-posedness and global attractor for abstract differential equations with state-dependent delay, *J. of Differential Equations* , **302** (2021), 753-806.

[4] Hernandez, E., Fernandes, D., Efendiev, M., Wu, J., Finite fractal dimensional global attractor for abstract differential equations with state-dependent delay, *J. of Mathematical Analysis and Applications*, **519** (1) (2023), 126743.

On the Geographic Spread of Chikungunya between Brazil and Florida: A Multi-patch Model with Time-delay

Jaqueline Godoy Mesquita, A. Gondim, X. Huo, S. Ruan

Universidade Estadual de Campinas, Brazil

Chikungunya (CHIK) is a viral disease transmitted to humans through the bites of Aedes mosquitoes infected with the chikungunya virus (CHIKV). CHIKV has been imported annually to Florida in the last decade due to Miami's crucial location as a hub for international travel, particularly from Central and South America including Brazil, where CHIK is endemic. This paper presents a comprehensive mathematical model for the geographic spread of CHIKV, incorporating pivotal factors such as human movement, temperature, rainfall, vertical transmission, and incubation period. Central to the model is the integration of a multi-patch framework, considering human movement between endemic Brazilian states and Florida. We establish crucial correlations between the mosquito reproduction number Rm and the disease reproduction number R0, thereby advancing our understanding of CHIKV transmission dynamics in complex multi-patch environments. Through numerical simulations, validated with real population, temperature and rainfall data, it is possible to understand the disease dynamics under many different scenarios and make future projections, offering crucial insights for devising effective control strategies.

Practical Stability for Measure FDEs via Generalized ODEs

Lucas Ozaki Mizuguti, Everaldo de Mello Bonotto, Suzete Maria Silva Afonso

Universidade Estadual Paulista, Brazil

In this talk we present the concept of practical stability in the realm of generalized ODEs and using the correspondence between a generalized ODE and a measure FDE, we prove that the measure FDEs whom satisfy some suitable hypothesis, are uniformly practically stable and practically asymptotically stable. Our goal was to investigate the direct method of Lyapunov and its converse.

Existence of at least two positive and periodic solutions for nonlinear delay differential equations

Marcela Nascimento, Pierluigi Benevieri

Universidade de São Paulo, Brazil

We study a nonlinear delay differential equation of the form

$$u''(t) + cu'(t) + \lambda a(t)g(u(t), u(t-\gamma)) = 0,$$

where $\gamma > 0$ is given, $a : \mathbb{R} \to \mathbb{R}$ is an L^1 map, T-periodic for a given $T > 0$ and g is a continuous function, sublinear at infinity and superlinear at zero. We prove the existence of at least two positive and T-periodic solutions when $\int_0^T a(t)dt < 0$ e $\lambda > 0$ is sufficiently large. We extend this result to the equation

$$u''(t) + cu'(t) + \varepsilon u(t) + \lambda a(t)g(u(t), u(t-\gamma)) = 0$$

with $\varepsilon > 0$ sufficiently small. The approach is topological using the coincidence degree theory by J. Mawhin.

Stability of scalar delay equations: stories from the non-autonomous world

Pablo Amster

FCEyN - Universidad de Buenos Aires, Argentina

In this talk, we shall introduce some simple criteria that guarantee the absolute and conditional stability of the trivial solution for a scalar equation with a variable nonnegative delay. Applications to well-known population dynamics models are presented to illustrate the results.

Continuation theorems for periodic systems with nonlinear time-dependent differential operators

Pierluigi Benevieri, Guglielmo Feltrin

Universidade de São Paulo , Brazil

The lecture propose some continuation theorems for the periodic problem

$$\begin{cases} x_i' = g_i(t, x_{i+1}), & i = 1, \dots, n-1, \\ x_n' = h(t, x_1, \dots, x_n), & \\ x_i(0) = x_i(T), & i = 1, \dots, n, \end{cases}$$

providing a unified framework that improves and extends earlier contributions by Jean Mawhin and collaborators to second-order differential problems governed by nonlinear time-dependent differential operators of the form

$$\begin{cases} (\phi(t,x'))' = f(t,x,x'), \\ x(0) = x(T), \quad x'(0) = x'(T). \end{cases}$$

The proof is based on the topological degree theory. This is a joint work with Guglielmo Feltrin, University of Udine, Italy.

Floquet Theory for Differential Equations with Generalized Piecewise Constant Arguments

Ricardo Torres Naranjo

Universidad Austral de Chile, Chile

We present a version of the classical Floquet–Lyapunov theorem for ω-periodic nonautonomous linear impulsive and nonimpulsive differential equations with piecewise constant arguments of generalized type, i.e., systems of the form

$$\begin{aligned} x'(t) &= A(t)x(t) + B(t)x(\gamma(t)), \quad t \neq t_k, \\ \Delta x|_{t=t_k} &= C_k x(t_k^-), \qquad\qquad\quad t = t_k, \end{aligned}$$

where $A(t)$ and $B(t)$ are piecewise continuous ω-periodic matrix functions, (C_k) is p-periodic, and $\gamma(t)$ is a generalized piecewise constant argument given by $\gamma(t) = \zeta_k$ for $t \in [t_k, t_{k+1})$ with $t_k \leq \zeta_k \leq t_{k+1}$, satisfying the so-called (ω, p)-property

$$t_{k+p} = t_k + \omega, \qquad \zeta_{k+p} = \zeta_k + \omega, \quad \forall k \in \mathbb{N} \cup \{0\}.$$

We show that such a system is kinematically similar to an autonomous linear ODE

$$y'(t) = Py(t).$$

We also provide explicit formulas for the Floquet normal form of the fundamental matrix. These results yield practical tools for analyzing stability and periodicity, thereby enhancing the qualitative study of IDEPCAG systems.

References

[1] Ricardo Torres, "Floquet-Lyapunov theorem for nonautonomous linear periodic differential equations with piecewise constant deviating arguments," *Nonlinear Analysis: Hybrid Systems*, 56, 101574, ISSN 1751-570X, 2025.

[2] Ricardo Torres, Manuel Pinto, "A variation of parameters formula for nonautonomous linear impulsive differential equations with piecewise constant arguments of generalized type," *Electron. J. Qual. Theory Differ. Equ.*, 2024, No. 20, 1–25.

[3] Manuel Pinto, "Cauchy and Green matrices type and stability in alternately advanced and delayed differential systems". *Journal of Difference Equations and Applications*, 17(2), 2011, 235–254.

Almost periodic solutions for dynamic equations on isolated time scales

Ronaldo Murakami

Universidade Estadual de Campinas, Brazil

In this work, we introduce a general concept of almost periodicity for functions defined on isolated time scales. Our concept is consistent with the existing concepts of almost periodicity on quantum calculus and on $\mathbb{Z}$. Also, we prove important properties such as the equivalence between different definitions for almost periodic functions, as well as results ensuring the existence of almost periodic solutions for dynamic equations on time scales under certain properties. We present several examples to illustrate our definition and main results. All the results can be found in [3].

References

[1] Bohner, M. & Mesquita, J. G. - *Almost periodic functions in quantum calculus*, Electronic Journal of Differential Equations, v. 2018, n. 197, p. 1-11, 2018.

[2] Bohner, M. & Mesquita, J. G. & Murakami, R. - *Almost periodicity on isolated time scales and applications*, Preprint, 2025

Nonlinear Dynamical Systems

Organizer: Juliana Fernandes S. Pimentel (UFRJ/Brazil), Maykel Boldrin Belluzi (USP/Brazil) & Phillipo Lappicy (UFRJ/Brazil)

Differential and variational approach to first order Mean Field Games in a generalized form

Antonio Siconolfi

La Sapienza University of Rome, Italy

We investigate time-dependent, first-order Mean Field Games on the flat torus comparing, in a broad and general framework, the classical differential formulation — given by a Hamilton–Jacobi equation coupled with a continuity equation — with a variational approach based on fixed points of a multivalued map acting on probability measures over trajectories.

We prove existence of fixed points for very general Hamiltonians. When the Hamiltonian is differentiable with respect to the momentum, we show that the evaluation curve of any such fixed point solves a continuity equation driven by a suitable vector field.

Our analysis provides a unified framework that bridges the differential and variational viewpoints in Mean Field Games, showing how aggregate optimality conditions naturally lead to continuity-equation descriptions under the sole assumption of differentiability of H in p.

Shadowing properties for infinite dimensional Morse-Smale dynamical systems

Carlos Takaessu, Alexandre N. Carvalho, José M. Arrieta

Universidade Federal de São Carlos, Brazil

In this talk, we will present results concerning the Shadowing property. Our main goal is to extend known results about Shadowing from finite-dimensional compact manifolds to infinite-dimensional spaces. As far as we know, the Lipschitz Shadowing property in nonlinear systems typically requires some form of finite dimensionality, even when the phase space is infinite dimensional. For instance, if $\mathcal{T}$ is a Morse–Smale dynamical system defined on a Hilbert space X that possesses an inertial manifold $\mathcal{M}$ (which is finite dimensional), then $\mathcal{T}$ exhibits the Shadowing property in a neighborhood of the attractor $\mathcal{A}$. Motivated by this, we show that it is still possible to obtain the Shadowing property in an infinite-dimensional setting, without relying on the existence of an inertial manifold or reducing the problem to a finite-dimensional case.

Plate models with Balakrishnan-Taylor damping

Eduardo Henrique Gomes Tavares

Dongguan University of Technology, China

In this talk, we present some recent results related to a fourth-order model proposed by Balakrishnan and Taylor in 1989. This model features a natural dissipation, where the coefficient is the energy functional itself. This characteristic reveals interesting stability and dynamical properties.

Existence of an inertial manifold for a nonlocal quasilinear problem

Estefani Moraes Moreira, Xiaoqing Yang, Alexandre N. Carvalho
Universidade de São Paulo, Brazil

In this talk, we consider the existence of a global attractor for a nonlocal quasilinear problem set in L^2. Moreover, we will show that the global attractor can be given as a graph of a Lipschitz function defined on a finite dimensional space.

The CR Yamabe Flow and Curvature Control on a complete Sasakian manifold

Flávio Almeida Lemos
Universidade Federal de Ouro Preto, Brazil

We study the CR Yamabe flow on a complete Sasakian manifold (M^{2n+1}, θ_0), $2n+1 \geq 3$, focusing on curvature estimates and comparison principles. The flow evolves the contact form by $\partial_t \theta(t) = -R(x,t)\theta(t)$, where R is the Tanaka–Webster scalar curvature. Using the Jerison–Lee transformation law and Ho's evolution formula, we show that R satisfies a nonlinear parabolic equation of reaction–diffusion type. Applying the parabolic comparison principle for Hörmander operators, we construct explicit ODE barriers that yield two-sided bounds for $R(x,t)$. These barriers provide effective curvature control, ensuring uniform boundedness under natural geometric assumptions. In particular, negative curvature remains bounded for all time, while positive curvature is controlled up to a finite blow-up time determined by the ODE model. Our results contribute to the analytical understanding of geometric flows that preserve CR structures on noncompact manifolds.

Recursive Properties and Global Attractors in Impulsive Dynamical Systems

Ginnara Mexia Souto, Everaldo de Mello Bonotto, Daniela Paula Demuner
Universidade Federal do Espírito Santo, Brazil

The theory of impulsive systems describes the evolution of processes in which continuous dynamics are interrupted by abrupt changes of state. In this talk, we focus on recursive properties of impulsive dynamical systems, such as minimality, non-wandering points, and the Birkhoff cente. Additionally, we explore the connections between these recursiveness properties and the global attractor. We also discuss a class of discrete systems associated with impulsive systems and how these properties are transferred from one to the other.

Strongly order preserving multivalued nonautonomous dynamical systems

Jacson Simsen
Universidade Federal de Itajubá, Brazil

This talk is devoted to present abstract results about monotone nonautonomous multivalued semiflows and their associated pullback attractors. For this kind of dynamical systems we are able to characterize the upper and lower bounds of the attractor as complete trajectories belonging to the attractor, so that all the internal dynamics is confined in this region, which can be described as an interval due to the orderly nature of the processes. The presentation will be based on the paper [1].

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References

[1] José A. Langa, Jacson Simsen, Mariza Stefanello Simsen and José Valero, Strongly order preservingmultivalued nonautonomous dynamical systems, Revista Matemática Complutense, https://doi.org/10.1007/s13163-025-00547-3

Differential equations to investigate the dynamics of SIR models with demography, random transmission, and non-autonomous vaccination

Javier López-de-la-Cruz

Universidad Politécnica de Madrid, Spain

Epidemics have influenced the humanity over history, bringing about consequences as catastrophic as the ones caused by wars and causing extinctions of entire populations. Apart from the human losses, epidemics also have important social consequences and have a huge negative impact on the economy.

Many infectious diseases, which have caused epidemics over history, are not eradicated yet and new ones are arising over the years. Hence, it is essential to set up mathematical models to describe the evolution of the diseases and then understand their evolution and make decisions to control the spread of the disease as soon as possible, trying to minimize the consequences.

The first known paper about mathematical modeling of epidemics was published in 1760 (see [1]), where D. Bernoulli studied the spread of smallpox by means of a system of ordinary differential equations. However, the study of mathematical models in Epidemiology was not developed until the very beginning of the twentieth century, when W. H. Hamer formulated in 1906 a discrete mathematical model to describe the transmission of measles (see [2]). Afterward, in 1911, R. Ross provided a mathematical model to predict the spread of an outbreak of malaria (see [3]), where he proved that it was enough to partially reduce the population of mosquitoes to eradicate the epidemics.

Nevertheless, in 1927 Kermack and McKendrick established a mathematical model to describe the spread of an infectious disease (see [4]) which has been the base of mathematical epidemiology ever since. This classical model, called SIR, is a compartmental model, i.e., it divides the population into three groups. The first group contains the susceptible individuals (S), people who are not infected but can contract the disease when contacting an infected. The second group contains the infected individuals (I). The third group contains the recovered individuals (R), people who are recovered with immunity or who pass away because of the disease.

