2024 chapter ICMC SUMMER MEETING

29-31 JANUARY | **SÃO CARLOS - SP BRAZIL**

LIST OF SESSIONS

- *Free Boundaries problems and related topics*
- *Nonlinear Dynamical Systems*
- *Conservation Laws and Transport Equations*
- *Domain perturbation for PDEs and applications*
- *Integral and Functional Differential Equations*
- *Dispersive Equations*
- *Elliptic Equations*
- *Linear Partial Differential Equations*
- *Harmonic Analysis and Related Topics*
- *Dynamical Systems via Ordinary Differential Equations*

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✓ Plenary Lectures ✓ Special Sessions ✓ Poster Sessions

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Welcome

It is a pleasure to welcome you to the *ICMC Summer Meeting on Differential Equations - 2024 Chapter* and to São Carlos. We wish you a pleasant stay and that you enjoy the meetings.

Scientific committee

Eduardo Teixeira (University of Central Florida/USA)

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Session Organizers

Jean Silva (UFMG/Brazil) & Wladimir Neves (UFRJ/Brazil): Conservation Laws and Transport Equations.

Marcia A. G. Scialom (UNICAMP/Brazil) & Mahendra Panthee (UNICAMP/Brazil): Dispersive Equations.

Marcone C. Pereira (USP/Brazil) & Alessandra Verri (UFSCar/Brazil): Domain Perturbation for PDEs and Applications.

Ederson Moreira dos Santos (USP/Brazil) & Marcos T. O. Pimenta (UNESP/Brazil): Elliptic Equations.

João Vitor da Silva (UNICAMP/Brazil) & Disson dos Prazeres (UFS/Brazil): Free Boundaries Problems and Related Topics.

Guilherme da Silva (USP/Brasil), Lucas Oliveira (UFGRS/Brazil) & Tiago Picon (USP/Brazil): Harmonic Analysis and Related Topics.

Pierluigi Benevieri (USP/Brazil), Everaldo de Mello Bonotto (USP/Brazil) & Jaqueline G. Mesquita (UnB/Brazil): Integral and Functional Differential Equations.

Patrícia Yukari Sato Rampazo (UFF/Brasil) & Nicholas Braun Rodrigues (UFSCar/Brasil): Linear Equations.

Juliana Fernandes da Silva Pimentel (UFRJ/Brazil), Maykel Boldrin Belluzi (USP/Brazil) & Phillipo Lappicy (Universidad Complutense de Madrid/Spain): Nonlinear Dynamical Systems.

Alex Carlucci Rezende (UFSCAr/Brazil) & Tiago Carvalho (USP/Brazil): Dynamical Systems via Ordinary Differential Equations.

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ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Maps

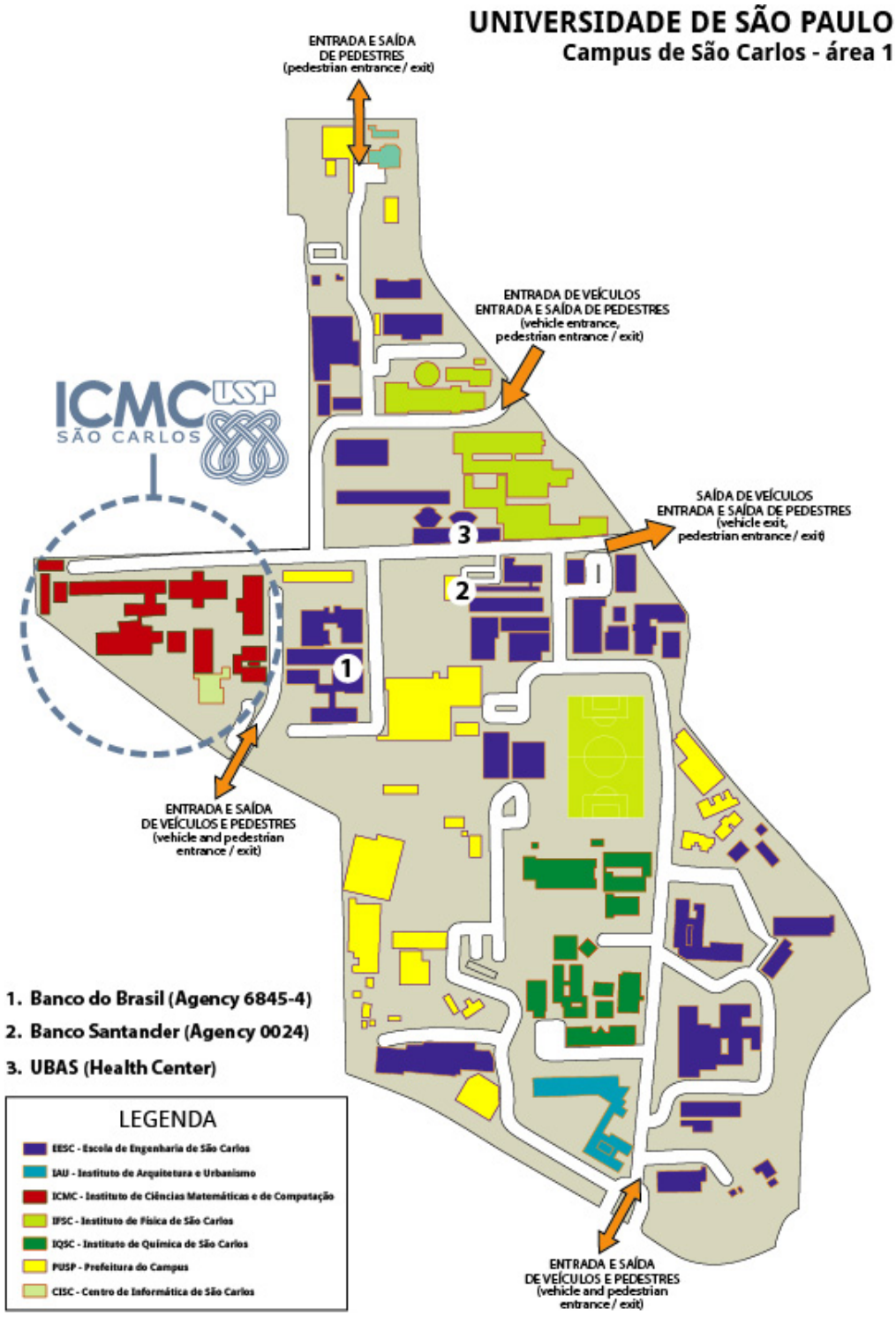

Figure 1: Campus map

Figure 2: ICMC map

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

General Information

Conference site

The meeting will take place at Buildings 4, 5 and 6. These buildings are indicated in the map on page 6.

Plenary talks and Session on Nonlinear Dynamical Systems will take place at the Auditorium (Building 6).

Session "Dynamical Systems via Ordinary Differential Equations" will take place in room 4001 (Building 4). All other sessions will take place in Building 5.

Poster session will take place at the Ground floor of the Library.

The Coffee Room is on the ground floor of the Library (1, map on page 6).

Registration

The registrations will be made in the following schedule:

Sunday, January 28^{th}: From 16:00 to 18:00 in the entrance of the ICMC Auditorium (Building 6).

Monday, January 29^{th}: From 8:00 to 8:50 in the entrance of the ICMC Auditorium (Building 6).

Those who cannot register on Sunday or Monday can also do it during the week at any time.

We will provide you with a badge at registration. Please wear your badge at the event to access the event rooms.

The Events Office will set up a help desk at the entrance of the Auditorium and will be at your disposal for any questions and information, also the Events Office (3, map on page 6) will be at your disposal.

Registration Fees

Student Fee: R$ 100,00 (reais).

Regular Fee: R$ 200,00 (reais).

Financial support

The financial support from the local organizing committee will be available on Monday, January 29, from 11:30 to 13:30 and on Tuesday, January 30, from 11:30 to 12:30, at the help desk at the entrance of the Auditorium (Building 6, map on page 6). In order to receive your support, it is mandatory to completely fill out the on-line registration form available at `summer.icmc.usp.br/user_summer/` .

Meals and refreshments

There are several restaurants near the campus. A selection of restaurants, coffee shops, and hotels next to ICMC (walk distance) can be found at `https://icmc.usp.br/e/e0f70`.

Social events

Tuesday, January 30: Photo of the meeting at 11:30 at ICMC.

Tuesday, January 30: Conference Banquet at 20:00 at Nanmi Restaurante Lounge e Bar.

Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

Money exchanges

In case you need to exchange your money, we recommend:

- Confidence Câmbio at Shopping Center Iguatemi. The working hours are from 10:00 to 20:00 (Mon-Fri) and from 10:00 to 18:00 (Sat).
- JIS Câmbio at 1931, São Sebastião Street. The working hours are from 9:30 to 17:30 (Mon-Fri).

Smoking

Smoking is prohibited inside any of the ICMC buildings also in the canteen and on the ground floor of the library.

Computer and wireless LAN use

The University provides access to wireless internet connection via **eduroam**. If you do not possess an eduroam account you can access another of our wireless connection through the following steps:

1. Enable wireless on your device.
2. Join the ICMC-GUEST wireless network.
3. Open a browser and try to visit any website.
4. You will be redirected to a login page. Enter the login and password as follows:

   User Name: summer2024

   Password: 2024summer
5. You may freely browse the internet after logging in. You may occasionally need to re-authenticate using the above procedure.

Contents

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Plenary Lectures

Plenary Lectures

Sums of squares of complex vector fields

Alberto Parmeggiani
Università di Bologna

C^∞ hypoellipticity is far from being understood, especially that of degenerate operators of the kind sums of squares of complex vector fields. In recent times, J.J.Kohn discovered complex vector fields that, despite finite-type generation of the complexified tangent space at any point, nevertheless lost many derivatives, a phenomenon also discovered by C. Parenti and myself at about the same time for operators with multiple transversal characteristics. In this talk, I will survey Kohn's results and some extensions that I have given, along with some speculations.

An overview on the Born-Infeld equation

Denis Bonheure
Université Libre de Bruxelles

I will give an overview of the recent advances made concerning the understanding of the electrostatic Born-Infeld energy functional. The associated PDE – usually called Born-Infeld Equation - is a quasilinear (and singular) elliptic equation with source (usually a measure). This fascinating equation can also be regarded as the mean curvature equation in Lorentz-Minkowski space promising a rich interplay between analysis, PDEs and geometry. Special attention will be given to the associated regularity theory and unsolved issues in this direction.

Polarization sets of involutive structures at Levi non-degenerate points

Gerardo A. Mendoza
Temple University

Let M be a manifold (or an open set in some Euclidean space). An involutive structure on M is a subbundle $\mathcal{V}$ of the complexification of the tangent bundle of M such that the Lie bracket $[X, Y]$ of any vector fields X, Y with values in $\mathcal{V}$ is again a vector field with values in $\mathcal{V}$. Associated with $\mathcal{V}$ there is a complex of first order differential operators. (An important class of example of this kind of objects are CR structures.) Fixing a metric on $\mathcal{V}$ (and a density on M) one can construct Laplacians (Hodge Laplacians) in each degree. These need not be elliptic operators, however they may be hypoelliptic. Conditions which imply hypoellipticity in various degrees and its failure in others have been known for many years (Kohn and many others). I will discuss this, but with emphasis on where the failure of hypoellipticity takes place, assuming non-degeneracy of the Levi form. Namely, in the degrees where hypoellipticity fails, only certain components can carry singularities.