Unfortunately, the classical SIR model imposes several strong assumptions. For instance, it assumes that the total population remains constant. However, infectious diseases do not always disappear quickly, as seen with COVID-19, which causes the population size to fluctuate due to external factors. Another major limitation is the assumption that the transmission rate is deterministic and constant, which is highly unrealistic since it is closely tied to people's movements, which are subject to randomness. Additionally, the model does not account for vaccination, an essential factor in controlling epidemics.

To set up more realistic models and avoid the drawbacks pointed out above, in this talk (based on [5]) we will introduce demography and non-autonomous vaccination in the classical SIR model. Nevertheless, we will go further and we will also perturb randomly some parameters, as the transmission coefficient, motivated by facts observed in real epidemics. After presenting carefully the resulting models, given by differential systems, we will investigate the dynamics of the disease, which will allow us to find conditions under which the disease

either is eradicated or becomes endemic. Apart from that we will interpret the results from the biological point of view. Finally, we will illustrate the theoretical results with several numerical simulations.

References

[1] D. Bernoulli. Essai d'une nouvelle analyse de la mortalité cause par la petite vèrole, et des aventages de l'inoculation pour la prévenir. In: Mémoires de mathématique et de physiques tires des registres de l'Academie Royale des Sciences (1760), pp. 1–75.

[2] W. H. Hamer. The milroy lectures on epidemic disease in England - The evidence of variability and persistence of type. Lancet (1906).

[3] R. Ross. The prevention of malaria. 2nd ed. London: John Murray (1911).

[4] W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. Proc R Soc Lond, vol. 115 (1927), pp. 700–21.

[5] J. López-de-la-Cruz, S. Merchán, F. Rivero and J. Rodrigo. SIR models with demography, random transmission coefficient and non-autonomous vaccination rate. Computational and Applied Mathematics, vol. 45, no. 47 (2026), pp. 1–21.

Hyperbolic evolution equations with time-dependent generators: A well-posedness framework

Paulo N. Seminario-Huertas

Universidad Politécnica de Madrid, Spain

In this talk, we will review key results from the literature concerning hyperbolic Cauchy problems with time-dependent families of generators. In particular, we will address the uniqueness of weak solutions and their connection to smooth and classical solutions. Finally, these results will be applied to a plate equation.

References

[1] T. Kato, Linear evolution equations of "hyperbolic" type, *J. Fac. Sci. Univ. Tokyo Sect. I* 17 (1970), 241–258.

[2] T. Kato, Linear evolution equations of "hyperbolic" type. II, *J. Math. Soc. Japan* 25 (1973), 648–666.

[3] T. Kato, Linear and quasi-linear equations of evolution of hyperbolic type, in *Hyperbolicity*, Lectures given at the Centro Internazionale Matematico Estivo (CIME), held in Cortona (Arezzo), Italy, June 24-July 2, 1976, 125–191, Springer, Heidelberg, 2011.

[4] P.L. Dattori da Silva, T.F. Ma, E.M. Maravi-Percca and P.N. Seminario-Huertas, A non- homogeneous weakly damped Lamé system with time-dependent delay, *Math. Meth. Appl. Sci.* 46 (8) (2023), 8793–8805.

[5] P. Marín-Rubio and P. N. Seminario-Huertas, Remarks on non-homogeneous Cauchy problems with time-dependent generators, *Adv. Oper. Theory* 10 (45) (2025), 1–30.

Hyperbolicity for dynamics on Banach spaces: Generalized (C, λ)-structure

Sergey Tikhomirov
PUC-Rio, Brazil

We introduce a notion of generalized (C, λ)-structure for nonlinear diffeomorphisms of Banach spaces. The main differences to the classical notion of hyperbolicity are that we allow the hyperbolic splitting to be discontinuous and in invariance condition assume only inclusions instead of equalities for both stable and unstable subspace. These aspects allows us to cover Morse-Smale systems and generalized hyperbolicity.

We suggest that the generalized (C, λ)-structure for infinite-dimensional dynamics plays a role similar to "Axiom A and strong transversality condition" for dynamics on compact manifolds. For diffeomorphisms of reflexive Banach space we showed that generalized (C, λ)-structure implies Lipschitz (periodic) shadowing and is robust under C^1-small perturbations. Assuming that generalized (C, λ)-structure is continuous for diffeomorphisms of arbitrary Banach spaces we obtain a weak form of structural stability: the diffeomorphism is semiconjugated from both sides with any C^1-small perturbation.

A panoramic view of exponential attractors

Stefanie Sonner, Radoslaw Czaja
Radboud University Nijmegen, The Netherlands

We provide a unifying framework for the construction of exponential attractors for infinite dimensional dynamical systems that allows us to generalize, improve and compare existing methods that are commonly used to construct exponential attractors. For autonomous deterministic systems we formulate necessary and sufficient conditions for the existence of discrete exponential attractors in terms of a covering condition for iterates of the absorbing set under the time evolution of the semigroup. The parameters in the covering condition determine the estimate for the fractal dimension of the exponential attractor and the exponential rate of attraction. We then verify the covering condition for existing approaches to construct exponential attractors where the most general setting concerns quasi-stable semigroups in complete metric spaces. Generalizing previous notions and methods used in the literature on exponential attractors then allows us to compare widely used approaches. To conclude, we mention generalizations of the constructions for non-autonomous and random dynamical systems.

Stabilization of Timoshenko-Boltzmann-Fourier systems

Tais Saito Tavares
Henan Normal University, China

In this talk, we present characterization results for certain Timoshenko beam systems with hybrid damping, specifically, thermoviscoelastic models. To achieve this, we consider memory kernels that do not require dissipative conditions on their derivatives and use tools from the theory of semigroups of linear operators.

Dynamic behavior of pantograph delay differential equations

Tomás Caraballo, Wenlong Sun, Peter E. Kloeden
Universidad de Sevilla, Spain

This talk is concerned with the pantograph delay differential equation on $\mathbb{R}^d$ $(d \geq 1)$ with initial time $t_0 \in \mathbb{R}$. First, the global well-posedness of the solution is proved. Then, a two-parameter semigroup on the weighted space $C_{\gamma_0}(\mathbb{R}^-, \mathbb{R}^d)$ is constructed, which is not formed naturally from the well-posedness of the solution due to the peculiarities of the delay term in the pantograph system. Finally, the existence of a pullback attractor

and a forward attractor is established by the existence of a compact set which is uniformly attracting for the two-parameter semigroup associated to the system. The analysis of pantograph equations requires a non-autonomous set-up due to the special nature of the proportional delay, and remained as an open problem for more than 20 years. Thanks to a suitable interpretation of the delay terms, we were able to handle the problem in an appropriate framework.

Dynamics of a random age-structured population model

Xiaoying Maggie Han
Auburn University, USA

In this talk, we first introduce a random age-structured population model in which the adult birth rate is described by a stochastic process formulated through a canonical noise process. We then establish well-posedness, the cocycle property, and asymptotic behaviors of the solutions. Finally, we show the asymptotic smoothness and bounded dissipativity of the cocycle generated by the solutions, which further implies that the fractal dimension of the random attractor is finite.

Dispersive Equations

Organizer: Mahendra Panthee (UNICAMP/Brazil) & Argenis Jose Mendez Garcia (UNICAMP/Brazil)

Boundary Effects on the Controllability of Coupled KdV Systems

Ademir Fernando Pazoto, Fernando Gallego, Ivonne Rivas
Universidade Federal do Rio de Janeiro, Brazil

We study the exact boundary controllability of a nonlinear coupled system of two Korteweg-de Vries equations on a bounded interval. The model describes the interactions of two weakly nonlinear gravity waves in a stratified fluid. Due to the nature of the system, six boundary conditions are required. However, to study the controllability property, we consider a different combination of the control inputs, with a maximum of four. Firstly, the results are obtained for the linearized system through a classical duality approach and some hidden regularity properties of the boundary terms. This approach reduces the controllability problem to the study of a spectral problem, which is solved by using the Paley–Wiener method introduced by Rosier (1997). Then, the issue is to establish when a certain quotient of entire functions still turns out to be an entire function. It can be viewed as a problem of factoring an entire function that, depending on the control configuration, leads to the study of a transcendental equation. Finally, by using the contraction mapping theorem, we derive the local controllability for the full system.

On decay and regularity of solutions to the 2D modified Zakharov-Kuznetsov equation

Alexander Muñoz Garcia
Universidade de Sao Paulo, Brazil

In this talk we discuss relation between regularity and decay for solutions of the two-dimensional modified Zakharov-Kuznetsov equation in the anisotropic weighted Sobolev spaces $Z_{s,(r_1,r_2)} := H^s(\mathbb{R}) \cap L^2((1+|x|^{2r_1}+|y|^{2r_2})dxdy)$. Our main result describes how the regularity index s and the directional decay indices r_1, r_2 are optimally related in the fractional case. More precisely, for a solution with regularity s, any extra decay beyond $s/2$, say $\min\{r_1, r_2\} = s/2 + \alpha$, is translated into an improvement of regularity of size 2α in the cases

- $\frac{3}{4} < s < 1$ and $0 \leq \alpha \leq \frac{1-s}{2}$.
- $s > \frac{9}{4}$ and $\alpha \geq 0$.

This talk is taken from a joint work with J. Jiménez and E. A. Bustamante from Universidad Nacional de Colombia Sede Medellín.

Dynamics for the Logarithmic Schrödinger Equation on a Tadpole Graph

Andrés Gerardo Pérez Yépez

Universidade de São Paulo, Brazil

In this talk, we will discuss some dynamical aspects of the nonlinear logarithmic Schrödinger equation (NLS-log) on a tadpole graph, namely, a graph consisting of a circle with a half-line attached at a single vertex. By considering Neumann-Kirchhoff boundary conditions at the junction, we show the existence and the orbital stability of standing wave solutions with a profile determined by a positive single-lobe state. Via a splitting-eigenvalue method, we identify the Morse index and the nullity index of a specific linearized operator around a positive single-lobe state.

Global well posedness for inhomogeneous nonlinear Schrödinger equation with combined power-type nonlinearities

Andressa Gomes, Mykael Cardoso

Universidade Federal do Delta do Parnaíba, Brazil

In this talk we show results about the global well-posedeness for the inhomogeneous nonlinear Schrödinger equation (INLS) with combined power-type nonlinearities in energy space. More precisely, we study the case when a nonlinearity is $\dot{H}_x^1$-critical and the other nonlinearity is subcritical. For this purpose, we developed the stability theory for the INLS equation with one $\dot{H}_x^1$-critical nonlinearity. Then, we treat the energy-subcritical nonlinearity as a pertubation to the energy-critical INLS, which is globally wellposed. Finally, we established conditions that ensure blow-up in finite time to solution of the problem with double nonlinearity. This is a joint work with Prof. Mykael Cardoso (UFPI).

On the Dispersive Generalized Benjamin-Ono-Zakharov-Kuznetsov-Equation: Revisited

Argenis José Méndez Garcia

Universidade Estadual de Campinas, Brazil

We revisit the initial value problem for the dispersive generalized Benjamin-Ono-Zakharov-Kuznetsov equation and refine the understanding of its propagation of regularity phenomenon. This model exhibits the effect that localized higher smoothness of the initial data, prescribed on families of half-spaces, is transmitted instantaneously by the flow. Several aspects of this mechanism were first investigated in the joint work of Freire, Méndez and Riaño, *On some regularity properties for the dispersive generalized Benjamin-Ono-Zakharov-Kuznetsov Equation*, Journal of Differential Equations **322** (2022), 135-179, where certain structural obstructions prevented a full resolution of the problem. In the present paper, we develop a new commutator framework that overcomes these obstacles and significantly enlarges the range of admissible propagation regimes. A microlocal analysis of the associated wave front set further identifies the geometric regions in which the gain of regularity travels and clarifies the mechanism behind the infinite-speed transmission. Altogether, these results yield a broader and more robust description of regularity propagation for this family of anisotropic dispersive equations.

On the inhomogeneous biharmonic nonlinear Schrödinger equation

Carlos Manuel Guzmán Jiménez

Universidade Federal Fluminense, Brazil

In this talk, we study the initial value problem for the inhomogeneous biharmonic nonlinear Schrödinger (IBNLS) equation

$$iu_t + \Delta^2 u + \lambda|x|^{-b}|u|^{\alpha}u = 0, \qquad \alpha, b > 0,\ \lambda = \pm 1.$$

We first present results on local and global well-posedness in the natural energy space. We then investigate the long-time behavior of solutions, establishing scattering for nonradial initial data in both the intercritical and the critical regimes. To achieve this, we combine refined analytical techniques with the well-known concentration–compactness and rigidity method of Kenig and Merle.

Periodic Waves for the Regularized Camassa-Holm Equation: Existence and Spectral Stability

Fábio Natali

Universidade Estadual de Maringá, Brazil

In this paper, we investigate the existence and spectral stability of periodic traveling wave solutions for the regularized Camassa-Holm equation. To establish the existence of periodic waves, we employ tools from bifurcation theory to construct solutions with the zero-mean property. We also prove that such waves may not exist for the well-known Camassa-Holm equation. Regarding spectral stability, we analyze the difference between the number of negative eigenvalues of the second variation of the Lyapunov functional at the wave, restricted to the space of zero-mean periodic functions, and the number of negative eigenvalues of the matrix formed from the tangent space associated with the low-order conserved quantities of the evolution model. Finally, we address the problem of orbital stability as a consequence of spectral stability.

Local Well-Posedness for a System of Modified KdV Equations in Modulation Spaces

Fidel Cuba Balvin, Xavier Carvajal, Mahendra Panthe

Universidad ESAN, Peru

We consider the initial value problem (IVP) associated to a system consisting modified Korteweg-de Vries (mKdV) type equations

$$\begin{cases} \partial_t v + \partial_x^3 v + \partial_x(vw^2) = 0, & v(x,0) = \phi(x), \\ \partial_t w + \alpha\partial_x^3 w + \partial_x(v^2 w) = 0, & w(x,0) = \psi(x), \end{cases}$$

The main interest is in addressing the well-posedness issues of the IVP when the initial data are considered in the modulation space $M_s^{2,p}(\mathbb{R}) \times M_s^{2,p}(\mathbb{R})$, $p \geq 2$. In this case when $0 < \alpha \neq 1$, we derive new trilinear estimates in these spaces an prove that the IVP is locally well-posed for data in $M_s^{2,p}(\mathbb{R}) \times M_s^{2,p}(\mathbb{R})$ whenever $s > \frac{1}{4} - \frac{1}{p}$. In deriving the trilinear estimates, the fact that the Fourier supports of the solution components v and w lie on distinct cubic curves, namely $\tau = \xi^3$ and $\tau = \alpha\xi^3$, introduces additional difficulties in handling the resonant case.

References

[1] Carvajal, X., Panthee, M., *Nonlinear Schrodinger equations with the third order dispersion on Modulation spaces*, Partial Differential Equations and Aplications. **3**, 59 (2022).
[2] Grunrock, A., *An improved local well-posedness results for the modified KdV equation*. Int. Math. Res. Not. **2004**, 3287–3308 (2004).
[3] Oh, T., Wang, Y., *On global well-posedness of the modified KdV equation in Modulation spaces*, Discrete & continuous Dynamical Systems, **41**(6): 2971-2992 (2021).

On the radius of analyticity of solutions to the BBM equation with damping

Mikaela Baldasso, Mahendra Panthee
Universidade Estadual de Campinas, Brazil

In this talk, we study the initial value problem for the Benjamin–Bona–Mahony (BBM) equation with damping, considering initial data in the analytic function space $G^{\sigma,1}(\mathbb{R})$ equipped with a hyperbolic cosine weight. We prove local well-posedness in this setting and analyze the behavior of the radius of spatial analyticity $\sigma(t)$ when the local solutions extend globally in time. By constructing an almost conserved quantity, we show that the radius of spatial analyticity analyticity remains uniformly bounded below by a positive constant for all time.

On the asymptotic dynamics for the L^2-Supercritical gKdV equation

Ricardo Carlos Freire, Claudio Muñoz
Universidad de Chile, Chile

We study the L^2-supercritical generalized Korteweg-de Vries equation (gKdV) with nonlinearities $p > 5$. While local well-posedness in H^1 is classical, the long–time dynamics in the supercritical regime remains largely unexplored beyond small data global solutions, the construction of multi-solitons for any power and self-similar blow-up near the critical power $p = 5$. We develop a unified description of the non-solitonic region for arbitrary H^1 solutions, both global and blowing up. Our analysis shows that the asymptotic L^2 and L^p dynamics in this region is completely determined by the growth rate of the L^2 norm of the gradient (or, equivalently, the critical H^{s_p} norm). In particular, we prove sharp far-field decay on both half-lines and establish normalized local vanishing along sequences of times, with improved estimates in the case of even-power nonlinearities. A key ingredient is a new virial method that compensates for the possible unboundedness of the H^1 norm by exploiting the conservation of mass and a careful localization of the nonlinear flux. This yields quantitative versions of decay phenomena previously known only in subcritical settings, and it applies without any smallness or proximity-to-soliton assumptions.

On dispersion generalized Benjamin-Ono equations equation

Thyago S.R. Santos
Universidade Estadual de Campinas, Brazil

We consider the Cauchy problem related to the family of k-dispersion generalized Benjamin-Ono (k-dgBO) equations:

$$u_t + D_x^\alpha u_x + \mu\, u^k u_x = 0, \quad (t,x) \in \boldsymbol{R} \times \boldsymbol{R}, \tag{1}$$

where $u = u(t,x)$ is real-valued, $\alpha \in [1,2]$, $\mu \in \{\pm 1\}$ and $k \in \mathbb{Z}^+$. Here, D_x^α represents the 1-dimensional fractional Laplacian operator in the spatial variable x. For $k \geq 4$, we establish local and global well-posedness

results for (1) in both the critical $\left(s=\frac{k-2\alpha}{2k}\right)$ and subcritical $\left(s>\frac{k-2\alpha}{2k}\right)$ regimes, addressing sharp regularity in homogeneous and inhomogeneous Sobolev spaces. Additionally, our method enables the formulation of a scattering criterion and a scattering theory for small data. We also investigate the case $k=3$ via frequency-restricted estimates, obtaining local well-posedness results for the initial value problem associated with the 3-dgBO equation and generalizing the existing results in the literature for the whole subcritical range. For higher dispersion, these local results can be extended globally even for rough data, particularly for initial data in Sobolev spaces with negative indices. As a byproduct, we derive new nonlinear smoothing estimates. This is a joint work with Luccas Campos (UFMG) and Felipe Linares (IMPA).

Linear Equations

Organizer: Nicholas Braun Rodrigues (UFSCar/Brazil) & Vinícius Novelli da Silva (USP/Brazil)

Poincaré type inequality for local Hardy-Sobolev spaces

Catarina Barbosa Machado, Tiago Henrique Picon
Universidade de São Paulo, Brazil

Inspired by the classical Poincaré inequality for Sobolev spaces on open balls, this work is concerned with the formulation of inequalities of this type when $0 < p \leq 1$ for higher order Hardy-Sobolev spaces $h^{k,p}(\mathbb{R}^N)$, with $k \in \mathbb{N}$. Namely, given an open ball with small radius and a function $f \in C_c^\infty(\mathbb{R}^N)$, our result constructs a special polynomial of order less than $k-1$ such that there exists a positive (and universal) constant C satisfying

$$\left(\int_{\mathbb{R}^N} \left[\sup_{0<t<1} \operatorname{fint}_{B_x^t} \left\{ \frac{1}{t^k} |f(y) - P_{x,t,f}(y)| \right\}^\alpha dy \right]^{p/\alpha} dx \right)^{1/p} \leq C \, \|f\|_{h^{k,p}},$$

where α is a parameter depending on k, N, and p. This is a joint work with Professor Tiago Picon (USP-RP).

Low regularity perturbations of non-autonomous parabolic problems

Francisco Javier Larcada-Sanchez, Aníbal Rodríguez-Bernal
Universidad Complutense de Madrid, Spain

We present general techniques for low regularity perturbations for evolution operators of linear non-autonomous parabolic problems, obtaining results to determine in which spaces of a scale of Banach spaces the problem has a solution and how it regularizes. This abstract results can be applied to specific cases in scales of Bessel potential spaces and Lebesgue spaces, addressing perturbations defined by interior and boundary potentials, or transport terms, where the spaces in which there is existence of solution and regularization can be explicitly determined.

This is a joint on-going project with Aníbal Rodríguez-Bernal (UCM-ICMAT).

Solvability of nonsingular closed 1-forms on compact manifolds

Gabriel Araújo, P. L. Dattori da Silva, B. de Lessa Victor, V. Novelli
Universidade de São Paulo, Brazil

To a nonsingular closed 1-form ω on a smooth manifold M, one naturally associates a subbundle of TM, which is integrable by Frobenius Theorem. A well-known construction then gives rise to a first-order differential complex, whose global solvability (meaning: closedness of the range in the Fréchet topology of smooth sections) we address in the first degree of the said complex when M is compact and connected. We provide a complete characterization of the latter propety in terms of certain number-theoretic approximation conditions on ω, loosely connected with the topology of the leaves of the foliation associated with ω (or, more precisely, how they embed into M). As a happy side effect of our techniques, we are able to fulfill the program started by A. Meziani (Comm. PDE, 2002) and settle, without further assumptions, the related problem of global hypoellipticity of the same differential operator.

Global hypoellipticity for real vector fields on torus

Igor Ambo Ferra, Luís Antônio Carvalho dos Santos, Rafael Fernando Barostichi
Universidade Federal de São Carlos, Brazil

In this talk we shall discuss the global hypoellipticity of the following system of vector fields on the torus $\mathbb{T}^4$:

$$X_1 = \partial_{t_1} + a_{11}(t)\partial_{x_1} + a_{22}(t)\partial_{x_2} \quad \text{and} \quad X_1 = \partial_{t_1} + a_{21}(t)\partial_{x_1} + a_{22}(t)\partial_{x_2},$$

where a_{jk} are smooth and real.

Subelliptic estimates for the complex Green operator on non-pseudoconvex domains

Joel Coacalle
Universidade de São Paulo, Brazil

In this talk, we present subelliptic estimates for the complex Green operator K_q at a given level q of the $\bar{\partial}_b$-complex. These estimates are established on a CR manifold that is not necessarily pseudoconvex but satisfies a finite type condition expressed in terms of commutators. We introduce a subtle comparability condition that applies to a family of manifolds, which includes a class of weak $Y(q)$ manifolds satisfying the $D(q)$ condition. The proof employs a microlocal decomposition to achieve the subelliptic estimates.

Asymptotic properties of solutions to a class of second-order evolution equations with time-dependent damping

Halit Sevki Aslan, Marcelo R. Ebert
Universidade de São Paulo, Brazil

We consider the following Cauchy problem for a semilinear evolution equation with scale-invariant time-dependent damping:

$$\begin{cases} u_{tt} + L_{w^2}u + \dfrac{\mu}{1+t}u_t = |D|^\theta |u|^p, & t > 0,\ x \in \mathbb{R}^n, \\ u(0,x) = 0, \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases}$$

where $\mu > 0$, $p > 1$, $\theta \in [0, b]$, and L_{w^2} is a sum of different powers of the Laplace operator

$$L_{w^2} \doteq (-\Delta)^a + (-\Delta)^b, \quad b \geq a > 0,$$

defined via the Fourier transform as $L_{w^2}u = \mathcal{F}^{-1}\big(\omega(\xi)^2 \hat{u}\big)$ with $w(\xi)^2 = |\xi|^{2a} + |\xi|^{2b}$.

Here $\mathcal{F}^{-1}$ denoting the Fourier transform and its inverse on the space of tempered distributions $\mathcal{S}'$. In particular, when a and b are positive integers, the corresponding Laplace operators are local, while fractional values of a and b give rise to nonlocal operators. We prove global (in time) existence results for small data solutions under suitable assumptions on the phase function ω^2, on the dissipation parameter μ (distinguishing between the regimes $\mu > 1$ and $\mu < 1$), and on the nonlinearity exponent $p > 1$. Our abstract framework is then applied to several concrete models with scale-invariant time-dependent damping, including the wave equation, σ-evolution equation, the Boussinesq equation, and evolution equations involving mixed nonlocal operators. In particular, in some of these cases we identify the critical exponent p and the threshold value of μ.

Real Involutive Systems on Compact Lie Groups

Luis Fernando Ragognette, Gabriel Araújo, Igor A. Ferra, Max R. Jahnke
Universidade Federal de Minas Gerais, Brazil

Consider G a connected compact Lie group and $\mathcal{V}$ an essentially real, left-invariant involutive structure. Our goal in this talk is to study the global solvability and also the cohomology groups of a differencial operator induced by the involutive structure $\mathcal{V}$.

We proved that global solvability in the first degree of the complex implies global solvability in all other degrees and, with the additional hypothesis that structure is Abelian, it is enough to check global solvability in any degree to verify that it is true for all others.

We have shown that each class in the q-cohomology group associated with $\mathcal{V}$ has a representative that is a q-form with constant coefficients, in particular, all these groups are finite-dimensional. We also consider the product of a compact connected Lie group with a torus and, in this particular setting, we show the existence of isomorphisms giving a further understanding of the cohomology groups associated with the differential complex of this involutive structure. For the last part, the main technique comes from the study of the cohomology groups of perturbations of the de Rham operator.

Dispersive character for weakly degenerate phase functions

Marcelo Rempel Ebert, Marcello D'Abbicco, Antonio Lagioia
Universidade de São Paulo, Brazil

In this talk we are interested in derive dispersive estimates for the solution to initial value problems for evolution equations in the form

$$\begin{cases} u_{tt} + L_{\omega^2} u + Au_t = 0, & t > 0,\ x \in \boldsymbol{R}^n, \\ (u, u_t)(0, x) = (u_0, u_1)(x), \end{cases} \tag{2}$$

or for models of first-order in time, as the generalized Schrödinger equation

$$\begin{cases} u_t \pm iL_\omega u + Au = 0, \quad t > 0,\ x \in \boldsymbol{R}^n, \\ u(0, x) = u_0(x), \end{cases} \tag{3}$$

where the dissipation term is given by $Af = \mathfrak{F}^{-1}(a(\xi)\,\hat{f})$, with $a > 0$ and $L_w f = \mathfrak{F}^{-1}(w(\xi)\,\hat{f})$.

Microlocal Analysis of the Pullback of Ultradistributions by Gevrey Maps

Renan Dantas Medrado
Universidade Federal de Alagoas, Brazil

We develop a theory of the pullback of ultradistributions via Gevrey maps, extending the notion of pullback of distributions by C^∞ functions. Using transversality conditions, we establish continuity and control over the s-wave-front sets. As an application, we study the Gevrey wave-front set of solutions to a locally integrable structure when restricted to maximally real submanifolds.

Elliptic PDEs with measure data: solutions in weighted Lebesgue spaces

Victor Biliatto, Joel Coacalle, Tiago Picon
Universidade de São Paulo, Brazil

In this talk, I go through a collection of results regarding the solvability of PDEs associated to elliptic linear differential operators with complex coefficients and measure data. I present conditions that ensure solvability in weighted Lebesgue spaces. The endpoint case of bounded solutions is particularly treated, as it relies on a new weighted L^1 Stein-Weiss type inequality on measures for a special class of vector fields. This is a joint work with Joel Coacalle (DM-UFSCar) and Tiago Picon (FFCLRP-USP).