The talk will begin with a brief review of the notions of wavefront set and its refinement by Dencker to polarization set. I will then explain which component will carry singularities, then show part of the proof of the statements about hypoellipticity. The involutive structure is not required to be locally integrable.

On the Radon-Carleman Problem in Uniformly Rectifiable Domains

Irina Mitrea

Temple University

This talk is focused on the Radon-Carleman Problem, dealing with computing and/or estimating the essential norm and/or the Fredholm radius of singular integral operators of double layer type associated with elliptic partial differential operators, on function spaces naturally intervening in the formulation of boundary value problems for the said operator in a given domain. The main goal is to monitor how the geometry of the domain affects the complexity of this type of study and to present a series of results in increasingly more irregular settings, culminating with that of uniformly rectifiable domains.

This is based on joint work with Dorina Mitrea and Marius Mitrea from Baylor University, which has recently appeared in volume V of our Geometric Harmonic Analysis research monograph series in Developments in Mathematics, Springer.

The Korteweg-de Vries equation over the Complex Numbers

Jerry L. Bona, Frederic Weissler

University of Illinois at Chicago

After rapidly reviewing the history of the Korteweg-deVries equation, I will discuss the issues of local and global well-posedness for complex-valued solutions on the circle. In particular, a large class of local-in time solutions develop a singularity in finite time. Hence global well-posedness can fail. In addition, local well-posedness can fail for two reasons. There exist initial values with no reasonable local-in time solution. Also, if complex-valued solutions are considered, continuous dependence fails at the zero solution. These results are of course completely at variance with the well-known results for real-valued solutions.

A dynamical system approach to a class of fully nonlinear equations and systems

Liliane Maia, Gabrielle Nornberg, Filomena Pacella

UNB - Universidade de Brasília

We will present some recent results obtained on the existence, nonexistence and classification of radial positive solutions of some weighted fully nonlinear equations and system of equations involving Pucci extremal operators. Our study is entirely based on the analysis of the dynamics induced by an autonomous quadratic system which is obtained after a suitable transformation. This method allows to treat both regular and singular solutions in a unified way, without using energy arguments. This is a work in collaboration with Gabrielle Nornberg (Universidad de Chile) and Filomena Pacella (Sapienza Università di Roma, Italy).

Differential equations in metric spaces: some perspectives and examples of application

Rosana Rodríguez-López
Universidade de Santiago de Compostela

In this talk, we focus on some aspects of the theory of differential equations in abstract metric spaces, showing some examples of application. The essence of the study is the use of a concept of a derivative for a function with values in an abstract metric space built in terms of the distance of this base space.

Panasyuk introduced metric dynamical systems through the term of approximation or quasi-differential equations (QDE), devoting the study to the case of locally compact metric spaces. Quasi-differential models were shown to be useful to explain the evolution of non-differentiable processes, in the sense that the existence of a derivative is not a requirement. The non-differentiability can be explained by several reasons, one of them could be the absence of a vector structure in the metric space. The extension by Panasyuk allows to recover classical differential equations for differentiable processes. This idea of approximation equations is related to the use of an approximation of first order of the process.

In connection with this notion, a theory of differential equations in metric spaces can be developed, and several classical methods find a suitable generalization in the context of semi-linearity, such as Euler polygonal method. The case of locally compact metric spaces is easier to handle, however, its consideration opens questions about how to deal with Banach spaces of infinite dimension and general non-locally compact metric spaces.

Some particular examples where we can find a semi-linear metric context lacking a vector structure are the set of compact real intervals (and its higher-dimensional generalizations), and the spaces of fuzzy intervals and fuzzy sets, which have been extensively used to model uncertain processes. The introduction of a general perspective where the metric selected is the only element required in order to define a derivative, and thus the concept of a solution to a differential equation, allows avoiding the use of particular notions of differences, which many times introduce restrictions that make the process of solvability difficult and subject to casuistry.

Dynamics and Wong-Zakai approximations of stochastic nonlocal PDEs with long time memory

Tomás Caraballo, Jiaohui Xu, José Valero
Universidad de Sevilla

In this talk, a combination of Galerkin's method and Dafermos' transformation is first used to prove the existence and uniqueness of solutions for a class of stochastic nonlocal PDEs with long time memory driven by additive noise. Next, the existence of tempered random attractors for such equations is established in an appropriate space for the analysis of problems with delay and memory. Eventually, the convergence of solutions of Wong-Zakai approximations and upper semicontinuity of random attractors of the approximate random system, as the step sizes of approximations approach zero, are analyzed in a detailed way.

Analysis of lattice neural models with uncertainty and time delays

Xiaoying Han, Prof. Dr. Peter E. Kloeden, Prof. Yejuan Wang
Auburn University

Lattice models arising from continuous-time artificial neural networks are introduced. One considers random weights of connections among neurons and unbounded distributed time delays, and the other considers time-dependent forcing and state-dependent nonlinear noise. For each system, the appropriate sequence space is introduced, and the lattice system is reformulated as a random equation on its corresponding space. Then the existence and uniqueness of random attractors or weak pullback mean random attractors will be discussed.

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Thematic Sessions

Conservation Laws and Transport Equations

Organizer: Jean Silva (UFMG/Brazil) & Wladimir Neves (UFRJ/Brazil)

Invariant measures for stochastic nonlinear partial differential equations in the space of almost periodic functions

Claudia Espitia, Hermano Frid, Daniel Marroquin
Unicamp

In this talk we present some results about two stochastic partial differential equations generalizing some previous results to a more general class of oscillatory solutions. More specifically, we treat stochastic conservation laws and stochastic degenerate parabolic-hyperbolic equations considering a wider notion of periodic solutions. For these equations, we study the well-posedness and the long-time behavior of almost periodic solutions under the assumption of Lipschitz continuity of the flux and the viscosity functions, and some non-degeneracy conditions. As a main objective, for each equation we show the existence and uniqueness of an invariant measure in a separable subspace of the space of Besicovitch almost periodic functions.

These results correspond to a joint work with Prof. H. Frid and Prof. D. Marroquin.

Doubly nonlinear isotropic degenerate fractional Parabolic hyperbolic equation

Gerardo Huaroto
Universidade Federal de Alagoas

We are concerned with the well-posedness theory for solutions to the initial-boundary value problems. More precisely, we study the following problem: one seeks for a function $u(t,x)$ satisfying

$$\begin{cases} \partial_t u + \nabla \mathbf{f}(u) + (-\Delta)^s_{\Omega,p} A(u) = 0 & \text{in } Q_T, \\ u|_{t=0} = u_0 & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_T, \end{cases} \tag{1}$$

where $Q_T := (0,T)\times\Omega$, for any real number $T>0$, and $\Omega \subset \mathbb{R}^n$ is a bounded open set having smooth (C^2) boundary Γ. Moreover, $\mathbf{f}:\mathbb{R}\to\mathbb{R}^n$ is called the flux function and $A:\mathbb{R}\to\mathbb{R}$ is a nondecreasing function. The initial data $u_0 \in L^\infty(\Omega)$, and the boundary condition $u_b \in L^\infty(\Gamma_T)$, where $\Gamma_T = (0,T)\times\Gamma$.

The mathematical model and analysis of the nanoparticle-stabilized foam displacement

Grigori Chapiro, Tatiana Danelon, Pavel Paz, Giulia Fritis, Luis Fernando Lozano
UFJF

This work proposes a mathematical model to study the foam displacement in porous media stabilized by nanoparticles. We consider a simplification of the Stochastic Bubble Population balance model in local equilibrium, with nanoparticle dependence inspired by the experimental data from the literature. It consists of a non-strictly hyperbolic system of conservation laws, which is solved

for the generic initial and injection conditions. We investigate the existence of a global solution as a sequence of waves following the Conservation Laws Theory. When the solution is composed of two or more waves, we present necessary and sufficient conditions to guarantee the compatibility of these wave sequences. The analytical solution for the nanoparticle-stabilized foam displacement in porous media allowed us to quantify the effect of nanoparticles on foam displacement, focusing on the breakthrough time and cumulative water production. In agreement with the literature, when only gas is injected, the breakthrough time and the water production increase with the nanoparticle concentration. Although, we also observe that the effect of nanoparticles is less pronounced for high nanoparticle concentration. Counterintuitively, during gas-water co-injection for a certain parameter range, adding nanoparticles changes the mathematical solution qualitatively, yielding a negligible effect on water production. We discuss the most favorable conditions for observing the action of nanoparticles in laboratory experiments.

Existence of Flows for generalized SQG equations with vorticity in Besov spaces

Henrique Borrin de Souza
UNICAMP

In this lecture, we shall explore an extension of the theory for existence of flow for vector fields whose derivative are a convolution of a singular kernel with a integrable function , by considering vector fields whose derivative more singular at the origin. Such vector fields naturally appear in generalized SQG equations, albeit our result does not rely on the space dimension. More precisely, we prove that if the function convolved with the kernel is in an appropriate Besov space, then we have local well-posedness for the associated flow.

H-measure and its concentration effects in different stochastic settings.

Jean Silva, Luis Fernando Salvino
Universidade Federal de Minas Gerais

In this talk, we analyse the concentration effects of H-measures generated by functions with different "self-averaging" behavior. This is a joint work with Luis Fernando Salvino (UFMG).

On the existence of source-solutions to the multi-dimensional Burgers equation

João Fernando Nariyoshi
Universidade de São Paulo

Recently, D. Serre and L. Silvestre [*Arch. Rat. Mech. Anal.* **234** (2019) 1391–1411] made a profound breakthrough in the study of the multidimensional Burgers' equation $u_t + uu_{x_1} + \cdots u^d u_{x_d} = 0$ by obtaining dispersive estimates for its entropy solutions. This enabled them to show that the multidimensional Burgers equation is well-posed if the initial data is in $L^p(\mathbb{R}^d)$, thus answering positively a conjecture of M. Crandall.

Then, D. Serre [*Arch. Rat. Mech. Anal.* **239** (2021) 95–116] investigated the existence of solutions to the multidimensional Burgers' equation if the initial data was a bounded measure, instead. This question is motivated by the description of the asymptotic behavior of solutions with

$L^1(\mathbb{R}^d)$-initial data. Nonetheless, the results therein were not entirely conclusive: Even though one could construct approximate solutions and prove their compactness, it was not established that the limit functions fitted the initial condition.

In this talk, we will explain why this is so, and we will provide some negative results regarding the existence of such solutions. In spite of this, we are still able to obtain some information on the decay of solutions.

Aprinciple of elliptic stability

Lauren Maria Mezzomo Bonaldo, Wladimir Neves, Talita Ribeiro
Universidade Federal do Rio de Janeiro

The study of systems of conservation laws in various spatial variables continues to be terra incognita to this day. However, the particular case of two spatial dimensions, in addition to being physically relevant, has recently shown to be possible to be treated mathematically. This possibility is still restricted to so-called irrotational, isentropic flows described in self-similar variables. For example, in so-called irrotational fluids, discontinuities of the "vortex sheet" type, known to be present in flows with high Reynolds numbers, are not admissible. We propose in this work to extend the general pressure law and employ the Hodge-Helmholtz decomposition to study an isentropic (possibly irrotational) flow model of gas in self-similar variables where the pseudovelocity is not necessarily potential. As a rule, this is a free boundary problem since part of the solution domain of the partial equation is elliptical (subsonic regime), and another part is hyperbolic (supersonic regime), with the boundary depending on the solution itself. The type is determined by the Mach number M, M< 1 is an elliptical region, and M> 1 is a hyperbolic region. We prove an ellipticity principle, already known for polytropic gases, for more general state laws, including Chaplygin gas, where we prove that within a parabolic-elliptical region of a sufficiently smooth solution, it must be elliptical in fact, M must be bounded above from 1 by a domain-dependent function. In particular, we will see that there are no open parabolic regions.