Elliptic Equations

Organizer: Ederson Moreira dos Santos (USP/Brazil) & Gustavo Ferron Madeira (UFSCar/Brazil)

Existence of positive solution for a fractional equation with a supercritical nonlinearity

Augusto César dos Reis Costa, Rafael A. D. Guimarães
Universidade Federal do Pará, Brazil

In this work, we study the following fractional problem

$$\begin{cases} \mathcal{L}u - \mu\dfrac{u}{|x|^{2s}} = \lambda|u|^{r-2}u \quad \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \\ u = 0 \quad \text{in } \mathbb{R}^n\backslash\Omega, \end{cases}$$

where $n > 2s$ with $s \in (0,1)$, $M : \mathbb{R}^+ \to \mathbb{R}$ is a function with suitable conditions, $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary, $\lambda > 0$, $r > 2_s^*$, μ belongs to a special interval and

$$\mathcal{L}u = M\left(\int_{\mathbb{R}^n\times\mathbb{R}^n} \frac{|u(x)-u(y)|^2}{|x-y|^{n+2s}}\right)(-\Delta)^s u.$$

The existence result is obtained via Moser Iteration Method.

Improving a Spectral Inequality by Payne

Carlo Nitsch
University of Napoli Federico II, Italy

A celebrated inequality by Payne relates the first eigenvalue of the Dirichlet Laplacian to the first eigenvalue of the buckling problem. Motivated by the goal of establishing a quantitative version of this inequality, we show that Payne's original estimate—which is not sharp—can in fact be improved. Our result provides a refined spectral bound and opens the way to further investigations into quantitative enhancements of classical inequalities in spectral theory.

Nonlinear Schrödinger-Poisson systems in dimension two: the zero mass case

Cristina Tarsi, Federico Bernini, Giulio Romani
Università degli studi di Milano, Italy

We provide an existence result for a Schrödinger-Poisson system in the whole plane, in the case of zero mass, by considering the corresponding logarithmic Choquard equation

$$-\Delta u = \left(\frac{1}{2\pi}\ln\frac{1}{|x|} * F(u)\right) f(u) \quad \text{in } \mathbb{R}^2$$

where f is a positive continuous nonlinearity with subcritical or critical growth in the sense of Trudinger-Moser, and $F(t) := \int_0^t f(s)\,ds$. The absence of the mass term combined with the logarithmic behaviour of the kernel make the analysis delicate. However, differently from the (local) Schrödinger equations with zero mass, the presence of the logarithmic kernel allows to recover a standard Sobolev type space as suitable functional framework, thanks to a careful splitting in positive and negative part of the logarithm. The existence of a mountain pass-type solution is established by means of a careful analysis of appropriate Cerami sequences.

Multiple nodal solutions for Kirchhoff-Choquard type equations

Eduardo de Souza Böer, Eudes M. Barboza, Olímpio H. Myagaki, Claudia R. Santana
Universidade de São Paulo, Brazil

In this work, we study a class of planar Kirchhoff-Choquard equations involving a sign-changing logarithmic kernel and an exponential nonlinearity. Our main goal is to construct solutions with k nodes for any natural number k, thereby establishing the multiplicity of solutions for this class of problems. To achieve this, we employ a gluing method.

Weighted Robin eigenvalue problems and nonlinear elliptic equations with quadratic growth in the gradient

Francesco Della Pietra, Giuseppina di Blasio, Giuseppe Riey
Napoli Federico II, Italy

In the talk I will describe an existence result for Robin boundary value problems modeled on

$$\begin{cases} \Delta u + |\nabla u|^2 + \lambda f(x) = 0 & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} + \beta u = 0 & \text{on } \partial\Omega \end{cases}$$

where Ω is a bounded, sufficiently smooth open set in $\mathbb{R}^N$, $f(x)$ belongs to the Marcinkiewicz space $M^{\frac{N}{2}}$ and $\beta > 0$, under a smallness assumption on the datum λ. In order to study such problem, I will show several properties of the weighted, singular Robin eigenvalue problem

$$\lambda_{1,f,\gamma}(\Omega) = \inf_{\psi \in H^1,\, \int_\Omega f\psi^2 = 1} \left\{ \int_\Omega |\nabla \psi|^2 dx + \gamma \int_{\partial\Omega} \psi^2 \right\}.$$

Ground states of the Schrödinger–Newton system on $\mathbb{R}^2$ with a point interaction

Gustavo de Paula Ramos
Universidade de São Paulo, Brazil

This work is concerned with the existence of ground states of the following normalized *Schrödinger–Newton system* with a point interaction:

$$\begin{cases} -\Delta_\alpha u + \phi u + \omega u = \beta u|u|^{p-2} & \text{on } \mathbb{R}^2; \\ \Delta\phi = 2\pi|u|^2 & \text{on } \mathbb{R}^2; \\ \|u\|_{L^2}^2 = c, \end{cases}$$

where $\alpha, \beta \in \mathbb{R}$; $p \in]2, \infty[$ and $c > 0$ are given parameters; $-\Delta_\alpha$ denotes the Laplacian of point interaction centered at $0 \in \mathbb{R}^2$ with inverse scattering length $-2\pi\alpha$ and the unknowns are $u\colon \mathbb{R}^2 \to \mathbb{C}$, $\phi\colon \mathbb{R}^2 \to \mathbb{R}$ and $\omega \in \mathbb{R}$. More precisely, the *Laplacian of point interaction* $-\Delta_\alpha$ models the action of a point defect at the origin in the sense that it is the L^2-self-adjoint operator that best approaches the behavior of $-\Delta + \frac{1}{\alpha}\delta_0$ on C_c^∞. In this talk, we review recent developments regarding variational PDEs with a point interaction, we recall the relevant variational framework and we sketch the main ideas that appear in the proof of the existence of ground states of the aforementioned system.

Classification of singular solutions to the fractional Yamabe equation

João Henrique Andrade, Azahara DelaTorre, João Marcos do Ó, Jesse Ratzkin, Juncheng Wei
Universidade de São Paulo, Brazil

We study positive solutions of the critical, conformally invariant fractional Yamabe equation on the twice-punctured sphere and its Euclidean and cylindrical formulations. For fractional orders $s \in (0,1)$ sufficiently close to one, we prove that every singular solution is a fractional Delaunay solution. The proof has two components: a local compactness argument applied to the normalized difference of two solutions, which yields convergence to a linear homogeneous limit; and a global classification step that uses sharp kernel bounds, weighted norms adapted to the Emden–Fowler transform, and an ODE-type barrier preventing nontrivial limits under noncompact recentering. A crucial step is obtaining a sharp supremum estimate from the Morse index, a property of the spherical bubble. We also establish the nondegeneracy of fractional Delaunay solutions for $s \sim 1$ near one.

Existence and non-existence results for a class of systems under concave-convex nonlinearities

João Pablo Pinheiro Da Silva
Universidade Federal do Pará, Brazil

In this work, we are interested in studying the following class of problems:

$$(\mathcal{P}_{\lambda\mu}) \qquad \begin{cases} -\Delta u = f_\lambda(x,u,v) & \text{in } \Omega \\ -\Delta v = g_\mu(x,u,v) & \text{in } \Omega \\ 0 \not\equiv u \geq 0,\ 0 \not\equiv v \geq 0 & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $\lambda > 0$, $\mu > 0$, $t \mapsto f_\lambda(x,t,t)$ and $t \mapsto g_\mu(x,t,t)$ have concave-convex type nonlinearities. We present results related to the existence and non-existence of solutions for problem $(\mathcal{P}_{\lambda\mu})$.

Existence of normalized solutions for a Schrödinger system with $L2-$supercritical growth

João Rodrigues Santos Júnior, Claudianor Oliveira Alves, Mayara Silvia Silva
Universidade Federal do Pará, Brazil

In this paper we are concerned with the existence of normalized solutions to the following class of elliptic systems

$$\begin{cases} -\Delta u - \lambda_1 u = \frac{\partial Q}{\partial u}(u,v), & \text{in } \Omega, \\ -\Delta v - \lambda_2 v = \frac{\partial Q}{\partial v}(u,v), & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \\ \int_\Omega |u|^2 dx = a_1^2 \text{ and } \int_\Omega |v|^2 dx = a_2^2, \end{cases}$$

where $a_1,\ a_2 > 0$, Ω is a bounded smooth domain in $\mathbb{R}^3$ and Q is a 4-homogeneous function that satisfies some technical conditions.

Ordered solutions for degenerate Kirchhoff problems

Kaye Silva, Francesca Faraci
Universidade Federal de Goiás, Brazil

We study a parametrized Kirchhoff type equation with two degeneracy points. The existence of two $H_0^1(\Omega)$-norm ordered solutions is established for small value of the parameter via a careful analysis of the fiber maps associated to the energy functional. As a consequence we show existence of multiple or even infinitely many solutions to degenerate Kirchhoff equations. Due to the simplicity of the conditions, many applications are given.

Multiple positive solutions for quasilinear nonlocal problem via topological, variational and set-valued methods

Leszek Gasinski
University of the National Education Commission, Poland

We study a nonlocal quasilinear problem driven by the $p-$Laplacian operator of a nonvariational type, without assuming any kind of monotonicity on the data. The nonlocal term depends on the L^q-norm of the unknown function, where p and q are independent exponents and the weight function can be sign changing. The multiplicity of positive solutions is established through a combination of variational methods, truncation techniques, set-valued analysis, and fixed-point results.

On the asymptotically linear problem for an elliptic equation with an indefinite nonlinearity

Mayra Soares, Mónica Clapp, Alberto Saldaña, Cristian Morales-Encinos
Universidade de Brasília, Brazil

We study the semilinear elliptic problem

$$-\Delta u = Q_\Omega |u|^{p-2}u \quad \text{in } \mathbb{R}^N,$$

where $Q_\Omega = \chi_\Omega - \chi_{\mathbb{R}^N\setminus\Omega}$ for a bounded smooth domain $\Omega \subset \mathbb{R}^N$, $N \geq 3$, and $1 < p < 2^*$. This equation arises in the study of optical waveguides and exhibits indefinite nonlinearity due to the sign-changing weight Q_Ω. We prove that, for $p > 2$ sufficiently close to 2, positive solutions are nondegenerate and the problem admits a unique least energy solution. Our approach combines a detailed analysis of an associated eigenvalue problem involving Q_Ω with variational methods and blow-up techniques in the asymptotically linear regime. We also provide a comprehensive study of the spectral properties of the corresponding linear problem, including the existence and qualitative behavior of eigenfunctions, sharp decay estimates, and symmetry results. In particular, we establish analogues of the Faber–Krahn and Hong–Krahn–Szegö inequalities in this non-standard setting.

A Critical Neumann problem with anisotropic $p-$Laplacian

Olimpio Hiroshi Miyagaki, Gustavo Ferron Madeira, Alânnio Barbosa Nóbrega
Universidade Federal de São Carlos, Brazil

We are concerned with the existence of solution of the problem

$$\begin{cases} -\Delta_p^H u + |u|^{p-2}u = \lambda |u|^{q-2}u + |u|^{p^*-2}u \quad \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \\ a(\nabla u)\cdot \nu = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Delta_p^H u = \operatorname{div}(a(\nabla u))$, with $a(\xi) = H^{p-1}(\xi)\nabla H(\xi)$, $\xi \in \boldsymbol{R}^N$, $N \geqslant 3$, is the anisotropic p-Laplacian with $1 < p < N$, $\lambda > 0$ is a parameter, and $p < q < p^* = pN/(N-p)$. Further, Ω is a C^1 bounded domain inside a convex open cone. To succeed with a variational approach, where the strong convergence of a bounded (PS) subsequence needs to be proved, one has to deal with anisotropic norms in the absence of a Tartar's type inequality, unlike the isotropic p-Laplace case. This is overcome by proving the a.e. convergence of its gradients. Furthermore, the solution of (P) is shown to belong to $C^{1,\alpha}(\Omega)$ from classical elliptic regularity theory, and is positive from a Harnack inequality, since any solution of (P) is bounded. This in turn is a consequence of a result we prove which assures that any $W^{1,p}$-solution of critical Neumann problems with the anisotropic p-Laplacian operator on bounded Lipschitz domains in $\mathbb{R}^N$ $(N \geqslant 3)$ is bounded.

Harmonic Analysis and Related Topics

Organizer: Guilherme da Silva (USP/Brazil), Tiago Picon (USP/Brazil)
& Charles Ferreira dos Santos (USP/Brazil)

Composition operators in Weighted Hardy spaces

Ben-Hur Eidt
Universidade de São Paulo, Brazil

In this talk, our main subject is operator theory on the weighted Hardy space $H^p(\mathbb{D}, \omega)$. After reviewing the basic framework, we focus on the role played by outer functions in this setting and provide a necessary and sufficient condition for the well-definedness of certain composition operators in the weighted context. We then present and discuss how the complex method of interpolation can be applied to weighted Hardy spaces; in particular, we will show an expected result that resembles the Coifman–Rochberg–Weiss commutator theorem.

On homeomorphisms between the spheres of ℓ_∞^k and ℓ_2^k

Bruno de Mendonça Braga
IMPA, Brazil

Bill Johnson asked in a MathOverFlow post in 2011 whether one can find a sequence of homeomorphisms $(f_k)_k$ between the unit spheres of the finite dimensional Banach spaces ℓ_∞^k and ℓ_2^k such that the Lipschitz constant of the f_k's are uniformly bounded. In this talk, I will discuss some recent developments on this question which provide a negative answer to Bill's question under some extra assumptions on the homeomorphisms $(f_k)_k$. The talk is based on ongoing work together with Gartland, Lancien, Motakis, Pernecká, and Schlumprecht.

Modular forms and irrationality proofs

Cynthia Bortolotto, Lucas Oliveira
IMPA, Brazil

Apéry's proof of the irrationality of $\zeta(3)$ is based on a remarkable recurrence satisfied by certain rational approximations. Beukers later reinterpreted this argument using integrals of modular forms over congruence subgroups. In this talk, we study Beukers' modular–forms approach and use it to construct new families of rational approximations arising from congruence groups.

Sums related to Dirichlet characters

Caio Bueno
Universidade Federal de Minas Gerais, Brazil

Let $f : \mathbb{N} \to \mathbb{C}$ be an arithmetic function taking values in $\{-1, +1\}$. In 2016, Tao used a Fourier-analytic reduction to prove that the discrepancy

$$\sup_{n,d\in\mathbb{N}} \left| \sum_{j=1}^{n} f(jd) \right|$$

of f is infinite. This problem was known as the Erdős Discrepancy Problem. After the resolution of this conjecture, a natural line of research is to investigate the behaviour of this sum when restricted to a subset of integers.

In this talk, we will discuss recent results related to upper bounds for partial sums of Dirichlet characters, modified in a simple way to eliminate its zeros, and restricted to the k-free numbers (i.e., n is k-free if no prime power p^k divides n). In addition, we will comment on a recent result concerning the existence of a limiting logarithmic distribution for these partial sums.

Random Schrödinger operators on manifolds

Konstantin Merz, Jean-Claude Cuenin, Eduard Stefanescu
ETH Zurich, Switzerland

We consider random Schrödinger operators of Anderson-type on smooth and closed Riemannian manifolds with complex potentials. Our main results are new eigenvalue estimates depending only on the L^q-norm of the potential. Compared to the deterministic setting considered in [1], our bounds yield a sharper localization of eigenvalues, particularly for those whose size is bounded by the inverse square of the randomization length scale. Our proof relies on an abstraction of the techniques recently developed in the Euclidean setting [2]. The talk is based on joint work in progress with Jean-Claude Cuenin and Eduard Stefanescu.

References

[1] Jean-Claude Cuenin. Eigenvalue bounds for Schrödinger operators with complex potentials on compact manifolds. *Forum Mathematicum*, 2025. In press. Available online at https://doi.org/10.1515/forum-2024-0564.

[2] Jean-Claude Cuenin and Konstantin Merz. Random Schrödinger operators with complex decaying potentials. *Anal. PDE*, 18(2):279–306, 2025.

Hardy inequalities on Hardy spaces

Luís Márcio Salge, Tiago Picon, Joel Coacalle
Universidade do Estado do Rio de Janeiro, Brazil

In this work, we revisit Hardy inequalities on the real line in the framework of Hardy spaces, i.e.,

$$\left(\int_0^\infty \left|\int_0^x f(y)y^\alpha dy\right|^r x^{(\mu-\alpha-1)r}dx\right)^{\frac{1}{r}} \leq C\|f\|_{H^p(\mathbb{R})}, f \in (H^p \cap L^q)(\mathbb{R}).$$

Furthermore, we establish new variants of such inequalities for local Hardy spaces.

Functional calculus associated with problems with dynamic boundary conditions

Muriel Andreane Dalcy, Rafael Fernando Barostichi, Pedro Tavares Paes Lopes
Universidade de São Paulo, Brazil

This work aims to study the holomorphic functional calculus associated with the problem

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \operatorname{div}(a_\Omega \cdot \operatorname{grad}(u)) + b_\Omega \cdot \operatorname{grad}(u) + c_\Omega u + f(u), && t > 0, x \in \Omega,\\
&\frac{\partial u}{\partial t} = \operatorname{div}_\Gamma(a_\Gamma \cdot \operatorname{grad}_\Gamma(u)) + b_\Gamma \cdot \operatorname{grad}_\Gamma(u) - a_\Omega \cdot \operatorname{grad}(u) \cdot \nu + c_\Gamma u + g(u), && t > 0, x \in \partial\Omega,\\
&u = u_0, && t = 0, x \in \overline{\Omega},
\end{aligned}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded and connected open set with C^∞ boundary $\Gamma = \partial\Omega$. Our main goal is to show that the sectorial operator associated with the above problem has a bounded H_∞ functional calculus and, for this purpose, we will utilize tools from the Boutet de Monvel calculus, as well as concepts from Functional Analysis.

Sectorial operators generated by global pseudodifferential operators on Lp and continuous functions

Pedro Tavares Paes Lopes, Weymar Andres Astaiza Sulez
Universidade de São Paulo, Brazil

We show that, under a suitable parameter-ellipticity condition, global pseudodifferential operators with vector-valued symbols belonging to classes introduced by Nicola and Rodino generate sectorial operators on L^p spaces, including the limiting cases $p = 1$ and $p = \infty$, as well as on spaces of continuous functions. We also discuss some possible applications of this result.

On norm-attaining positive operators between Banach lattices

Vinícius Miranda, José Lucas P. Luiz
Universidade de São Paulo, Brazil

In this talk, we present recent results considering the norm-attainment of positive operators between Banach lattices. By considering an absolute version of James boundaries, we prove that: If E is a reflexive Banach lattice whose order is given by a basis and F is a Dedekind complete Banach lattice, then every positive operator from E to F is compact if and only if every positive operator from E to F attains its norm. An analogue result considering that E is reflexive and the order in F is continuous and given by a basis was proven.

A function space approach to primes and zeta-zeros

Waleed Noor
Universidade Estadual de Campinas, Brazil

The Nyman-Beurling criterion for the Riemann hypothesis (RH) is a classical Hilbert space reformulation of RH from the 1950s. In this talk we describe how translating these results in the language of Hilbert spaces of analytic functions provides a new path to the study of the Riemann ζ-function and the distribution of primes. This new approach allows for the employment of recent tools from operator theory, complex analysis and linear dynamics to arrive at some interesting conclusions regarding the zero-free regions of the Riemann ζ-function and the distribution of primes.

Dynamical Systems via Ordinary Differential Equations

Organizer: Alex Carlucci Rezende (UFSCar/Brazil) & Márcio Ricardo Alves Gouveia (IBILCE UNESP/Brazil)

Periodic Orbits, Invariant Tori and Hidden Attractors in Nonequilibrium Differential Systems

Alisson de Carvalho Reinol, Marcelo Messias
Universidade Tecnológica Federal do Paraná, Brazil

The study of chaotic dynamics has traditionally focused on attractors organized around unstable equilibrium points. However, the recent discovery of "hidden attractors" in differential systems without equilibrium points challenges this classical intuition. This work presents an investigation into the dynamics of a class of three-dimensional quadratic systems with no equilibrium points. We address the fundamental question: without a fixed point to stretch and fold the phase space, what is the skeleton organizing the chaotic dynamics? To this end, we employ Averaging Theory to prove the bifurcation of a linearly stable periodic orbit. Furthermore, we verify the non-degeneracy conditions required by the KAM Theorem for the reduced averaged model to provide analytical evidence for the existence of a family of nested invariant tori surrounding this periodic orbit. Our results show that the route to chaos in these systems is driven by the progressive breakdown of these invariant tori, which is confirmed by numerical studies.

Singularly perturbed saddle boundary equilibrium of planar piecewise smooth vector fields

Bruno de Souza Rangel, Tiago Carvalho
Universidade Federal de São Carlos, Brazil

In this work, we investigate planar piecewise smooth vector fields defined on two distinct regions of the phase space. In each region, the dynamics are governed by a smooth vector field, while the separating boundary is given by a switching manifold represented by a curve passing through the origin. At this singular point, the first smooth vector field exhibits a fold, whereas the second possesses a classical saddle equilibrium. Since this equilibrium lies on the switching manifold, it is referred to as a boundary equilibrium. To analyze this configuration, we apply the Sotomayor–Teixeira regularization method, which provides a smooth approximation of the original discontinuous system. Within this framework, a singular perturbation approach is employed, leading to the identification of a non-normally hyperbolic curve. In order to achieve a complete understanding of the dynamics in a curve, a sequence of blow-up transformations is performed, allowing for a desingularization of the system and a detailed characterization of the local behavior.

Canard Cycles in a Discontinuous Van der Pol Oscillator: Regularization and Persistence

Clayton Eduardo Lente da Silva, Márcio Ricardo Alves Gouveia
Universidade Federal de Rondonópolis, Brazil

This work analyzes the phenomenon of canard cycles in a discontinuous variant of the Van der Pol oscillator, described by a second-order differential equation. The discontinuity, introduced by a sign function in the

damping term, models physical systems with asymmetric behavior – such as circuits with diodes, mechanical systems with dry friction, or biological models with directional responses. Our goal is to demonstrate the realizability and persistence, via fine-tuning of parameters, of the various canard cycles numerically observed in the literature. To this end, we adopt a geometric approach based on regularization of the discontinuity. We argue that previous analyses, which did not employ regularization, are insufficient to establish the robust persistence of certain canards. The regularized framework extends the effective parameter space, revealing richer and previously unforeseen solution regimes. We investigate in detail the canard cycles in the regularized system for exponentially small variations of the bifurcation parameter, and study the double limit where the regularization parameters and the singular parameter tend to zero, thereby recovering the limit periodic sets of the original discontinuous system. The work consolidates and extends geometric techniques for the rigorous analysis of canards in discontinuous systems, opening new avenues for the dynamic interpretation of such models.

Number of crossing limit cycles in planar piecewise polynomial vector fields

Douglas Duarte Novaes

Universidade Estadual de Campinas, Brazil

Motivated by the classical Hilbert's Sixteenth Problem, we extend some main developments obtained for Hilbert's number in the polynomial setting to the piecewise polynomial context. Specifically, we study the growth of the maximum number of crossing limit cycles in planar piecewise polynomial vector fields of degree n, denoted by $H_c(n)$. The best previously known general lower bound is $H_c(n) > 2(n-1)$. In this work, we show that $H_c(n)$ grows at least as fast as $n^2/4$. Furthermore, we prove that $H_c(n)$ is strictly increasing whenever it is finite, and that in such cases this maximum can be realized by piecewise polynomial systems whose crossing limit cycles are all hyperbolic.

Limit Cycles in Continuous–Discontinuous Piecewise Systems in the plane

Ingrid Sofia Meza Sarmiento

Universidade Federal de Itajubá, Brazil

These last decades piecewise differential systems have been studied intensively, mainly due to their applications. Inside the study of the dynamcis of these differential systems the limit cycles, i.e. the isolated periodic orbits inside the set of all periodic orbits of the system, play a main role.

In this work we consider continuous-discontinuous piecewise differential systems in the plane $\mathbb{R}^2$ separated by the analytic curve $y = \sin x$ and formed by two arbitrary linear Hamiltonian systems. Continuous in the sense that the first component of the two Hamiltonian systems coincide on the curve $y = \sin x$, and discontinuous in the sense that the second component of the two Hamiltonian systems are distinct on the curve $y = \sin x$. We prove that such piecewise differential systems can have four limit cycles. In fact, we solve the extension of the 16th Hilbert problem to our class of continuous-discontinuous piecewise differential systems, i.e. for this class of differential systems we provide an upper bound for their maximum number of limit cycles that these differential systems can exhibit. Moreover, we prove that this upper bound is reached.

Secular evolution of a rigid body with fluid-filled cavity in the two-body problem

Lucas Ruiz dos Santos

Universidade Federal de Itajubá, Brazil

We modify a three-dimensional Hamiltonian model of a celestial body composed of a rigid mantle and a fluid-filled cavity by incorporating core-mantle viscous friction. We study the secular rotational evolution of the system and apply singular perturbation theory to derive the averaged dynamics of the mantle's spin under this viscous coupling. We analyze the stability of the equilibria in the secular system across the full range of parameters.

Limit cycles in a class of piecewise polynomial differential systems having the unit circle as their switching manifold

Luiz Fernando Gonçalves, Ronaldo Garcia, Bruno Freitas

Universidade Federal de Goiás, Brazil

We investigate the existence of limit cycles in a class of planar piecewise smooth differential systems having the unit circle as their switching manifold. The vector field inside the circle is assumed to be linear and Hamiltonian, while the vector field outside is given by $\dot{z} = z^2$. We provide an upper bound for the number of crossing limit cycles such systems can possess, as well as for some of their perturbations.

Hyperbolicity of renormalization for dissipative gap mappings

Márcio Gouveia, Trevor Clark

Universidade Estadual Paulista, Brazil

In this talk, we study the dynamics of gap mappings, which are discontinuous interval mappings with two strictly increasing branches that have a gap between their ranges. They are one-dimensional dynamical systems, which arise in the study of certain higher dimensional flows, for example the Lorenz flow and the Cherry flow. We will present some results such as hyperbolicity of renormalization acting on C^3 dissipative gap mappings, and show that the topological conjugacy classes of infinitely renormalizable gap mappings are C^1 manifolds.

Selfconsistent transfer operator for systems coupled on heterogeneous networks

Matteo Tanzi, Herbert MC Maquera, Tiago Pereira

King's College London, England

We study the dynamics of large heterogeneous network dynamical systems composed of nonlocally coupled maps. These systems are modeled using graphons, describing infinite limits of dense graphs, allowing for a rigorous analysis as the network size tends to infinity. We construct suitable strong and weak functional spaces to formulate a fixed-point problem for the infinite-dimensional system and prove the existence of solutions under mild conditions. We introduced a new operator framework for infinite-dimensional dynamical systems with heterogeneous interactions. Our results establish convergence of the finite-size system to the graphon limit in a suitable norm. This work combines operator theory and graph limits tools to offer a framework for understanding emergent behaviors in complex networks.

A singularly perturbed approach to a class of non-smooth vector fields

Rodrigo Donizete Euzébio, Rony Cristiano, Richard McGehee, Adimar Moretti Jr
Universidade Federal de Goiás, Brazil

In this work, we address a class of non-smooth vector fields defined on a closed region of the plane, which are non-smooth along two disjoint straight lines and singularly perturbed. We first provide an approximation of the considered differential system such that the phase portrait becomes invariant and the system becomes smooth. Moreover, the proposed approximation extends the critical manifold of the non-smooth system so that it becomes smooth and asymptotic to the former boundaries. Within this framework, we are able to apply classical results from Fenichel theory to study the dynamics of the approximating differential systems. The results establish the shape of the new critical manifold and ensure the existence of a periodic orbit arising after a canard explosion phenomenon. They also compare the trajectories of the obtained smooth systems with those of the original non-smooth system and prove that they are equivalent on compact portions of the phase portrait.