A strong precompactness criterion and the homogenization of conservation law non homogeneous

Luís Fernando Salvino, Jean Carlos da Silva
Universidade Federal de Minas Gerais

This study deals of a strong precompactness criterion for a sequence of bounded Young measures $(\nu_x^k)_{k\in\mathbb{N}}$ that satisfies the inequality

$$\operatorname{div}\left(\int_p^\infty sgn(\lambda-p)(f(kx,\lambda)-f(kx,p))\,d\nu_x^k(\lambda)\right)\leq 0 \text{ em } \mathcal{D}'(U),\forall p\in\mathbb{R}.$$

Furthermore, we will apply this criterion to the homogenization of a Cauchy problem for non-homogeneous scalar conservation laws.

Optimal boundary control of the isothermal semilinear Euler equation for gas dynamics on a network

Marcelo Bongarti, Michael Hintermüller

Weierstraß Institute, Berlin, Germany

The analysis and boundary optimal control of the nonlinear transport of gas on a network of pipelines is considered. The evolution of the gas distribution on a given pipe is modeled by an isothermal semilinear compressible Euler system in one space dimension. On the network, solutions satisfying (at nodes) the Kirchhoff flux continuity conditions are shown to exist in a neighborhood of an equilibrium state. The associated nonlinear optimization problem then aims at steering such dynamics to a given target distribution by means of suitable (network) boundary controls while keeping the distribution within given (state) constraints. The existence of local optimal controls is established and a corresponding Karush-Kuhn-Tucker (KKT) stationarity system with an almost surely non-singular Lagrange multiplier is derived.

Numerical solutions for a general time variable-coefficient Burgers equation

Richard De la Cruz

Universidad Pedagógica y Tecnológica de Colombia

Burgers equation, $u_t + uu_x = 0$, is a model for nonlinear wave propagation and it is one of the simplest nonlinear scalar conservation law. Physically, the Burgers equation is a model of sticky particles and means that the velocity of a particle only changes in case of collision, and when two particles collide, they stick and form a heavier cluster. The Burgers equation was originally proposed by Bateman in 1915 as a simple model for the system of conservation laws of gas dynamics and later analyzed in detail by Burgers as a model equation of strong hydrodynamic turbulence. As the time variable coefficients can provide more useful models in many complicated physical situations [1, 2, 3] as to model propagation of a long shock-wave in a two-layer shallow liquid, the purpose of this talk is to show some numerical results on the existence of Riemann solutions to the general time variable-coefficient Burgers equation, $u_t + \alpha(t)uu_x = -\sigma(t)u$ where $\alpha(t) > 0$ and $\sigma(t) \geq 0$ for all $t \geq 0$, $\alpha, \sigma \in C([0, \infty))$.

References

[1] W.P. Hong, On Bäcklund transformation for a generalized Burgers equation and solitonic solutions, Phys. Lett. A 268, 81-84 (2000).

[2] M. Wang, J. Zhang, E. Li, X. Xin, The generalized Cole-Hopf transformation to a general variable coefficient Burgers equation with linear damping term, App. Math. Lett. 105, 106299 (2020).

[3] T. Xu, C-Y. Zhang, J. Li, X-H. Meng, H-W. Zhu, B. Tian, Symbolic computation on generalized Hopf-Cole transformation for a forced Burgers equation model with variable coefficients from fluid dynamics, Wave Motion 44, 262-270 (2007).

On a Characterization of the Rellich–Kondrachov Theorem on Groups and the Bloch spectral cell equation

Vernny Ccajma, Wladimir Neves, Jean Silva

Universidade Federal do Rio de Janeiro

This Talk is concerned with the Rellich–Kondrachov Theorem on Groups. We establish some conditions which characterize in a precise manner important properties related to this theorem and

the Sobolev spaces on groups involved on it. The main motivation to study the Rellich–Kondrachov Theorem on Groups comes from the Bloch spectral cell equation, which is an eigenvalue-eigenfunction problem associated with the asymptotic limit of the anisotropic Schrödinger equation.

The Buckley-Leverett system. Revisited.

Wladimir Neves , André de Oliveira Gomes
Universidade Federal do Rio de Janeiro

In this talk, we discuss about the important issue of existence of solutions for the Buckley-Leverett system, that is to say, we consider the following system

$$\begin{cases} \partial_t u + \mathrm{div}(g(u)\,\mathbf{v}) = 0, \\ h(u)\mathbf{v} = -\nabla p, \\ \mathrm{div}(\mathbf{v}) = 0. \end{cases} \tag{2}$$

The first equation is a conservation law that expresses the mass balance for the density u of a fluid evolving according to a nonlinear continuity equation. The second equation expresses the Darcy's law, that is an empirical law which de- scribes the dynamics of the velocity field v of the flow in relation with the scalar function p, usually called pressure, exerted on it. The third and last equation is the incompressibility condition for the velocity field of the flow.

Vanishing adsorption admissibility criterion for contact discontinuities in the polymer model

Yulia Petrova, Dan Marchesin, Bradley Plohr
IMPA

In the talk we discuss the admissibility criteria for solutions to a Riemann problem of a non-strictly hyperbolic system of conservation laws modelling chemical flooding process in oil recovery. We introduce the vanishing adsorption criterion for contact discontinuities and prove that this criterion, which derives from a physical effect, justifies the admissibility criteria adopted previously by Keyfitz-Kranzer, Isaacson-Temple, and de Souza-Marchesin for models such that the fractional flow function depends monotonically on chemical concentration. Another interesting feature is that the adsorption criterion selects the undercompressive contact discontinuities required to solve the general Riemann problem in an example model with non-monotone dependence. The talk is based on joint work with D. Marchesin and B. Plohr (arxiv:2211.10326).

Domain Perturbation for PDEs and Applications

Organizer: Alessandra Verri (UFSCar/Brazil) & Marcone C. Pereira (USP/Brazil)

Spectral analysis in sheared waveguides

Alessandra Verri
UFSCar

Let $\Omega \subset \mathbb{R}^3$ be a waveguide which is obtained by translating a cross-section in a constant direction along an unbounded spatial curve. Consider $-\Delta_\Omega^D$ the Dirichlet Laplacian operator in Ω. In this talk we show that, under the condition that the tangent vector of the reference curve admits a finite limit at infinity, the essential spectrum of $-\Delta_\Omega^D$ can be found. Furthermore, sufficient conditions to ensure the existence of a non-empty discrete spectrum for $-\Delta_\Omega^D$ are presented. In particular, we show that the number of discrete eigenvalues can be arbitrarily large since the waveguide is thin enough.

Influence of the bound states in the Neumann Laplacian in a thin waveguide

Carlos Ronal Mamani Mamani, Alessandra A. Verri
Universidad Nacional de Moquegua - Perú

In this talk, we consider $-\Delta_\Omega^N$ be the Neumann Laplacian operator restricted to a twisted waveguide Ω. Our first goal is to find the effective operator when Ω is "squeezed." However, since, in this process, there are divergent eigenvalues, we consider $-\Delta_\Omega^N$ acting in specific subspaces of the initial Hilbert space. The strategy is interesting since we find different effective operators in each situation. In the case where Ω is periodic and sufficiently thin, we also obtain information regarding the absolutely continuous spectrum of $-\Delta_\Omega^N$ (restricted to such subspaces) and the existence and location of band gaps in its structure.

Contribution of Fractional Operator in Differential Equations

Gunvant Achutrao Birajdar
Institute of Chemical Technology, Mumbai

This talk will focus on applying various fraction operators in physical phenomena like subdiffusion, superdiffusion, convection, etc., with various fractional differential operators. Moreover, I provided some suitable examples of physical situations.

On thin domains with randomly oscillating boundary

Jean Carlos Nakasato, Fabio Prates Machado, Marcone Corrêa Pereira
University of Sao Paulo

In this work, we study the asymptotic behaviour of solutions of a Poisson equation in a thin domain with randomly oscillating boundary. In each case of oscillations, namely critical, subcritical and supercritical, we determine the different effective problems and rates of convergence.

Rate of convergence for reaction-diffusion equations with nonlinear boundary conditions and $\mathcal{C}^1$ variation of the domain

Leonardo Pires, Marcone C. Pereira
Universidade Estadual de Ponta Grossa

In this talk, we present the compact convergence approach to deal with the continuity of attractors of some reaction-diffusion equations under smooth perturbations of the domain subject to nonlinear Neumann boundary conditions. We define a family of invertible linear operators to compare the dynamics of perturbed and unperturbed problems in the same phase space. All continuity arising from small smooth perturbations will be estimated by a rate of convergence given by the domain variation in a $\mathcal{C}^1$ topology.

Roughness-induced effects on a reaction-diffusion problem in a thin domain

Marcone C. Pereira, Igor Pazanin, Jean Carlos Nakasato
Universidade de São Paulo

We discuss a reaction-diffusion problem in a thin domain endowed with the Robin-type boundary condition describing the reaction catalyzed by the upper wall. Motivated by the microfluidic applications, we allow the oscillating behavior of the upper boundary and analyze the resonant case where the amplitude and period of the oscillation have the same small order as the domain's thickness. Depending on the magnitude of the reaction mechanism, we rigorously derive three different asymptotic models via the unfolding operator method. In particular, we identify the critical case in which the effects of the domain's geometry and all physically relevant processes become balanced.

Spectra of Domains with Holes or of Extremal Shapes

Shuichi Jimbo
Hokkaido University

I talk about the asympttotics of the eigenvalues of several elliptic operators in domains with a thin hole or ones of extremal shape. Several decades ago Swanson gave one method to see asymptotics of eigenvalues when domains perturb regularly or singularly. By making an (potentially) apporoximate eigenfunction and using it as a test function in a weak formulation of the equation, one may see a difference of the true eigenvalue and the limit value in the limitting process of domain deformation. It looks simple and naive but it is applicable to many kind perturbation problem of operators. I deal with several spectral perturbation of several operators. I present the perturbation formula for the eigenvalues of 2nd order elliptic equation or system.

Free Boundaries Problems and Related Topics

Organizer: João Vitor da Silva (UNICAMP/Brazil) & Disson dos Prazeres (UFS/Brazil)

Regularity in Diffusion Models with Gradient Activation

Aelson Sobral (UFPB), Damião Araújo (UFPB), Eduardo Teixeira (UCF)
Universidade Federal da Paraíba

We establish sharp regularity estimates for solutions of highly degenerate fully nonlinear elliptic equations. These are free boundary models in which a nonlinear diffusion process drives the system only in the region where the gradient surpasses a given threshold. Our main result concerns the existence of a universal modulus of continuity for Du, up to the free boundary. We elucidate the practical applications of these methods, showcasing their relevance and integration with other research trends.

This is a joint work with Damião Araújo (UFPB-Brazil) and Eduardo Teixeira (UCF-USA).