Bifurcations of piecewise smooth Lotka-Volterra systems with intermittent discontinuities

Tiago Carvalho
Universidade Estadual Paulista, Brazil

In this talk we will analyze the bifurcation scenario of 2D piecewise smooth Lotka-Volterra systems $Z = (X_1, X_2)$ where the smooth vector field X_1 is imposed when $y \leq \alpha_0$ with y downstairs. Also, X_1 is suspended when $y \geq \alpha_1$ with y upstairs. In the switching region we consider the Filippov's rules. This protocol can be applied, for example, in a disease treatment where the treatment is imposed when the quantity of health cells is small than a number α_0 and this intermittent treatment is suspended when the quantity of health cells is bigger than a number $\alpha_1 > \alpha_0$. We will search for bifurcations involving equilibria, limit cycles, kinds of basin of attraction and other minimal sets.

Invariant Measures for Generalizations of Viana Maps with Higher Order Critical Set

Vanderlei Horita
Universidade Estadual Paulista, Brazil

In this talk, we introduce skew-products of smooth functions having a unique critical point of degree greater than 2 over a strongly expanding map of the circle and prove that these systems admit two positive Lyapunov exponents. This extends an analogous result of Viana who considered, in the seminal paper (Viana in Inst Hautes Études Sci Publ Math 85:63–96, 1997), the quadratic case. For these systems we prove existence and uniqueness of absolutely continuous invariant measures. As a consequence of our approach, we obtain super-polynomial decay of correlations.

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Poster Session

Poster

Organizer: Everaldo de Mello Bonotto

Classification of Geometric and Algebraic Configurations of Invariant Parabolas and Lines in Quadratic Differential Systems

Alex Carlucci Rezende, Regilene Oliveira, Dana Schlomiuk, Nicolae Vulpe
Universidade Federal de São Carlos, Brazil

We study a family of planar quadratic differential systems exhibiting three distinct infinite singularities and admitting invariant parabolas and lines. Through a combined geometric and algebraic analysis, based on affine transformations and affine-invariant polynomials, we provide a complete classification of all admissible configurations of these invariant curves. Our results yield an algorithmic framework that identifies 144 distinct configuration types, offering new insights into the geometric structure, symmetry properties, and integrability of such quadratic systems.

Dulac functions in the study of monodromic singularities

Ana Livia Rodero
Universidade de São Paulo, Brazil

In this work, we show how to construct, under certain assumptions, a Dulac function in a neighborhood (which can be punctured) of a monodromic singularity of a real analytic planar autonomous system. We use the existence of a real analytic invariant curve passing through the singularity to identify candidates for a Dulac function, and we apply these results to study a degenerate polynomial monodromic family.

The presented results are part of [1], developed jointly with Prof. Isaac A. García and Prof. Jaume Giné, both from Universitat de Lleida.

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References

[1] García I.A., Giné J., Rodero A.L., *Dulac functions and monodromic singularities*, J. Math. Anal. Appl. 547 (2025) 129309.
https://doi.org/10.1016/j.jmaa.2025.129309

The method of characteristics for first-order PDE

Ana Paula Schramm Steuernagel, Jan Metzger
Universidade Estadual de Campinas, Brazil

This research presents the method of characteristics for solving first-order partial differential equations (PDE), with a focus on nonlinear equations. The reduction of a partial differential equation to a system of ordinary differential equations provides a clear geometric interpretation of how information propagates from the initial data. The central result, the Local Existence Theorem, guarantees the existence of a local C^2 solution to the initial value problem under suitable hypotheses.

A study on solutions of measure neutral functional differential equations with state-dependent delays

André Luis Martins Tomaz da Silva, Jaqueline Godoy Mesquita, Renato Huzak
Universidade de Brasília, Brazil

Our objective is to study the class of equations defined as follows

$$\begin{cases} x(t) = x(t_0) + N(t)x_{\rho(t,x_t)} - N(t_0)x_{\rho(t_0,x_{t_0})} + \int_{t_0}^{t} f(x_{\rho(s,x_s)}, s)dg(s) \\ x_{t_0} = \phi, \end{cases} \tag{4}$$

where $\sigma > 0$, $t_0 \in \mathbb{R}$, $g\colon [t_0, t_0+\sigma] \to \mathbb{R}$ is a non-decreasing and left-continuous function, $\mathcal{B} \subset G((-\infty, 0], \mathbb{R}^n)$ is a proper Banach space, $\phi \in \mathcal{B}$, $x\colon (-\infty, t_0+\sigma] \to \mathbb{R}^n$, $\rho\colon \mathcal{B} \times [t_0, t_0+\sigma] \to \mathbb{R}$, the symbol $x_{\rho(x_s,s)}$ denotes the function $x_{\rho(x_s,s)}(\theta) = x(\theta + \rho(x_s, s))$ defined on $(-\infty, 0]$, $f\colon \mathcal{B} \times [t_0, t_0+\sigma] \to \mathbb{R}^n$ are functions. Our study aims to show that the solutions of (4) can be related to the solutions of generalized ODEs.

Dynamics for composition operators on weighted Bergman spaces

Artur de Aquino Blois, Osmar Rogerio Reis Severiano
Universidade Estadual de Campinas, Brazil

Linear dynamics has been a very active area of research in the past few years, more specifically the dynamics for composition operators which has intrinsic connections to topological dynamics. In this work we aimed to completely characterize dynamical properties for affine composition operators on the Hilbert space $\mathcal{A}^2_\alpha(\mathbb{C}_+)$.

Normalized solutions for a nonlinear Schrödinger equations with potential and sum of nonlinearities

Carolina Santana Tomaz, Olimpio Hiroshi Miyagaki
Universidade Federal de São Carlos, Brazil

In this work, we are interested in presenting the existence of normalized solutions to the Schrödinger equation

$$-\Delta u + V(x)u + \lambda u = |u|^{2^*-2}u + \beta|u|^{p-2}u, \quad \text{in } \mathbb{R}^N$$

where $N \geq 3$, V is a fixed potential and $p \in (2, 2^*)$. The existence of a solution $(u, \lambda) \in H^1(\mathbb{R}^N) \times \mathbb{R}^+$ with a prescribed norm will be ensured under various technical assumptions on the potential $V\colon \mathbb{R}^N \to \mathbb{R}$. Such solutions will be obtained as a critical point of a functional constrained to the sphere S_ρ in L^2 and λ will be a Lagrange multiplier. The results are proved using Variational Methods.

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Existence of pullback attractors for a semilinear parabolic equation on time-varying domains

Daniel Alberto Morales Ramirez, Gleiciane da Silva Aragão
Universidade de São Paulo, Brazil

Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be a nonempty bounded open set with C^2 boundary $\partial\Omega$. We consider the function $r : \mathbb{R} \times \overline{\Omega} \to \mathbb{R}^n$ such that $r \in C^1(\mathbb{R} \times \overline{\Omega}, \mathbb{R}^n)$ and $r(t, \cdot) : \overline{\Omega} \to \overline{\Omega}_t$ is a C^2-diffeomorphism, for all $t \in \mathbb{R}$, with $\Omega_t = r(t, \Omega)$ (see Figure 1). For $\tau \in \mathbb{R}$, we define

$$Q_\tau = \bigcup_{t\in(\tau,\infty)} \{t\} \times \Omega_t \quad \text{and} \quad \Sigma_\tau = \bigcup_{t\in(\tau,\infty)} \{t\} \times \partial\Omega_t.$$

We are interested in studying the existence of pullback attractors for the following non-autonomous semilinear parabolic problem with nonlinear Neumann boundary conditions

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) - \Delta u(t,x) + \beta u(t,x) = f(t,u), & (t,x) \in Q_\tau \\ \dfrac{\partial u}{\partial n_t}(t,x) = g(t,u), & (t,x) \in \Sigma_\tau \\ u(\tau,x) = u_\tau(x), & x \in \Omega_\tau \end{cases} \tag{5}$$

where $\beta > 0$, $n_t(x)$ is the unit outward normal vector at $x \in \partial\Omega_t$, $u_\tau : \Omega_\tau \to \mathbb{R}$ and f, $g : \mathbb{R}^2 \to \mathbb{R}$ are nonlinear functions.

Initially, we apply a coordinate transformation technique of [5] to rewrite the original problem (5) as an auxiliary non-autonomous problem on the fixed domain Ω. Then, we show the existence and uniqueness of solutions to this problem in a fractional power space, whose properties are studied in depth in [6]. Using the Moser-Alikakos iteration technique, given in [1], and based on the work [3], we show that the solution of the problem in the fixed domain possesses exponential decay.

In particular, with the exponential estimate of the solution, we apply the results from [4] to prove that the pullback attractors exist in some fractional power space. This work extends the results obtained in [2] to the case of nonlinear Neumann boundary conditions.

References

[1] N. D. Alikakos, An application of the invariance principle to reaction-diffusion equations, Journal of Differential Equations 33(1979) 201-225.

[2] G. S. Aragão, F. D. M. Bezerra, and L. G. Mendonça, Pullback dynamics for a semilinear parabolic equation with homogeneous Neumann boundary conditions and time-varying domains, Submitted for publication (2024).

[3] M. Belluzi, T. Caraballo, M. J. D. Nascimento, and K. Schiabel, Smoothing effect and asymptotic dynamics of nonautonomous parabolic equations with time-dependent linear operators, Journal of Differential Equations 314(2022), 808-849.

[4] A. Carvalho, J. A. Langa, and J. Robinson, Attractors for infinite-dimensional non-autonomous dynamical systems, Springer, Berlin, 2013.

[5] P. E. Kloeden, J. Real, C. Sun, Pullback attractors for a semilinear heat equation on time-varying domains, Journal of Differential Equations 246(2009), 4702-4730.

[6] S. M. Oliva, A. L. Pereira, Attractors for parabolic problems with nonlinear boundary conditions in fractional power space, Dynamics of Continuous, Discrete and Impulsive System 9(2002), 551-562.

Existence and Uniqueness Theorems for Caputo Fractional Differential Equations

Débora Vieira Araújo Damasceno, Ginnara Mexia Souto

Universidade Federal do Espírito Santo, Brazil

Fractional calculus extends the operations of differentiation and integration to non-integer orders, allowing for the modeling of phenomena with memory and non-locality in various areas such as physics and engineering.

Within this theory, fundamental operators stand out: the Riemann-Liouville fractional integral operator J^α, defined for $\alpha > 0$ by $J_a^\alpha f(x) := \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt$, and the Riemann-Liouville fractional differential operator D^α, given by $D_a^\alpha f := D^m J_a^{m-\alpha} f$ with $m \in \mathbb{N}$, where $m-1 \leq \alpha < m$. Among fractional operators, the Caputo derivative is notable because it allows initial conditions in terms of integer-order derivatives, making it suitable for initial value problems. This work presents the main existence and uniqueness results for differential equations involving the Caputo derivative, establishing sufficient conditions to guarantee unique solutions using tools from Analysis.

On a Hamiltonian System in $\mathbb{R}^N$ Involving the Grushin Operator

Diego Jonathan Bezerra Silva, Claudianor Oliveira Alves

Universidade Federal de Campina Grande, Brazil

In this work, we present some recent results involving the Grushin operator and Hamiltonian Systems. We consider the following class of problems:

$$\begin{cases} -\Delta_\gamma u + u = W_2(z)|v|^{p-1}v & \text{in} \quad \mathbb{R}^N, \\ -\Delta_\gamma v + v = W_1(z)|u|^{q-1}u & \text{in} \quad \mathbb{R}^N \\ u > 0,\ v > 0, \text{ a.e} \quad \text{in} \quad \mathbb{R}^N \end{cases} \tag{$\mathcal{P}_W$}$$

where W_1 and W_2 are continuous functions with a positive infimum that satisfy certain technical conditions. We employ an approach based on [1], introducing the necessary adaptations due to the presence of the Grushin operator. The dual method combined with the Mountain Pass Theorem is used to obtain a nontrivial positive solution in both the periodic and asymptotically periodic cases. To apply this method, a thorough understanding of the operator $(-\Delta_\gamma + I)^{-1} : L^p(\mathbb{R}^N) \to \mathcal{W}_\gamma^{1,p}(\mathbb{R}^N)$ and an Agmon–Douglis–Nirenberg (ADN) type result for the Grushin operator are essential. We obtain the latter based on the works [3, 4]. Furthermore, several compactness results are required, such as Lions' Lemma and a symmetric and compact version of Lions' Lemma in a specific situation, which we derive based on [2].

References

[1] Alves, C. O., Carrião, P. C., and Miyagaki, O. H., On the existence of positive solutions of a perturbed Hamiltonian system in $\mathbb{R}^N$. *Journal of Mathematical Analysis and Applications* **276** (2), 673–690, 2002.

[2] Alves, C. O. and de Holanda, A. R. F., A Berestycki–Lions type result for a class of degenerate elliptic problems involving the Grushin operator. *Proceedings of the Royal Society of Edinburgh. Section A. Mathematics* **153** (4), 1244–1271, 2023.

[3] Metafune, G., Negro, L., and Spina, C., L^p estimates for Baouendi–Grushin operators. *Pure and Applied Analysis* **2** (3), 603–625, 2020.

[4] Zhao, N., Wang, Z., Li, P., and Liu, Y., Geometric topics related to Besov type spaces on the Grushin setting. Preprint, available at https://arxiv.org/abs/2401.03409 arXiv:2401.03409 [math.AP], 2024.

Existence of positive solutions for a Kirchhoff-type elliptic problem involving a critical growth and a concave-convex nonlinearity

Eduardo Dias Lima, Olímpio Hiroshi Miyagaki, Edcarlos Domingos da Silva

Universidade Federal de São Carlos, Brazil

In this work, we prove the existence and multiplicity of positive solutions for a Kirchhoff-type elliptic problem with critical growth in $\mathbb{R}^N$. More specifically, we consider the following nonlocal elliptic problem:

$$\begin{cases} -m\left(\|\nabla u\|_2^2\right)\Delta u + V(x)u = \lambda a(x)|u|^{q-2}u + \gamma b(x)|u|^{2^*-2}u \text{ in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases}$$

where the parameters $\lambda, \gamma > 0$, $1 < q < 2 < 2(\sigma+1) < 2^* := 2N/(N-2)$ and $N \geq 3$. The functions a, b, m and V satisfy some additional conditions. Hence, our main objective is to prove that the above problem has at least one ground state solution and at least one bound state solution whenever $\lambda \in (0, \lambda^*)$, for some suitable $\lambda^* > 0$. The main idea is to use the minimization method on the Nehari manifold together with the nonlinear Rayleigh quotient. In our setting, the main difficulty lies in ensuring the existence of nontrivial solutions when applying the Nehari method, taking into account the Lagrange Multipliers Theorem.