A free boundary problem governed by a non-standard growth operator

Claudia Lederman (Universidad de Buenos Aires and IMAS -UBA- CONICET, Argentina), Fausto Ferrari (University of Bologna, Italy)
Universidad de Buenos Aires, Argentina

We will present recent results on a free boundary problem for an operator with non-standard growth.

The study of PDE's of this type is motivated by their application in the modelling of different phenomena, such as non-Newtonian fluids, non-linear elasticity and image reconstruction.

The nonlinear degenerate/singular nature of the equations we consider leads to challenging difficulties that will be discussed in this talk.

Global regularity for a class of fully nonlinear PDEs with unbalanced variable degeneracy

Elzon Cézar Bezerra Júnior, João Vítor da Silva, Giane Casari Rampasso, Gleydson Chaves Ricarte
Universidade Federal do Cariri

We establish the existence and sharp global regularity results ($C^{0,\gamma}$,$C^{0,1}$ and $C^{1,\alpha}$ estimates) for a class of fully nonlinear elliptic PDEs with unbalanced variable degeneracy. The model case in question is given by

$$\begin{cases} \left[|Du|^{p(x)} + \mathfrak{a}(x)|Du|^{q(x)}\right] \mathcal{M}^{+}_{\lambda,\Lambda}(D^2u) &= f(x) \quad \text{in} \quad \Omega \\ u(x) &= g(x) \quad \text{on} \quad \partial\Omega. \end{cases}$$

for a bounded, regular and open set $\Omega \subset \mathbb{R}^n$, and appropriate continuous data $p(\cdot), q(\cdot)$, $f(\cdot)$ and $g(\cdot)$. Such sharp regularity estimates generalize and improve, to some extent, earlier ones via geometric treatments. Our results are consequences of geometric tangential methods and make use of compactness, localized oscillating and scaling techniques. In the end, our findings are applied in the study of a wide class of nonlinear models and free boundary problems.

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Regularity theory for solutions to a class of doubly nonlinear evolution PDEs

Giane Casari Rampasso, Pêdra D.S. Andrade, João Vitor da Silva, Makson S. Santos
Universidade Federal de Itajubá

In this work, we investigate the interior regularity of solutions to a class of evolution equations governed by doubly nonlinear operators. More precisely, we preduce sharp regularity estimates in Hölder spaces for weak solutions of the called inhomogeneous (m, p)-Laplacian equation.

Almost minimizers of a Bernoulli problem governed by a nonstandard growth operator

Hernán Vivas, João Da Silva, Analía Silva
Universidad Nacional de Mar del Plata

We establish optimal Lipschitz regularity for non-negative almost minimizers of the one-phase Bernoulli-type functional

$$\mathcal{J}_{\mathrm{G}}(u,\Omega) := \int_{\Omega} \left(\mathrm{G}(|\nabla u|) + \chi_{\{u>0\}}\right)\, dx$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain and G is a Young function with $\mathrm{G}' = g$ satisfying the Lieberman's classical conditions.

Uniform Lipschitz estimates on the interface for solutions to two-phase free boundary problems governed by non-uniformly elliptic operator

Jefferson Abrantes dos Santos, Sergio Henrique Monari Soares
Universidade Federal de Campina Grande

We will deal with a two-phase free boundary problem involving a degenerate non-uniformly elliptic operator with Φ-Laplacian type growth. We prove Lipschitz regularity for minimizers by controlling the negative phase density along the free boundary. It is also shown that the region where the local Lipschitz regularity fails is contained in the contact set between the positive and negative free boundaries and there the negative phase is cusp free. As an application, we prove Lipschitz regularity for a two-phase free boundary problem driven by the infinity Laplacian operator by studying the behavior of an ℓ-two-phase free boundary problem as $\ell \to 0^+$.

Optimal regularity for the obstacle problem for p-Laplacian type equations

João Vitor da Silva, Elzon Cézar Bezerra Júnior, Romário Tomilhero Frias
UNICAMP - UNIVERSIDADE ESTADUAL DE CAMPINAS

In this Lecture we show existence/uniqueness of weak solutions of an obstacle problem for a quasi-linear operator with unbounded source terms. In our results, we obtain sharp gradient estimates, namely, $C^{1,\alpha}_{loc}(B_1)$ for the solution to an explicit and universal regularity exponent. Our results are relevant even for the simplest model case governed by the $p-$Laplacian with Hölder continuous coefficients

$$\begin{cases} \text{div}\left(|\nabla u|^{p-2}\mathfrak{A}(x)\nabla u\right) &= f(x) & \text{in} & \{u>\varphi\}\cap B_1 \\ \text{div}\left(|\nabla u|^{p-2}\mathfrak{A}(x)\nabla u\right) &\leq f(x) & \text{in} & B_1 \\ u(x) &\geq \varphi(x) & \text{in} & B_1 \\ u(x) &= 0 & \text{on} & \partial B_1, \end{cases}$$

where $f \in L^q(\Omega)$ for $q > n$ and $q \geq \frac{p}{p-1}$ $(1 < p < \infty)$, $\mathfrak{A} \in C^{0,\sigma}(\Omega, \mathbb{R}^{n\times n})$ (for some $\sigma \in (0,1]$) with $\mathfrak{A}$ a $(\lambda,\Lambda)-$uniformly elliptic matrix, and $\varphi \in C^{1,\beta}(\Omega)\cap \mathfrak{X}_{p,q}$, for some $\beta \in (0,1]$ where

$$\mathfrak{X}_{p,q} := \left\{ v \in W^{1,p}(\Omega); \quad \text{div } \mathfrak{a}(x,\nabla v) \in L^q(\Omega) \right\}.$$

For some specific scenarios, we show the non-degeneracy of solutions, which provides crucial information about the free boundary of solutions. Our regularity estimates improve and extend, to a certain extent, results previously obtained for the obstacle problem governed by the $p-$Laplacian with bounded source term (cf. [1] and [3]). Furthermore, we gave special emphasis to the study of the linear and non-homogeneous case, i.e., $p = 2$ and $f \neq 0$, which was not available in the literature and it plays a decisive role in analysing the non-linear case (cf. [2]).

This is a joint work with Elzon C.B. Júnior (UFCA) and Romário T. Frias (Unicamp).

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Hölder Regularity Theory for Nonloncal Operators

Juan Pablo Cabeza
Universidad de Chile

In this talk we discuss some regularity results for viscosity solutions of nonlocal uniformly elliptic second order differential equations of gradient terms and fully nonlinear type.

Regularity for an optimal partition problem with volume constraint

Makson Santos, Pêdra Andrade, Ederson Moreira dos Santos, Hugo Tavares
Instituto Superior Técnico - Universidade de Lisboa

We study the existence and regularity of optimal partitions for a problem with volume and inclusion constraints. In particular, we prove that an optimal partition is connected and the eigenfunction

associated with each set is locally Lipschitz continuous, which implies that the optimal sets are at least open sets. We show that there is a variational formulation to our problem that does not involve subsets, only functions, and we prove the desired properties for the minimizes. In addition, we show some qualitative properties for the free boundaries of such partitions.

Almgren-type monotonicity formulas

Mariana Smit Vega Garcia, Blair Davey
Western Washington University

In this talk, we will explore the celebrated Almgren's monotonicity formula. This beautiful result with far-reaching consequences states that if u is harmonic in the unit ball, then a certain frequency function $N(r)$ is non-decreasing. Moreover, $N(r) = k$ for all $r < 1$ if, and only if, u is homogeneous of degree k. We will then discuss some of the many applications of this formula, and recent developments connected to it. This is joint work with Blair Davey.

Calderón-Zygmund estimates for the fully nonlinear obstacle problem with super-linear Hamiltonian terms and unbounded ingredients

Romário Tomilhero Frias, João Vitor da Silva
UNICAMP

In this talk, we demonstrate the existence and uniqueness of L^p-viscosity solutions for a fully nonlinear obstacle problem with super-linear (and sub-quadratic) gradient growth, unbounded ingredients, and an irregular obstacle. Furthermore, we present Calderón-Zygmund estimates, specifically $W^{2,p}_{\text{loc}}$ regularity estimates (with $\frac{n}{2} < p < \infty$) for such a solution.

Integral and Functional Differential Equations

Organizer: Everaldo de Mello Bonotto (USP/Brazil), Jaqueline G. Mesquita (UnB/Brazil) & Pierluigi Benevieri (USP/Brazil)

Existence of positive solution for a second-order nonlinear problem with mixed conditions

Adriano Leandro da Costa Peixoto
Universidade de São Paulo

In this work, we present the existence of a positive solution to a second-order nonlinear problem with mixed boundary conditions. The proofs of the main results are based on the Mawhin's coincidence degree.

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Mild solutions to Cauchy problems using Henstock Kurzweil integrals

Aldo Pereira, Edgardo Alvarez, Rogelio Grau, Jaqueline G. Mesquita
Universidad de La Serena

In this work, we prove the existence and uniqueness for the solution of a fractional–order impulsive differential equation. The approach used is the concept of regulated functions and the Henstock–Kurzweil integration, on which we define a fractional calculus and the Laplace Transform.

Relations among extinction, permanence and persistence of stochastic process

Antonio Veloso, Rodolfo Collegari, Marcia Federson
Universidade de São Paulo

In a biological context, the extinction, permanence, and persistence of a species carry specific meanings, and translating these concepts into population dynamics is crucial for accurately describing a phenomenon. We offer insights into the relations of these concepts in the light of stochastic calculus. Our analysis takes place in the domain of Hilbert-valued stochastic processes. Due to the generality of our hypothesis, the outcomes cover a broad range of specialized models within population dynamics. To illustrate, we provide representative examples.

This work was supported by Fapesp.

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Decreasing and exponential stability for generalized ordinary differential equations

Eduard Toon, Fernanda Andrade da Silva
Universidade Federal de Juiz de Fora

The theory of generalized ordinary differential equations (generalized ODEs, for short) has been shown to be a very powerful theory once several types of equations can be regarded as them. In the present lecture, we introduce a new concept of stability, called decreasing stability, and deal with some Lyapunov techniques on decreasing and exponential stability.

Generalized equations in the framework of stochastic problems

Fernanda Andrade da Silva
Universidade de São Paulo

If on one hand the adequate modeling of deterministic problems involving functions of unbounded variation is better done by means of integral equations with integrals in the sense of Kurzweil-Henstock or Kurzweil-Henstock-Stieltjes, on the other hand, stochastic problems are usually investigated from the point of view of the Itô integral, when one investigates its integral form. For instance, the integral form of stochastic differential equations involving Brownian motions is naturally thought of in the setting of the Itô integral. In contrast, the integral form of differential equations involving Kurzweil-Henstock integrable functions are easily dealt within the framework of generalized ordinary differential equations.

In this work, we consider the Itô-Henstock integral for functions taking values in the (separable) Hilbert space of Hilbert-Schmidt operators. We present a new integral on this space, which contains the Itô-Henstock integral of Hilbert-Schmidt operators. In this approach, the classical Kurzweil integral is modified to obtained our new integral, which we call the Kurzweil-belated integral. This integral contains the Itô-Henstock integral (and, hence, the classical Itô integral). In order to provide a similar environment of the generalized ordinary differential equations for stochastic differential equations, we defined a new class of generalized stochastic equations (GSEs) by means of our Kurzweil-belated integral in the same way Kurzweil used to define generalized ordinary differential equations with his integral. Thus, the family of classical stochastic differential equations become special cases of GSEs.

We also present some properties of the GSEs as existence and uniqueness of solutions and some stability concepts.

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Well-posedness and asymptotic behavior of difference equations with a time-dependent delay and applications

Jaqueline Godoy Mesquita
Universidade de Brasília

In this lecture, we investigate the well-posedness and asymptotic behavior of difference equations of the form

$$x(t) = Ax(t - \tau(t)), \qquad t \geq 0,$$

where the unknown function x takes values in $\mathbb{R}^d$ for some positive integer d, A is a $d \times d$ matrix with real coefficients, and $\tau\colon [0, +\infty) \to (0, +\infty)$ is a time-dependent delay.

We provide our investigations for three spaces of functions: continuous, regulated, and L^p. We compare our results for these three cases, showing how the hypotheses change according to the space that we are treating. Finally, we provide applications of our results to difference equations with state-dependent delays for the cases of continuous and regulated function spaces, as well as to transport equations in one space dimension with time-dependent velocity. This is a joint work with Professor Guilherme Mazanti, INRIA, France.

Relations between different attractors related to impulsive dynamical systems

José M Uzal
Universidad Complutense de Madrid

In this talk we introduce impulsive dynamical systems and we study the existence of its global attractor. Moreover, we investigate the relationship between the global attractor of impulsive systems, the global attractor of the related continuous dynamical system which comes from the impulsive system. This is based on a joint work with Everaldo M. Bonotto (ICMC-USP).

Convergence for non-autonomos semidynamical systems with impulses

Manuel Francisco Zuloeta Jimenez, Everaldo de Mello Bonotto, Daniela Paula Demuner
Universidade Tecnológica Federal do Paraná

The present paper deals with impulsive non-autonomous systems with convergence. We show that the structure of the Levinson center of a compact dissipative system is preserved under homomorphism in impulsive convergent systems. Also, we present some criteria of convergence using Lyapunov functions.

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An abstract version of the Gronwall lemma

Pablo Amster, Julián Epstein
Buenos Aires

May the Gronwall Lemma be regarded as a maximum principle? In this talk, an abstract version of the celebrated inequality is obtained by means of the spectral bound of a compact operator defined on a Banach lattice. As a consequence, uniqueness and continuous dependence results for the general semilinear problem $L(u) = N(u)$ are established.

Bifurcation results for a class of second order equations

Pierluigi Benevieri, Guglielmo Feltrin
Universidade de São Paulo - USP

In this paper we prove a general bifurcation theorem for a parametrized T-periodic boundary value problem

$$\begin{cases} (\phi(x'))' = F(\lambda, t, x, x'), \\ x(0) = x(T), \quad x'(0) = x'(T), \end{cases} \tag{3}$$

in which the following conditions hold:

(H_1) $\phi : \mathbb{R}^n \to \mathbb{R}^n$ is a homeomorphism with $\phi(0) = 0$;

(H_2) $F : \boldsymbol{R} \times [0,T] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a Carathéodory function, that is,

- i) for almost every $t \in [0,T]$, $F(\cdot, t, \cdot, \cdot)$ is continuous;
- ii) for any $(\lambda, x, y) \in \boldsymbol{R} \times \mathbb{R}^n \times \mathbb{R}^n$, $F(\lambda, \cdot, x, y)$ is measurable;
- iii) for any $\rho > 0$ and any compact interval I, there exists $g \in L^1([0,T], \boldsymbol{R}^n)$ such that, for almost every $t \in [0,T]$, every $\lambda \in I$ and every $x, y \in \mathbb{R}^n$, with $\|x\| \leq \rho$ and $\|y\| \leq \rho$, we have
$$\|F(\lambda, t, x, y)\| \leq g(t);$$

(H_3) $F(0, t, x, y) = f_0(x, y)$, that is, $F(0, \cdot, \cdot, \cdot)$ is an autonomous vector field. We also assume that f_0 is continuous.

Nonlinear Dynamical Systems

Organizer: Juliana Fernandes S. Pimentel (UFRJ/Brazil), Maykel Boldrin Belluzi (USP/Brazil) & Phillipo Lappicy (UFRJ/Brazil)

Nonuniform exponential dichotomies for Parabolic PDEs and applications

Alexandre do Nascimento Oliveira Sousa, José A. Langa, Rafael Obaya
UFSC

This study investigates nonuniform exponential dichotomies and pullback and forward attractors within the context of evolution processes linked to nonautonomous partial differential equations (PDEs). Introducing a novel notion of nonuniform exponential dichotomy, we examine the pullback and forward dynamics of parabolic PDEs by utilizing nonuniform exponential dichotomies and comparison. Additionally, we present a technique for generating examples of PDEs with nonuniform exponential dichotomies through continuous separation in strongly monotone dynamical systems.

Bifurcation and hyperbolicity for a nonlocal quasilinear parabolic problem.

Alexandre N. Carvalho, José Arrieta, Estefani M. Moreira, José Valero
Universidade de São Paulo

We study a one-dimensional nonlocal quasilinear problem of the form $u_t = a(\|u_x\|^2)u_{xx} + \nu f(u)$, with Dirichlet boundary conditions on the interval $[0, \pi]$, where $0 < m \leq a(s) \leq M$ for all $s \in \mathbb{R}^+$ and f satisfies suitable conditions. We give a complete characterization of the bifurcations and of the hyperbolicity of the corresponding equilibria. With respect to the bifurcations we extend the existing results to the case when $a(\cdot)$ is not necessarily monotone increasing and show that bifurcations may be pitchfork or saddle-node, subcritical or supercritical. We also give a complete characterization of hyperbolicity specifying necessary and sufficient conditions for its presence or absence.

Smoothing and finite-dimensionality of uniform attractors in Banach spaces

Arthur Cunha, Alexandre Carvalho, Hongyong Cui, José Langa
Universidade Federal da Bahia - UFBA

The aim of this talk is to find an upper bound for the fractal dimension of uniform attractors in Banach spaces. The main technique is essentially based on a compact embedding of some auxiliary Banach space into the phase space and a corresponding smoothing effect between these spaces. Our bounds on the fractal dimension of uniform attractors are given in terms of the dimension of the symbol space and the Kolmogorov entropy number of the embedding. A dynamical analysis on the symbol space is also given, showing that the finite-dimensionality of the hull of a time-dependent function is fully determined by the tails of the function, which allows us to consider more general nonautonomous terms than quasi-periodic functions. As application, we show that the uniform attractor of a reaction-diffusion equation is finite-dimensional in L^p with $p \geq 2$.

Global Attractor for a Class of Abstract Differential Equations with State-Dependent Delay

Denis Fernandes, Eduardo Hernandez, Messoud Efendiev, Jianhong Wu

York University

It is well known that, in general, differential equations incorporating nonlinear terms with state-dependent delay are not well-posed in the space of continuous functions, making the study of this class of differential equations more complex and challenging.

In this talk, I will present results concerning the existence of a finite fractal dimensional global attractor for the multivalued semigroup generated by a class of abstract differential equations with state-dependent delay when considering the space of continuous functions as the phase space. The class under consideration takes the form

$$u'(t) = Au(t) + F(u(t), u(t - \sigma(t, u_t))),\ t \geq 0, \tag{4}$$

$$u_0 = \varphi \in C([-p, 0]; X), \tag{5}$$

in which $A : D(A) \subset X \to X$ is the generator of an analytic C_0-semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ defined on a Banach space $(X, \|\cdot\|)$. The analysis of this class considers both subsets and the entire space $C([-p, 0]; X)$ as phase spaces for the initial conditions.

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A index theory for multivalued dynamical systems with application to a reaction diffusion equation with discontinuous nonlinearity

Estefani M. Moreira, José Valero

Universidade de São Paulo

We present first a theory of existence of index pairs for multivalued semiflows. We apply this result to a reaction-diffusion equation having a discontinuous nonlinearity which gives rise to a differential inclusion governed by the Heaviside function.

Time-dependent differential processes and their relationship with the fractal dimension theory.

Heraclio Ledgar Lopez Lazaro, Alexandre Nolasco de Carvalho, Tomás Caraballo , Arthur Cavalcante Cunha

ICMC-USP

We present a method to estimate the fractal dimension of families of negatively invariant time-parameterized compact sets with respect to maps acting on families of time-parameterized Banach spaces. These methods allow us to estimate the fractal dimension of pullback attractors associated with systems of differential equations defined on non-cylindrical domains.

On global attractors for autonomous evolution problems with variable exponents

Jacson Simsen

UNIFEI - Universidade Federal de Itajubá

Abstract: In this talk I will give an overview on the results which we have obtained during the last 13 years about existence and upper semicontinuity of global attractors for parabolic problems with variable exponents.

The chemostat model with real random perturbations on Monod and Haldane consumption functions

Javier López-de-la-Cruz

Universidad Politécnica de Madrid

The chemostat is a laboratory device very used in practice due to its large number of applications, such as waste water treatment, antibiotic production and fermentation models, to name the most popular applications. It consists on three different tanks, the feed bottle, the culture vessel and the collection vessel. A substrate is stored in the first tank and provided, by means of a pump, to the second tank, where it is consumed by some population of microorganisms that are growing. In addition, in order to keep the volume constant in the culture vessel, another flow is pumped from the culture vessel to the third tank.

Then, the main goal for scientists is to understand the evolution with respect to the time of the concentrations of both the substrate and the microorganisms in the culture vessel, which is described by the following differential system called chemostat model

$$\frac{ds}{dt} = D(s_{\text{in}} - s) - \mu(s)x, \tag{6}$$

$$\frac{dx}{dt} = -Dx + \mu(s)x, \tag{7}$$

where $s = s(t)$ and $x = x(t)$ denote the concentrations of substrate and species at time t, respectively, $s_{\text{in}} > 0$ represents the concentration of substrate in the feed bottle, $D > 0$ is the dilution rate and $\mu = \mu(s)$ is the consumption function describing how the substrate is consumed by the microorganisms.

The chemostat model (6)-(7) has been widely investigated in the literature. However, it assumes restrictions that are very strong. For instance, the consumption function $\mu = \mu(s)$ is tipically supposed to be continuously differentiable on its domain such that the corresponding initial value problem (i.e., the chemostat model with certain initial values) is well-posed.

However, practitioners claim that the smooth deterministic consumption function $\mu = \mu(s)$ is derived as approximations of dotted lines obtained from experimental data in the laboratory, thanks to measurements in real devices. This means that the value $\mu(s)$ cannot be totally determined in real life (in a pure deterministic way) for every value s.

In order to obtain more realistic chemostat models, in this talk we will explain how to perturb randomly the consumption function when it is given by both the Monod and the Haldane expressions (the most common ones when modeling real experiments in laboratories).

To this end, instead of using the usual Standard Wiener process, we will motivate the use of a different noise, which produces more realistic models and allows us to obtain results that are much more interesting from both the biological and the mathematical points of view. In addition, we will provide several numerical simulations to illustrate the achievements and clarify the ideas presented throughout the talk.