The Lyapunov constants method in piecewise systems

Gabriela Lye Watanabe

Universidade Federal de São Carlos, Brazil

The Lyapunov constants method is a classical and widely used tool for detecting the bifurcation of limit cycles from centers in dynamical systems under small perturbations. In this work, we will explain how to use the Lyapunov constants to calculate limit cycles in piecewise systems, and we will apply this method to examples on the plane.

Critical Schrödinger-Bopp-Podolsky Systems: Solutions in the semiclassical limit

Heydy M. Santos Damian, Gaetano Siciliano

Universidade Federal de São Carlos, Brazil

In this work, we study the following critical Schrödinger–Bopp–Podolsky system, in $\mathbb{R}^3$,

$$-\varepsilon^2\Delta u + V(x)u + Q(x)\phi u = h(x,u) + K(x)|u|^4 u,$$

$$-\Delta\phi + a^2\Delta^2\phi = 4\pi Q(x)u^2,$$

where the unknowns are $u, \phi : \mathbb{R}^3 \to \mathbb{R}$, and $\varepsilon, a > 0$ are arbitrary parameters. The functions V, K, Q satisfy suitable assumptions as well as the nonlinearity h which is subcritical. Using variational methods, we are able to establish several main results: For any fixed $a > 0$, we show existence of "small" solutions in the semiclassical limit, namely whenever $\varepsilon \to 0$. We give also estimates of the norm of this solutions in terms of ε. Moreover, we show also that fixed ε suitably small, when $a \to 0$ the solutions found strongly converge to solutions of the Schrödinger-Poisson system.

Existence and Uniqueness for Differential Equations in Banach Spaces with State-Dependent Delay in Unbounded Phase Spaces

José Paulo Carvalho dos Santos, Michelle Pierre
Universidade Federal de Alfenas, Brazil

In this presentation, we investigate the existence and uniqueness of local and maximal solutions for a general class of abstract differential equations with state-dependent delays in phase spaces that allow for unbounded delays described by

$$\frac{du(t)}{dt} = Au(t) + F(t, u(t - \sigma(t, u_t))), \quad t \in [0, a], \tag{6}$$

$$u_0 = \varphi \in \mathcal{B}, \tag{7}$$

where $A\colon D(A) \subset Z \to Z$ is the generator of an analytic semigroup $(\mathcal{S}(t))_{t\geq 0}$, u_t denotes the history of $u(\cdot)$ at time t (that is, $u_t(\theta) = u(t + \theta)$), which belongs to an abstract phase space $\mathcal{B}$ defined axiomatically, and $F(\cdot), \sigma(\cdot)$ are suitable continuous functions. . We develop a framework for treating state-dependent delays via unbounded delay operators defined in space os Lipschitz functions, and establish our results without assumption that the nonlinear forçant terms are locally Lipschitz. To illustrate the applicability of the theory, we include an application to partial differential equations with state-dependent delay.

Atypical bifurcation for periodic solutions of nonlinear ODEs with indefinite weight

Juan Francisco Camasca Fernández, Pierluigi Benevieri
Universidade Estadual de Campinas, Brazil

We study the periodic boundary-value problem (BVP), associated with a nonlinear second order ordinary differential equation of the form

$$u'' + cu' + a(t)g(, u) = 0, \tag{8}$$

depending on a real parameter . We suppose that $a : \boldsymbol{R} \to \boldsymbol{R}$ is a locally integrable T-periodic function, $g : \boldsymbol{R} \times \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a continuous function and c is an arbitrary real constant. We obtain a global bifurcation result for T-periodic pairs $(, u)$, such that u is a T-periodic solution of (8) for the corresponding .

Global existence and regularity results for a class quasilinear non-uniformly elliptic problems with fast diffusion

Lucas da Silva , Jefferson Abrantes dos Santos, Gustavo Ferron Madeira
Universidade Federal de Campina Grande, Brazil

In this work, we investigate the existence, nonexistence, and regularity of solutions for a class of quasilinear non-uniformly elliptic problems characterized by a fast diffusion term at infinity. We establish Schauder and Calderón-Zygmund type regularity results, providing new insights into the smoothness of weak solutions associated with the operator

$$\mathcal{L}_p u = -\operatorname{div}\Big(p|\nabla u|^{p-2} e^{|\nabla u|^p} \nabla u\Big)$$

for $p > 1$. By employing variational techniques, sub and supersolution methods, and advanced functional analysis in Orlicz-Sobolev spaces, we prove that weak solutions belong to $C^1(\Omega)$ and, under certain conditions, are strong solutions in $W^{2,2}_{\text{loc}}(\Omega)$. Moreover, we establish an optimal threshold Λ_p determining the existence and nonexistence of positive strong solutions for a class of quasilinear elliptic problems.

Radial Symmetrization in L^p and Inequalities for Convex Functionals

Luis Urbines

Universidade Estadual de Campinas, Brazil

We investigate radial symmetrization and rearrangements of nonnegative functions in L^p and discuss their applications to variational problems and elliptic partial differential equations. Starting from the distribution function φ_u and the decreasing rearrangement u^* of a function $u \in L^p(E)$, we introduce the radial symmetrization $u^\#$ associated with a measurable set $E \subset \mathbb{R}^N$, within the classical framework of rearrangement theory (cf. [1, Ch. 2], [2, Ch. 1–2], [3, Ch. 1]). We provide a precise description of the level sets of u and $u^\#$ and show that the symmetrization preserves the measure of superlevel sets.

On this basis we establish a layer-cake type inequality

$$\int_{E^\#} f(u^\#)\,dy \le \int_E f(u)\,dx$$

for every nonnegative Borel function f, which in particular yields the preservation of L^p norms under radial symmetrization. We then prove a comparison theorem for two functions u and v vanishing at infinity, involving the quantity $(u - v - \lambda)^+$ and their rearrangements, in a form analogous to the classical rearrangement inequalities (cf. [2, Ch. 2], [3, Ch. 2–3]). Combining this comparison principle with a representation formula for convex functions Ψ in terms of positive parts $(t - \lambda)^+$, we obtain

$$\int_{E^\#} \Psi(u^\# - v^\#)\,dy \le \int_E \Psi(u - v)\,dx$$

for every convex function Ψ satisfying $\Psi(0) = 0$. As a consequence, the rearrangement operator

$$T : L^p(E) \to L^p(E^\#), \qquad T(u) = u^\#,$$

is 1-Lipschitz on L^p for every $1 \le p < \infty$, that is,

$$\|u^\# - v^\#\|_{L^p(E^\#)} \le \|u - v\|_{L^p(E)}.$$

These results provide a concise functional-analytic framework for the systematic use of symmetric rearrangements in the study of minimizers and radially symmetric solutions of elliptic boundary value problems (cf. [1, 2, 3]).

References

[1] G. Leoni, *A First Course in Sobolev Spaces*, 2nd ed., Graduate Studies in Mathematics, Vol. 181, American Mathematical Society, Providence, RI, 2017.

[2] B. Kawohl, *Rearrangements and Convexity of Level Sets in PDE*, Lecture Notes in Mathematics, Vol. 1150, Springer, Berlin, 1985.

[3] S. Kesavan, *Symmetrization and Applications*, Series in Analysis, Vol. 3, World Scientific, Singapore, 2006.

Homogenization of Mixed Local-Nonlocal PDE Systems

Luiza Camile Rosa da Silva, Marcone Pereira, Julio Rossi

Universidade de São Paulo, Brazil

Our main goal is to study the homogenization arising in a mixed problem involving the Laplacian and nonlocal integral operators, with two different non-singular kernels acting on distinct domains A_n and B_n, described by the system

$$\begin{cases} f_n(x) = \Delta u_n(x) + \displaystyle\int_{B_n} J(x-y)\big(v_n(y) - u_n(x)\big)\,dy, & x \in A_n, \\ \dfrac{\partial u_n}{\partial n}(x) = 0, & x \in \partial A_n. \end{cases} \tag{9}$$

and

$$f_n(x) = \int_{B_n} G(x-y)\big(v_n(y) - v_n(x)\big)\,dy + \int_{A_n} J(x-y)\big(u_n(y) - v_n(x)\big)\,dy, \qquad x \in B_n, \tag{10}$$

where the kernels are radial probability densities and $f_n \in L^2(\Omega)$. The ambient domain Ω is partitioned into two disjoint sets A_n and B_n, where $B_n = \Omega \setminus A_n$.

We consider two different configurations of the domain Ω. In the first one, A_n is the union of finitely many periodic, disjoint balls with radius $r_n = 1/n$. In the second configuration, A_n is given by the complementary of the union of such balls.

Assuming that the characteristic functions of A_n converge weakly, as $n \to \infty$, to a constant function $0 < X < 1$, we pass to the limit in (9)–(10). In the first configuration, the homogenized equation (in terms of X) exhibits the disappearance of the local term, whereas in the second configuration the local term persists.

Global attractors for a class of wave equations of the Klein-Gordon-Zakharov type

Luíza Gomes Accarini, Marcelo J. D. Nascimento, Everaldo M. Bonotto, José A. Langa

Universidade Federal de São Carlos, Brazil

In this work, we study the long-time dynamics of an autonomous coupled Klein-Gordon-Zakharov type system defined on a bounded smooth domain. We establish global well-posedness of solutions in a suitable phase space and we proceed to prove the existence of a global attractor for the semigroup generated by the system. We also discuss some properties studied for the global attractor (regularity, upper-semicontinuity and finite fractal dimension).

Global Solvability in Gevrey Classes for Certain Partial Differential Operators on the Torus

Maria Verônica Bartmeyer, Paulo L. Dattori da Silva, Rafael B. Gonzalez

Universidade Estadual de Maringa, Brazil

The existence of periodic solutions to vector fields of tube type is a well-understood question, either on the space of smooth functions or on the space of ultradifferentiable functions. There are conditions on the coefficients of the vector field that characterize the global solvability. Moreover, if we consider partial differential operators of tube type with higher derivatives, then this question is well-understood, but only on the space of smooth periodic functions. In this work, following [BDG], we analyze the existence of periodic solutions in Gevrey classes for certain partial differential operators of tube type with higher derivatives.

References

[1] A. Bergamasco, P. Dattori da Silva and R. B. Gonzalez. *Global solvability and global hypoellipticity in Gevrey classes for vector fields on the torus* J. Diff. Eq. 264 (5) (2018), 3500-3526.

On the number of limit cycles of regularized piecewise linear vector fields

Otavio Henrique Perez

Universidade de São Paulo, Brazil

Our goal is to investigate the number of limit cycles that can arise in linear and nonlinear regularizations of piecewise linear (PWL) systems. By employing the notions of slow divergence integrals and generic breaking mechanisms, we show that it is possible to construct systems with an arbitrary finite number of hyperbolic limit cycles in two settings: nonlinear regularizations with monotonic transition functions, and linear regularizations with non-monotonic transition functions. Therefore, it remains an open problem to find the maximum number of limit cycles in regularized PWL vector fields in linear regularizations with monotonic transition functions (which agrees with the classical Sotomayor–Teixeira process). This presentation gathers results from joint works with Peter de Maesschalk and Renato Huzak. Otavio Henrique Perez is supported by Sao Paulo Research Foundation grants 2021/10198-9 and 2024/00392-0.

Algorithm for the Injectivity of Planar Polynomial Maps

Paulo Donizete Pereira Machado, Francisco Braun, Francisco Braun

Universidade Federal de São Carlos, Brazil

Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that the Jacobian $\det DF(p)$ is nonzero for all $p \in \mathbb{R}^2$. We present a characterization of the global polynomial injectivity problem in relation to the monodromy problem in the plane, and we adapt a known monodromy algorithm from the works of Algaba, García, and Reyes [2, 1], obtaining one that decides whether a given polynomial map whose Jacobian $\det DF(p)$ is nonzero in the plane is globally injective. In this work, we also show, using the algorithm, that the celebrated example presented in 1994 by S. Pinchuk [3], in which the Jacobian $\det DF(p)$ is nonzero for all $p \in \mathbb{R}^2$, is not injective; this example proved that the Real Jacobian Conjecture in the plane is false.

Therefore, deciding when F is injective is quite a difficult problem. We now have an algorithm to do this, and we have implemented it in the *Mathematica* software. Apart from difficulties related to the number of blow-ups, the algorithm works perfectly.

References

[1] A. Algaba, C. García and M. Reyes, *Characterization of a monodromic singular point of a planar vector field*, Nonlinear Anal. **74** (2011), 5402–5414.

[2] A. Algaba, C. García and M. Reyes, *A new algorithm for determining the monodromy of a planar differential system*, Appl. Math. Comput. **237** (2014), 419–429.

[3] S. Pinchuk, *A counterexample to the strong real Jacobian conjecture*, Math. Z. **217** (1994), 1–4.

Optimal Surface for a Uniform Moser–Trudinger-Type Estimate on Musielak–Orlicz Spaces in 2D

Rodrigo Marques Faustino da Silva, Jefferson Abrantes dos Santos, Uberlândio Batista Severo
Universidade Federal de Campina Grande, Brazil

In this work, we establish a Moser–Trudinger type inequality in Musielak–Orlicz spaces. For the unit ball $B_1(0) \subset \mathbb{R}^2$ and the Musielak spaces $L^{\Psi(\cdot)}(B_1(0))$ and $W^{1,\Phi(\cdot)}_{0,\mathrm{rad}}(B_1(0))$, with

$$\Psi(x,t) := e^{t^{\frac{2}{1-\alpha(x)}}}, \qquad \Phi(x,t) := \tfrac{1}{2}t^2 + t^2 \log^{\alpha(x)}(|t| + e),$$

where α is radial, continuous and Hölder continuous at the origin, we prove a continuous embedding and determine the optimal surface. Our result recovers the sharp constants of Moser and Hencl as particular cases, thus establishing a double-phase setting. Extending it to the whole space $W^{1,\Phi(\cdot)}_0(B_1(0))$ remains an open problem due to the lack of a Pólya–Szegö type inequality.