Stabilization of a nonautonomous logistic equation by moving the refuge region

José M. Arrieta, Neus Cónsul
Universidad Complutense de Madrid

We consider nonnegative solutions of a nonautonomous degenerate logistic equation of the type $u_t - \Delta u = \lambda u - n(t,x)u^p$ with $p > 1$, in a bounded domain $\Omega \subset \boldsymbol{R}^n$ where the refuge region $K(t) = \{x \in \Omega : n(t,x) = 0\}$ is moving. We analyze how the motion of this region affects the boundedness of the solutions of the equations. We will provide examples where for small velocity of the set $K(t)$ the solutions grow without bound as $t \to +\infty$ while for large velocity the solutions are stabilized and become bounded as $t \to +\infty$.

On unbounded attractors for dynamical systems and applications

Juan Garcia Fuentes, Matheus Cheque Bortolan, Juliana Fernandes, Piotr Kalita
Universidad de Sevilla

Dynamical systems governed by dissipative semigroups contain structures that are invariants and attracts every trajectory of the phase space, well know as global attractors, and by its own definition of dissipation, these global attractors are bounded sets. Nevertheless, in case of working with slowly non-disipative semigroups, that is, its solutions can diverge to infinity as time tends to infinity, one can find also an invariant attracting structure, but in this case unbounded.

We follow the study of Chepyzhov and Goritskii [1], to provide abstract results on the unbounded attractor existence. We introduce the concept of a *unbounded B-attractor* for a semigroup, and find conditions under which a semigroup in a metric space possess an unbounded B-attractor.

Finally, we apply our result to a parabolic semilinear PDE, where the nonlinearity can be unbounded, as long as it grows linearly with a controlled growth constant, and find a unbounded B-attractor for it. Furthermore, under a Lipschitz condition on the nonlinearity, with small Lipschitz constant, we prove that this maximal B-attractor is, in fact, an unbounded attractor.

This is a joint work with Matheus Bortolan, Juliana Fernandes and Piotr Kalita, fully developed in [4].

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Results on semilinear evolution equations with time-dependent linear operators

Maykel Boldrin Belluzi
Universidade de São Paulo - USP

In this talk we discuss semilinear evolution equations of the form

$$u_t + A(t)u = F(t,u), \quad t > \tau,$$

where $\{A(t),\ t \in \mathbb{R}\}$ is a family of sectorial operators in a Banach Space X and F a nonlinearity taking values in X. This time-dependence found on the linear operators $A(t)$ brings additional challenges when compared to the case where the linear operator is fixed in time. For instance, the theory on linear semigroup used to solve semilinear problems needs to be replaced by the theory on linear processes, first developed by Sobolevskiĭ and Tanabe around the 60's [4,5].

We present some motivational examples involving this type of semilinear equations and we discuss the matter of local/global existence of solutions and existence of pullback attractors for the problem (found in [2,3]). We shall also present some recent results [1] related to perturbations of this semilinear equation

$$(u_\varepsilon)_t + A_\varepsilon(t)u_\varepsilon = F_\varepsilon(t,u_\varepsilon),$$

where ε is a parameter in $[0,\varepsilon_0)$.

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A new thermoelastic circular beam and its exponential stability

To Fu Ma, Irving Ramirez-Barreto

Universidade de Brasília

In this talk we study a thermoelastic Bresse system modeling vibrations of circular elastic beams. This new model features heat effects on shear force, bending moment and longitudinal displacements. Subjected to a nonlinear elastic foundation we discuss its asymptotic stability.

Generalized Brenier Principle and the Closure Problem of Landgren–Monin–Novikov Hierarchy for Vorticity Field

Vladimir Grebenev, Alexandre Grishkov

Federal Research Center of Information and Computational Technologies Russian Ac

Brenier's concept – a representation of solutions to the equations of ideal incompressible fluids in terms of probability measures on the set of Lagrangian trajectories in the case of their stochasticity, is a generalization of Arnold's principle of least action of finding smooth solutions of Euler's equations. In this work, the variational generalized Brenier principle (Brenier, J. Am. Math. Soc. 1989) is used to close the infinite chain of Landgren–Monin–Novikov equations for the n-point probability density functions (PDF) f_n of the vortex field of two-dimensional turbulence. In addition, within the framework of the statistical approach, an approximation of the variational problem with conditions at the ends posed by Shnirelman (Mat. Sat. 1985) for the Euler equation is proposed.

The aim of the paper is to apply Brenier's variational principle to close Landgren–Monin–Novikov (LMN) hierarchy to the n-point PDFs of the vortex field i.e. the Euler PDF function f_n^E. Brenier's variational principle (BVP) is a generalization of Arnold's principle of least action for constructing smooth solutions of Euler's equations. BVP describes solutions of Euler's equations in terms of probability measures (generalized flows) on a set of Lagrangian trajectories. The concept of generalized flows is natural for describing Lagrangian trajectories of hydrodynamic turbulence in terms of random processes. An interpretation of the generalized flow is given within the framework of a statistical description of hydrodynamic turbulence. The random vorticity field is taken as the main quantity of motion and the evolution of f_n^E is determined by the LMN hierarchy in the Euler formulation, which is derived from the statistical form of Euler's equations using Biot-Savart's law. Transition to the Lagrangian formulation is carried out by an equivalent notation of the LMN hierarchy in the form of equations along the characteristics (Lagrangian trajectories) of the LMN hierarchy. The statistical form of the BVP is derived using the characteristic equations and the minimization of the functional is performed over the variational variable f_{n+1}^E. In this case, according to the BVP, two boundary conditions, the initial and final distribution of the PDF f_{n+1}^E along the characteristics must be specified. As a result, f_{n+1}^E determined from the variational principle (optimal statistical ensemble implementations) allows you to close the f_n^E-equation of the LMN hierarchy. In this case, the statistical form of the BVP coincides with the Arnold variational principle (AVP) (considering formally the limit at $n \to \infty$) for the boundary value problem of a geodesic connecting two given fluid configurations. The formulation of such a problem was first proposed by Shnirelmann and studied using a discrete analogue of the group of diffeomorphisms for an ideal incompressible fluid, which is not classical in hydrodynamics. For the two-dimensional case, it is unknown whether there always exists a solution to the variational problem with conditions at the ends. In this paper, another approximation of this problem is proposed based on the statistical version of the Euler equation, i.e. LMN hierarchy, which is more focused on numerical experiments, which represents a separate topic of research.

Local well posedness, regularity and comparison for solutions of abstract parabolic problems without uniqueness.

Yessica Yulieth Julio Pérez, Tomás Caraballo, Alexandre Nolasco

Universidade de São Paulo, ICMC.

In this lecture, we will present a study of a one-dimensional nonlocal quasilinear problem of the form $\frac{\partial u}{\partial t} - a(l(u))\frac{\partial u^2}{\partial x^2} = \lambda f(u) + h(t)$ with Dirichlet boundary conditions on the interval $(0, 1)$. We use a change in the time scale for reformulate the problem into a semilinear one, thus study a general equation through semigroup theory. Our focus extends to establishing the existence of a classical solution for an abstract problem, particularly when the forcing term f has a weaker modulus of continuity than continuous Hölder. We will obtain comparison results that will be helpful in ensuring the existence of a pullback attractor.

Dispersive Equations

Organizer: Marcia A. G. Scialom (UNICAMP/Brazil) & Mahendra Panthee (UNICAMP/Brazil)

Well-posedness and stability of energy-critical non-isotropic fourth-order Schrödinger equation in high dimension

Andressa Gomes, Mahendra Panthee, Mykael Cardoso
Universidade Federal do Delta do Parnaíba - UFDPar

In this talk we consider the Cauchy problem of the non-isotropically perturbed nonlinear Schródinger equation

$$\begin{cases} iu_t\Delta u + a\sum_{i=1}^{k} u_{x_ix_ix_ix_i} + |u|^{p*}u = 0, \quad x \in \mathbb{R}^N, \quad t \in \mathbb{R}, \quad 1 \leq k < N, \\ u(x,0) = u_0(x) \in H^1(\mathbb{R}^N), \end{cases}$$

where a is a negative real constant and $p^* = \frac{8N}{(2N-k)(N-2)}$ is the energy-critical exponent. By using the Strichartz estimates developed in [1] and appropriate nonlinear estimates we stablish the well-posedness results. Next, we present a stability theory in $H^1(\mathbb{R}^N)$ for all $N \geq 3$. For the cases when $p^* < 1$ we obtain the new exotic Strichartz estimate by the fractional calculus technique presented in [2]. Finally, as a consequence of the stability result we prove a scattering criterion. This is a joint work with Prof. Mahendra Panthee (UNICAMP) and Prof. Mykael Cardoso (UFPI).

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Analytic solutions associated to the Schrödinger Korteweg de Vries system

Argenis J Mendez, Marcelo Nogueira
Universidade Estadual de Campinas

In this work, we study the analyticity of the solutions of the Schrödinger-Korteweg-de Vries system

$$\begin{cases} i\partial_t u + \partial_x^2 u = \alpha uv + \beta u|u|^2, \quad t, x \in \mathbb{R}, \\ \partial_t v + \partial_x^3 v + v\partial_x v = \gamma\partial_x(|u|^2) \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x). \end{cases}$$

In this context, $u = u(x,t)$ represents a complex-valued function, while $v = v(x,t)$ is a real-valued function, and the constants α, β, and γ are real.

The system describes the interplay between short-wave phenomena, represented by $u = u(x,t)$, and long-wave interactions, characterized by $v = v(x,t)$. This model finds application in fluid mechanics and plasma physics.

We prove that, given appropriate conditions on the parameters α, β, and γ, a solution to the Schrödinger-Korteweg-de Vries system displays a significant smoothing effect up to real analyticity when the initial data possess a singular point solely at $x = 0$. More specifically, for initial data (u_0, v_0) subject to certain regularity conditions, the solution (u, v) attains analyticity in both spatial and temporal variables.

Transversal spectral instability of periodic traveling waves for the generalized Zakharov-Kuznetsov equation

Fabio Natali

Universidade Estadual de Maringá

In this talk, we determine the transversal instability of periodic traveling wave solutions of the generalized Zakharov-Kuznetsov equation in two space dimensions. Using an adaptation of the arguments in [1] in the periodic context, it is possible to prove that all positive and one-dimensional L-periodic waves are spectrally (transversally) unstable. In addition, when periodic waves that change their sign exist, we also obtain the same property when the associated projection operator defined in the zero mean Sobolev space has only one negative eigenvalue. Some remarks concerning the nonlinear instablity of periodic waves that are transversally unstable are also established.

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Exact controllability and stabilization for linear dispersive PDE's on the two-dimensional torus

Francisco Javier Vielma Leal, Ademir Pastor

Universidad Tecnológica Metropolitana

In this talk we use the moment method to prove the exact controllability of a wide class of bidimensional linear dispersive PDE's posed on the two-dimensional torus $\mathbb{T}^2$. The control function is considered to be acting on a small vertical and horizontal strip of the torus. Our results apply to several well-known models including some bidimesional extensions of the Benajamin-Ono and Korteweg-de Vries equations. As a by product, the exponential stabilizability with any given decay rate is also established in the Sobolev space $H^s_p(\mathbb{T}^2)$, with $s \geq 0$, by constructing an appropriated feedback control law.

Global well-posedness for the nonlinear Schrödinger equation with third order dispersion

Mahendra Panthee, Xavier Carvajal

Universidade Estadual de Campinas

In this talk, we consider the initial value problem (IVP) associated to the cubic nonlinear Schrödinger equation with third-order dispersion

$$\partial_t u + i\alpha\partial_x^2 u - \partial_x^3 u + i\beta|u|^2 u = 0, \quad x, t \in \mathbb{R},$$

for given data in the Sobolev space $H^s(\mathbb{R})$. This IVP is known to be locally well-posed for given data with Sobolev regularity $s > -\frac{1}{4}$ and globally well-posed for $s \geq 0$ [**?**]. For given data in $H^s(\mathbb{R})$, $0 > s > -\frac{1}{4}$ no global well-posedness result is known. We derive an *almost conserved quantity* for such data and obtain a sharp global well-posedness result. Our result answers the question left open in [X. Carvajal, *Local well-posedness for a higher order nonlinear Schrödinger equation in Sobolev spaces of negative indices*, Electronic J Diff Equations **2004** (2004) 1–10].

A note on C^2 Ill-posedness results for the Zakharov system in arbitrary dimension

Raphael A. dos Santos, Leandro Domingues

Universidade Federal do Rio de Janeiro

This work is concerned with the Cauchy problem for a Zakharov system with initial data in Sobolev spaces $H^k(\boldsymbol{R}^d)\times H^l(\boldsymbol{R}^d)\times H^{l-1}(\boldsymbol{R}^d)$. We recall the well-posedness and ill-posedness results known to date and establish new ill-posedness results. We prove C^2 ill-posedness for some new indices $(k,l)\in \boldsymbol{R}^2$. Moreover, our results are valid in arbitrary dimension. We believe that our detailed proofs are built on a methodical approach and can be adapted to obtain similar results for other systems and equations.

Evolution of the radius of analyticity for the periodic mKdV equation

Renata O. Figueira, Mahendra Panthee

University of Campinas

We shall consider the Cauchy problem on the torus for the modified KdV equation in the defocusing case, where the initial data belongs in a class of analytic functions on the line that can be extended holomorphically in a symmetric strip around x-axis.

From the result about local well-posedness in these classes of functions, we guarantee the analytic regularity of the solutions in space variable without shrinking the width of the strip for short times, which means that the uniform radius of spatial analyticity remains the same till some lifespan $0 < T_0 \leq 1$. This talk is devoted to discuss the evolution of the radius of spatial analyticity $\sigma(t)$ when the local solution extends globally in time and prove that for any time $T \geq T_0$ it is bounded from below by cT^{-1}.

This works is in collaboration with Mahendra Panthee and supported by FAPESP.

Energy Decay for Classes of Nonlocal Dispersive Equations

Ricardo Freire

Universidad de Chile-DIM

We consider the long-time dynamics of large solutions to a special class of evolution equations. Using virial techniques, we describe regions of space where every solution in a suitable Sobolev space must decay to zero along sequences of times. Moreover, in the case of interior regions, we prove decay for a sequence of times. The classes of nonlocal dispersive equations which we will treat are as follows:

$$\begin{cases} \partial_t u + L_\alpha u + u\partial_x u = 0, \quad x,t\in\mathbb{R}, \\ u(x,0) = u_0(x), \end{cases}$$

where $\alpha > 0$, and the operator L_α is the Fourier multiplier operator by a real-valued odd function belonging to $(C^1(\mathbb{R})\cap C^\infty(\mathbb{R}^*))$. These classes contain, in particular, the following equations: the fractional KdV, Benjamin-Ono and the Intermediate Long Wave, for example.

Sharp global well-posedness for the higher order non-linear Scrödinger equation on modulations spaces

Xavier Carvajal, Rafael Antunes, Pedro Gamboa

Universidade Federal Do Rio De Janeiro

We consider the initial value problem (IVP) associated to a higher order nonlinear Schrödinger (h-NLS) equation

$$\partial_t u + ia\partial_x^2 u + b\partial_x^3 u + ic_1|u|^2 u + c_2|u|^2\partial_x u = 0, \quad x, t \in \boldsymbol{R},$$

for given data in the modulation space $M_s^{2,p}(\boldsymbol{R})$. Using ideias of Killip, Visan, Zhang, T. Oh and Wang, we prove that the IVP associated to the h-NLS equation is globally well-posed in the modulation spaces $M^{s,p}$ for $s \geq \frac{1}{4}$ and $p \geq 2$.

Linear Equations

Organizer: Patrícia Yukari S. Rampazo (UFF/Brazil) & Nicholas Braun Rodrigues (UFSCar/Brazil)

The double dispersion equation: a case of very uneffective damping

Antonio Lagioia, Marcello D'Abbicco
University of Bari

In this talk, we will discuss the effects of applying a damping term to wave equations or other related evolution equations. Dissipation can result in one or both of two different situations: the fundamental solution may exhibit oscillations with reduced amplitude (damped oscillations regime), or the oscillations may be suppressed by friction (overdamping regime).
We will specifically focus on the model

$$\begin{cases} u_{tt} + Au + (I+A)^{-1}Au_t = 0, \quad t \geq 0, x \in \mathbb{R}^n \\ u(0,x) = 0,\ u_t(0,x) = u_1(x), \end{cases}$$

where A is a differential operator in the form $A = (-\Delta)^k$ for a even integer $k \geq 2$. The case $k = 1$ is known as DDE (Double Dispersion Equation) and has been of great interest in recent years. We will present the results obtained concerning $L^p - L^q$ long time estimates for the solution of the linear problem considered and an application to some associated semi-linear problems.

Div-curl type estimates for elliptic differential operators

Catarina Barbosa Machado, Tiago Henrique Picon
UFSCar

In this work, we extend the classical div-curl inequality proved by Coifman, Lions, Meyer and Semmes to the local setup of higher order elliptic differential operators $A(x, D)$ with smooth coefficients. The tools include a generalized Poincaré-type inequality and an appropriated atomic decomposition on localizable Sobolev-Hardy spaces $h^{k,p}$ with $0 < p \leq 1$.

New tools and conditions for global regularity of the $\bar{\partial}$-Neumann operator

Dmitri Zaitsev
Trinity College Dublin

A fundamental question for a boundary value problem for a system of Partial Differential Equations is finding conditions for *global regularity*, i.e. when (weak) solutions are smooth whenever the data of the problem are smooth. Global regularity of the *$\bar{\partial}$-Neumann problem*, due to its implication of *condition R* is of further importance for the study of smooth boundary extension of proper holomorphic maps, generalizing the famous Fefferman's theorem. The goal of my lecture is to introduce a set of new tools in a new approach to global regularity that include:

1. a new multitype based on *distinguished nested sequences of* $(1,0)$ *subbundles*;

2. *formal (Nagano type) orbits* O and their *complexifications* V for certain *special subbundles* defined in terms of derivatives of the Levi form;

3. *k-jets relative to a pair (O,V), of briefly (k,O,V)-jets*, where O,V are as above or, more generally, any pair of formal submanifolds;

4. *relative contact orders* of real hypersurfaces with pairs (O,V) of formal submanifolds, generalizing usual *contact orders of with complex submanifolds*;

5. *supertangent vector fields* having higher than expected relative contact orders, for which we establish an important *Lie algebra property*

6. infiniteness of relative contact orders with (O,V) when O is *complex-tangential*, this may be regarded as the formal version of the important result by Diederich-Fornæss — a difficult key step in their proof of Kohn's ideal termination in the real-analytic case

In particular, our approach gives a new proof of a celebrated Catlin's theorem.

Propagation of regularity for noncommutative systems of vector fields defined on the torus

Igor Ambo Ferra, Luís Antônio Carvalho dos Santos
Universidade Federal do ABC

In this work we present a connection between properties of regularity of solutions and global solvabilty of the systems $L_j = \partial_{t_j} + a_j(t)\partial_x, j \in \{1,\dots,n\}$ in the torus $\mathbb{T}^n_t \times S^1_x$ and the sum of squares associated to this system. By using results about propagation of regularity we characterize these properties even when the system do not commute.

Global solvability and cohomology on compact manifolds

Luis Fernando Ragognette, Gabriel Araújo , Igor Ambo Ferra, Max Reinhold Jahnke
Universidade Federal de Minas Gerais - UFMG

We will discuss techniques to study diffrential complexes on a product of a compact manifold with the m-dimensional torus. In special, how to use a Partial Fourier transform to characterize global solvability in terms of a weak notion of global hypoellipticity and to obtain an isomorphism for the cohomology groups associated with the differential complex.

Sharp $L^p - L^q$ estimates for a class of dissipative wave equations

Marcelo Rempel Ebert, Marcelo D'Abbicco
Universidade de São Paulo

In this talk we will derive $L^p - L^q$ estimates, with $1 \le p \le q \le \infty$ (including endpoint estimates as $L^1 - L^1$ and $L^1 - L^\infty$), for the solution to

$$\begin{cases} u_{tt} - \Delta u + Au_t = 0, \quad t \in \boldsymbol{R}_+,\, x \in \boldsymbol{R}^n, \\ u(0,x) = 0, \\ u_t(0,x) = u_1(x), \end{cases}$$

for a general class of dissipation terms, where $Af = \mathcal{F}^{-1}(a(\xi)\,\mathcal{F}f(\xi))$, with $a \in \mathcal{C}^{n+1}(\boldsymbol{R}^n \setminus \{0\})$, and $a(\xi) > 0$ verifies conditions of Mikhlin-Hörmander type for M_p^q multipliers which may be different at low frequencies and at high frequencies; in particular $a(\xi)$ may also be inhomogeneous and anisotropic. We prove that the obtained estimates are sharp.

Global solvability for certain partial differential operators on the torus

Rafael Borro Gonzalez
UEM

We study the range of certain partial differential operators by using certain properties of its coefficients, such as: changes of sign, order of vanishing, and Diophantine conditions.

Some singular solutions on the Möbius band

Renato Andrielli Laguna, Sérgio Luís Zani
USP

Let $\mathbb{M}$ be the Möbius band, uniformized as $\mathbb{R}^2/G$ where G is the automorphism group generated by the isometry $(x, y) \mapsto (x + 1, -y)$. We show the existence of singular solutions u to the complex vector field

$$L \doteq \frac{\partial}{\partial x} + y\big(a(x,y) + b(x,y)i\big)\frac{\partial}{\partial y}, \qquad a, b \in \mathcal{C}^\infty(\mathbb{R}^2;\mathbb{R}), \qquad b \geq 0$$

where both L and u pass to the quotient. In other words, the passage of L to the Möbius band is not globally hypoelliptic. Some consequences for the question of whether non-orientable surfaces admit globally hypoelliptic vector fields are examined.

A survey about Vekua-type operators

Ricardo Paleari
Unespar

In this talk, I will talk about Vekua-type operators and some recent works related to this topic. In particular, the first one will be the case of this type of operator acting on the n-torus. Later, I will talk about Vekua operators acting on compact Lie groups, to emphasize the subtle theoretical differences in this case. Finally, I will present some possible questions for continuing these works, possibly on the setting of compact homogeneous spaces.

A comparison principle between certain Levi-flat compact CR manifolds and systems of real vector fields

Vinícius Novelli, Paulo D. Cordaro
University of São Paulo

In this talk, we discuss a comparison principle between Levi-flat CR structures on certain manifolds and an associated system of real vector fields. We are able to relate properties of global hypoellipticity of the associated differential operators, even when acting on forms of higher degree. We also show an isomorphism betweem the cohomology complexes of both structures, and discuss this relation in several models. This work is part of the author's PhD thesis, supervised by Paulo Cordaro.