Existence and Uniqueness of Weak Solutions of elliptic PDE

Rony Pastor Hurtado
PUC-Rio, Brazil

We develop key notions concerning Sobolev spaces, including approximation theorems (which ensure the density of smooth functions), extension theorems (allowing functions to be extended beyond their original domain), and the Trace Theorem, which associates to each function in a Sobolev space a well-defined boundary value. This theorem plays a central role in understanding boundary conditions in the weak setting. In particular, the characterization of functions with zero trace allows us to interpret the standard Sobolev space as the space of Sobolev functions that vanish on the boundary. The Hilbert space structure allows the application of powerful tools from functional analysis, in particular, the Lax-Milgram Theorem, whose hypotheses are verified through the Poincaré inequality (ensuring coercivity) and continuity estimates. The Lax-Milgram theorem is used to establish the existence and uniqueness of weak solutions to second-order linear uniformly elliptic partial differential equations. This theoretical framework provides a rigorous and unified approach to solving such equations in the weak sense.

Stability Analysis of Continuous Bioprocesses of Cell Growth Under Conditions of Non-Inhibition and Inhibition by Substrate

Samuel Conceição Oliveira
Universidade Estadual Paulista, Brazil

This study analyzes the steady-state solutions for two models describing cell growth in a continuous stirred-tank bioreactor. It investigates whether such steady states can be achieved and maintained in practice, analyzing the stability of these steady states when perturbations occur in the system parameters. The study is based on linearized stability analysis, following the Lyapunov method. The stability of the nonlinear system is based on the behavior of the eigenvalues of the linearized system around the steady-state point. Two cell growth models are studied: one that does not consider substrate inhibition (Monod model) and another that does consider this inhibition (Andrews model).

Qualitative Analytic via Lyapunov Coefficients in Analytic Planar Systems

Tamires Imaculada Santos Costa

Universidade Federal de São Carlos, Brazil

The study of Dynamical Systems is fundamental in Mathematics and in various applied fields, as it describes phenomena that evolve over time through ordinary differential equations (ODEs). The Qualitative Theory of ODEs, initiated by Poincaré, seeks to understand the geometric and topological properties of solutions without resorting to exact expressions. Among its central problems are the Center-Focus Problem and Hilbert's 16th Problem, both related to the Lyapunov Coefficients, which allow us to characterize the local behavior around singular points, identify centers and foci, and analyze the Hopf bifurcation. In this work, we present basic concepts of the qualitative theory and implement the computation of the Lyapunov coefficients, exploring their connection with the emergence of limit cycles and their importance for understanding fundamental problems of nonlinear dynamics.

Global Weak Solutions for a Simplified Ferrofluid Phase-Field Model

Tenilson Neves Silva, Gabriela Planas

Universidade Estadual de Campinas, Brazil

In this work, we study a simplified model proposed by Zhang, He, and Yang [1] to describe the two-phase flow of an incompressible and viscous ferrofluid in a smooth, bounded domain $\Omega \subset \mathbb{R}^3$. The model is based on a phase-field approach and couples the Allen–Cahn equation with mass conservation, the Navier–Stokes equations, and a magnetization equation. A linear magnetization law $M \approx \chi H$ is assumed, which allows neglecting the nonlinear the terms $\mu \operatorname{curl}(M \times H)$ and $\beta M \times (M \times H)$ are neglected. Assuming that convection and reaction dominate in the domain, the term $\frac{1}{2} \operatorname{curl}(U) \times M$ in the magnetization equation is also neglected. Additionally, the effective magnetic field H is assumed to be given [2].

References

[1] Zhang, G.-D., He, X. and Yang, X. - *A Unified Framework of the SAV-ZEC Method for a Mass-Conserved Allen–Cahn Type Two-Phase Ferrofluid Flow Model*, SIAM Journal on Scientific Computing 46, 2, (2024), B77-B106.

[2] Nochetto, R.H., Salgado, A.J., Tomas, I. - *A diffuse interface model for two-phase ferrofluid flows*, Comput. Methods Appl. Mech. Engrg. 309 (2016) 497–531.

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Programme

ICMC Summer Meeting on Differential Equations - 2026 Chapter

Schedule Overview

SUNDAY 02 from 15:30 to 18:00, Registration at the ICMC Auditorium (Building 6)

	MONDAY 02	TUESDAY 03	WEDNESDAY 04
08:00-08:40	Registration		
08:40-09:00	Opening		
Auditorium	PLENARY LECTURES		
Chair	José Arrieta	Ederson Moreira dos Santos	Matthias Hieber
09:00-10:00	**Matthias Hieber**	**Cristina Trombetti**	**Filomena Pacella**
10:00-10:30		Coffee Break and Poster Session	
10:30-11:30	**Maria Joana Torres**	**Abdelhamid Meziani**	**José Arrieta**
11:30-13:30		Lunch	
Auditorium	PLENARY LECTURES		
Chair	Filomena Pacella	Maria Joana Torres	Xiaoying Han
13:30-14:30	**Bernhard Lamel**	**Felipe Linares**	**Gabriela Planas**
14:30-15:00		Coffee Break	
15:00-17:30	**Sessions 1-10**	**Sessions 1-10**	**Sessions 1-10**
		SOCIAL EVENTS	
11:30		Photo	
20:00		Conference Dinner	

Session 1 – Conservation Laws and Transport Equations
Session 2 – Domain Perturbation for PDEs and Applications
Session 3 – Recent Trends in Nonlinear PDEs and Free Boundaries Problems
Session 4 – Integral and Functional Differential Equations
Session 5 – Nonlinear Dynamical Systems
Session 6 – Dispersive Equations
Session 7 – Linear Equations
Session 8 – Elliptic Equations
Session 9 – Harmonic Analysis and Related Topics
Session 10 – Dynamical Systems via Ordinary Differential Equations

THEMATIC SESSIONS

Room 5-004 — Session 1 – Conservation Laws and Transport Equations

Chair	Gerardo Huaroto/Wladimir Neves	Luis Salvino/João Nariyoshi	Iullia Petrova/Henrique Souza
15:00-15:30	**Iullia Petrova**	**Wladimir Neves**	**Luis Lozano**
15:30-16:00	**Henrique Souza**	**Eduardo Abreu**	**Luis Salvino**
16:00-16:30	**Wanderson Lambert**	**Gerardo Huaroto**	**Jean Silva**
16:30-17:00	**João Nariyoshi**	**Robson Junior**	**Alexandre Batista**
17:00-17:30	**Henrique da Costa**		

Room 5-102 — Session 2 – Domain Perturbation for PDEs and Applications

Chair	Alessandra Verri/Jean Nakasato	Alessandra Verri/Jean Nakasato
15:00-15:30	**Priscila Leal da Silva**	**Alessandra Verri**
15:30-16:00	**Willian Tokura**	**Jean Carlo Nakasato**
16:00-16:30	**Igor Leite Freire**	**Diana Carolina Suarez Bello**
16:30-17:00	**Vinicius Tavares Azevedo**	**Wanessa Ferreira Tavares**

Room 5-003 — Session 3 – Recent Trends in Nonlinear PDEs and Free Boundaries Problems

Chair	Ginaldo de Santana Sá	João Vitor da Silva	Disson dos Prazeres
15:00-15:30	**Giane Rampasso**	**Laura Ospina**	**Junior Bessa**
15:30-16:00	**Romário Tomilhero Frias**	**Ritabrata Jana**	**Luis de Miranda**
16:00-16:30	**Leandro Tavares**	**Sergio Monari**	**Juan Pablo Apaza**
16:30-17:00	**João Vitor da Silva**	**Ahmed Mohammed**	**Ginaldo de Sanatana Sá**

Room 5-101 — Session 4 – Integral and Functional Differential Equations

Chair	Pierluigi Benevieri	Jaqueline Mesquita	Everaldo Bonotto
15:00-15:30	**Jaqueline Mesquita**	**Pierluigi Benevieri**	**Pablo Amster**
15:30-16:00	**Ricardo Torres**	**Anatoli F. Ivanov**	**Denis Fernandes da Silva**
16:00-16:30	**Christian Pöetzsche**	**Lucas Ozaki Mizuguti**	**Marcela Nascimento**
16:30-17:00	**Andréa Cristina P. Arita**	**Adriano Leandro Costa Peixoto**	**Ronaldo Murakami Filho**
17:00-17:30	**Aldo Pereira**		

Auditorium — Session 5 – Nonlinear Dynamical Systems

Chair	Estefani Moraes Moreira	Carlos Takaessu	Eduardo Gomes Tavares
15:00-15:30	**Eduardo Gomes Tavares**	**Stefanie Sonner**	**Carlos Takaessu**
15:30-16:00	**Xiaoying Han**	**Javier López-de-la-Cruz**	**Ginnara Mexia Souto**
16:00-16:30	**Tais Saito Tavares**	**Tomás Caraballo**	**Paulo Nicanor Seminario**
16:30-17:00	**Antonio Siconolfi**	**Estefani Moraes Moreira**	**Jacson Simsen**
17:00-17:30	**Sergey Tikhomirov**	**Flávio Almeida Lemos**	

Room 5-104 — Session 6 – Dispersive Equations

Chair	Argenis José Mendez García	Fábio Natali	Ademir Pazoto
15:00-15:30	**Ademir Pazoto**	**Ricardo Carlos Freire**	**Alexander Muñoz Garcia**
15:30-16:00	**Fábio Natali**	**Carlos Manuel G. Jiménez**	**Thyago Santos**
16:00-16:30	**Andrés Gerardo P. Yépez**	**Fidel Cuba Balvin**	**Argenis José Mendez García**
16:30-17:00	**Andressa Gomes**	**Mikaela Baldasso**	

Room 5-103	Session 7 – Linear Equations		
Chair	Luis Ragognette	Vinicius Novelli da Silva	Nicholas Braun Rodrigues
15:00-15:30	**Gabriel Araújo**	**Francisco Javier**	**Joel Coacalle**
15:30-16:00	**Catarina Barbosa**	**Luis Ragognette**	**Marcelo Ebert**
16:00-16:30	**Halit Sevki Aslan**	**Victor Sandrin Biliatto**	**Igor Ambo Ferra**
16:30-17:00	**Renan Medrado**		

Room 5-002	Session 8 – Elliptic Equations		
Chair	Ederson Moreira dos Santos	Eugenio Massa	Gustavo Ferron Madeira
15:00-15:30	**Carlo Nitsch**	**Cristina Tarsi**	**Olimpio Hiroshi Miyagaki**
15:30-16:00	**Kaye Silva**	**João Henrique Andrade**	**Leszek Gasinski**
16:00-16:30	**Francesco Della Pietra**	**Mayra Soares**	**João Rodrigues Santos Jr**
16:30-17:00	**Gustavo de Paula Ramos**	**Augusto César dos Reis Costa**	**João Pablo Pinheiro da Silva**
17:00-17:30	**Eduardo Böer**		

Room 5-001	Session 9 – Harmonic Analysis and Related Topics		
Chair	Guilherme Silva	Victor Julio Alves	Charles dos Santos
15:00-15:30	**Konstantin Merz**	**Pedro Lopes**	**Bruno Braga**
15:30-16:00	**Waleed Noor**	**Caio Bueno**	**Ben Hur**
16:00-16:30	**Muriel Dalcy**	**Vinicius Colferai**	**Luis Marcio Salge**

Room 4-001	Session 10 – Dynamical Systems via Ordinary Differential Equations		
Chair	Alex Carlucci Rezende	Márcio Gouveia	Ingrid Meza Sarmiento
15:00-15:30	**Clayton Eduardo Lente da Silva**	**Bruno de Souza Rangel**	**Alisson Reinol**
15:30-16:00	**Vanderlei Minori Horita**	**Luis Fernando da Silva Gouveia**	**Tiago Carvalho**
16:00-16:30	**Ingrid Meza Sarmiento**	**Rodrigo Donizete Euzébio**	**Márcio Gouveia**
16:30-17:00	**Matteo Tanzi**	**Lucas Ruiz dos Santos**	**Douglas Duarte Novaes**

Ground floor of the Library 10:00-10:30	Poster Session		
	Alex Carlucci Rezende	**Ana Paula S. Steuernagel**	**André Luis Martins T. Silva**
	Ana Livia Rodero	**Artur Blois**	**Carolina Santana Tomaz**
	Daniel Alberto Morales Ramirez	**Débora Vieira A. Damasceno**	**Heydy Melchora Santos Damian**
	Eduardo Dias Lima	**Gabriela Lye Watanabe**	**José Paulo Carvalho dos Santos**
	Lucas da Silva	**Juan Francisco Camasca Fernández**	**Maria Verônica Bartmeyer**
	Luis Urbines	**Luiza Camile Rosa da Silva**	**Paulo Donizete Pereira Machado**
	Luíza Gomes Accarini	**Otavio Henrique Perez**	**Tamires Imaculada S. Costa**
	Rodrigo M F da Silva	**Samuel Conceição Oliveira**	
	Tenilson Neves Silva	**Rony Pastor Hurtado**	

	SOCIAL EVENTS
11:30	**Photo**
20:00	**Conference Dinner**

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

List of Speakers

List of speakers **(ordered by first name)**

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2026 CHAPTER

Sponsors

The ICMC Summer Meeting on Differential Equations - 2026 Chapter are promoted by Instituto de Ciências Matemáticas e de Computação (ICMC), Pró-Reitoria de Pós-Graduação da USP, Pró-Reitoria de Cultura e Extensão Universitária da USP, and Pró-Reitoria de Pesquisa e Inovação da USP.

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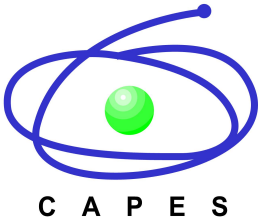

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