Elliptic Equations

Organizer: Ederson Moreira dos Santos (USP/Brazil) & Marcos T. O. Pimenta (UNESP/Brazil)

A Critical Neumann problem with anisotropic $p-$Laplacian

Alannio B. Nóbrega, Olimpio Hiroshi Miyagaki, Gustavo Ferron Madeira
Universidade Federal de Campina Grande

We are concerned with the existence of solution of the problem

$$\begin{cases} -\Delta_p^H u + |u|^{p-2}u = \lambda|u|^{q-2}u + |u|^{p^*-2}u & \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \\ a(\nabla u)\cdot\nu = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Delta_p^H u = \operatorname{div}(a(\nabla u))$, with $a(\xi) = H^{p-1}(\xi)\nabla H(\xi)$, $\xi \in \mathbb{R}^N$, $N \geqslant 3$, is the anisotropic p-Laplacian with $1 < p < N$, $\lambda > 0$ is a parameter, and $p < q < p^* = pN/(N-p)$. Further, $\Omega \subset \Sigma$ is a C^1 bounded domain inside a convex open cone Σ in $\mathbb{R}^N$ with $\partial\Omega \cap \partial\Sigma$ being a C^1-manifold, and ν is the unit outward normal to $\partial\Omega$. To succeed with a variational approach, where the strong convergence of a bounded (PS) subsequence needs to be proved, one has to deal with anisotropic norms in the absence of a Tartar's type inequality, unlike the isotropic p-Laplace case. This is overcome by proving the a.e. convergence of its gradients. Furthermore, the solution of (P) is shown to belong to $C^{1,\alpha}(\Omega)$, and is strictly positive in Ω. Such conclusions are achieved from classical elliptic regularity theory and a Harnack inequality, since the solution of (P) is bounded. This in turn is a consequence of a result in this paper which ensures that any $W^{1,p}$-solution of critical Neumann problems with the anisotropic p-Laplacian operator on bounded Lipschitz domains in $\mathbb{R}^N$ $(N \geqslant 3)$ is bounded.

Critical and Subcritical Fractional Hamiltonian Systems of Schrödinger Equations with Vanishing Potentials

Cláudia Santana, O. Miyagaki, E. Toon, P. Ubilla
UESC - UNIVERSIDADE ESTADUAL DE SANTA CRUZ

We analyze a class of fractional Hamiltonian systems of Schrödinger equations in $\mathbb{R}^N$ involving vanishing potential with critical and subcritical nonlinearity. We make use of variational methods to prove the existence of at least one pair of positive solutions. Our result in the critical case is new even for the Laplacian operator.

Schrödinger-Poisson system with zero mass in $\mathbb{R}^2$ involving $(2,q)$-Laplacian: existence, asymptotic behavior and regularity of solutions

Edcarlos D. Silva , J. Carvalho, J. C. de Albuquerque
UFG

In this talk we consider existence of positive least energy solution for the following class of planar elliptic systems in the zero mass case

$$\begin{cases} -\Delta u - \Delta_q u + \phi|u|^{r-2}u = \lambda|u|^{p-2}u, & \text{in } \mathbb{R}^2, \\ \Delta\phi = 2\pi|u|^r, & \text{in } \mathbb{R}^2, \end{cases}$$

where $\lambda \geq 0$, $1 < q < 2$, $q^* := 2q/(2-q) < r < \infty$ and $p \geq 2r$. Due to the nature of the problem, we deal with the logarithmic integral kernel. Our approach is based on Nehari manifold and a version of the Principle of Symmetric Criticality due to Palais. Furthermore, we study the asymptotic behavior of the solutions whenever the parameter λ goes to zero or infinity. Finally, we study regularity of solutions applying Moser iteration scheme.

This is a joint work with J. C. de Albuquerque - UFPE and J. Carvalho - UFS.

Advances on Choquard Equations

Eduardo de Souza Böer

USP - São Carlos

In the present talk we are going to discuss some results concerning sign-changing and unbounded Choquard potentials that include the logarithm. We will show results about different operators, such as fractionals and Kirchhoff, focusing in the main approach ideas. Finally, we will discuss some open problems and future possibilities in the area.

On Nonlinear Perturbations of a Periodic Integrodifferential Kirchhoff Equation with Critical Exponential Growth

Eudes Mendes Barboza, Yane de Araújo, Gilson de Carvalho

UFRPE - Universidade Federal Rural de Pernambuco

In this talk, we investigate the existence of solutions for a class of integrodifferential Kirchhoff equations. These equations involve a nonlocal operator with a measurable kernel that satisfies "structural properties" that are more general than the standard kernel of the fractional Laplacian operator. Additionally, the potential can be periodic or asymptotically periodic, and the nonlinear term exhibits critical exponential growth in the sense of Trudinger–Moser inequality. To guarantee the existence of solutions, we employ variational methods, specifically the mountain-pass theorem. In this context, it is important to emphasize that we have additional difficulties due to the lack of compactness in our problem, because we deal with critical growth nonlinearities in unbounded domains. Moreover, the Kirchhoff term adds complexity to the problem, as it requires suitable calculations for control the estimate the minimax level, representing the main challenge in this work. Finally, we consider two different approaches to estimate the minimax level. The first approach is based on a hypothesis proposed by D. M. Cao, while the second one involves a slightly weaker assumption addressed by Adimurthi and Miyagaki.

The extremal problem for Sobolev inequalities with upper order remainder terms

Flávio Almeida Lemos, Patricia Leal da Cunha

Universidade Federal de Ouro Preto

Given a smooth compact Riemannian n-manifold (M, g), we prove existence and compactness results of extremal functions for sharp Sobolev inequalities which are closely related to the embedding of $H^{1,q}(M)$ into $L^{qn/(n-q)}(M)$ where the L^q remainder term is replaced by upper order terms.

The goal is to discuss the existence of extremal functions of Sobolev type inequality modeled on smooth compact Riemannian manifolds.

Regularity for a class of fully nonlinear PDEs with double degeneracy and variable order

Giane Casari Rampasso, Elzon C. Bezerra Júnior, João Vitor da Silva, Gleydson C. Ricarte
Universidade Federal de Itajubá

The purpose of this work is to discuss interior regularity estimates for solutions of a class of problems governed by double degenerate fully nonlinear elliptic operators and variable order. One of the main signatures of this model is its interplay between two different kinds of degeneracy laws of variable order, according to the null set of a non negative modulating function. Our findings include Hölder continuity of the gradient and geometric non-degeneracy properties of solutions.

Positive stationary solutions of Kirchhoff equations with convection terms in a possibly non-coercive setting

Gustavo Ferron Madeira
UFSCar

We will comment on the existence of positive stationary solutions of a class of Kirchhoff equations. The nonlocal coefficient is non-autonomous and the source may depend on convection terms, ingredients precluding a variational approach in general. Furthermore, unlike the coercive Dirichlet problem a lack of coerciveness may take place under Neumann boundary condition. Our aim is to discuss an existence result on the positive stationary solutions in the Neumann BC case. Examples showing the existence of positive stationary solutions for several classes of source terms will be also discussed.

The method of the Nehari Manifold on cones

João R. Santos Junior
Universidade Federal do Pará

In this talk we develop an abstract theory of the Nehari method on cones. We apply our method to several elliptic problems involving p-laplacian and Kirchhoff operator as well as in some classes of problems arising in population dynamics.

This is a joint work with Denilson Pereira(UFCG) and Felipe Silva(UFPA).

Fractional Sobolev regularity for fully nonlinear elliptic equations

Makson Santos, Edgard Pimentel, Eduardo Teixeira
Instituto Superior Técnico - Universidade de Lisboa

We study high-order fractional Sobolev regularity for fully nonlinear, uniformly elliptic equations, in the presence of unbounded source terms. Our techniques are based on touching the solution with $C^{1,\alpha}$ cone-like functions to produce a decay rate of the measure of certain sets.

Chaotic dynamics in a model for suspension bridge-type structures

Maurizio Garrione, Fabio Zanolin
Politecnico di Milano

In this talk, we consider a degenerate plate-type model inspired by the dynamics of suspension bridges, consisting in a system of PDEs describing the evolution of both the longitudinal and the torsional oscillations. When both the oscillations are uni-modal, a coupled system of ODEs is obtained, for which we discuss the occurrence of rich and complex dynamics. In particular, under suitable assumptions on the external force acting on the longitudinal component, we show the existence of infinitely many periodic (harmonic and subharmonic) longitudinal motions whenever the torsional component is small. The goal is achieved by applying a rigorous analytical approach, based on the theory of linked twist maps.

The isoanisotropic problem to fundamental p-frequencies of membranes

Raul Fernandes Horta, Raul Fernandes Horta, Emerson Abreu, Marcos Montenegro
UFMG

This lecture concerns to anisotropic fundamental p-frequencies of membranes $\Omega \subset \mathbb{R}^2$ corresponding to functions $H\colon \mathbb{R}^2 \to \mathbb{R}$, which are defined by

$$\lambda_{1,p}^{H}(\Omega) = \inf \left\{ \int_{\Omega} H^p(\nabla u)\, dA : u \in W_0^{1,p}(\Omega), \|u\|_p = 1 \right\}, \tag{8}$$

where $p > 1$ and H belong the class of functions

$$\mathcal{H} = \left\{ H\colon \mathbb{R}^2 \to \mathbb{R} : H \text{ is nonnegative, convex and 1-homogeneous} \right\}.$$

This class is a subset of the Banach space of 1-homogeneous continuous functions endowed with the norm

$$\|H\| = \max_{|\xi|=1} H(\xi),$$

where $|\cdot|$ is the standard euclidean norm.

We present necessary and sufficient conditions for the existence of minimizers in (8) for degenerate $H \in \mathcal{H}$ and discuss on the related isoanisotropic problem.

By isoanisotropic problem we mean the study of existence and characterization of extremizers for the conditioned optimization problems

$$\begin{aligned} \lambda_{1,p}^{\max}(\Omega) &= \sup \left\{ \lambda_{1,p}^{H}(\Omega) : H \in \mathcal{H}, \|H\| = 1 \right\}, \\ \lambda_{1,p}^{\min}(\Omega) &= \inf \left\{ \lambda_{1,p}^{H}(\Omega) : H \in \mathcal{H}, \|H\| = 1 \right\}. \end{aligned}$$

Existence and Multiple of Solutions for a class Integro-Differential Equations with singular term via Variational and Galerkin Methods

Romildo N. de Lima, Romildo N. de Lima, Gelson C. G. dos Santos, Natan de Assis Lima
Universidade Federal de Campina Grande

In this work, we focus our attention on the singular and nonlocal equation

$$\begin{cases} -div(a(x)\nabla u) + \eta\psi(x)\displaystyle\int_\Omega \varphi u = \frac{\lambda}{(u^+)^\alpha} + (u^+)^p \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

for which we prove the existence of solutions. Here $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $u^+ = \max\{u, 0\}$, $\eta \in \mathbb{R}, \lambda > 0, \alpha \in (0, 1)$, $p \in (0, 2^*)$ and a, ψ, φ are functions whose properties will be timely introduced.

Problems involving the fractional g-Laplacian with Lack of Compactness

Sabri Bahrouni
University of Monastir

In this talk, we present a comprehensive exploration of compact embedding within a subspace of the fractional Orlicz-Sobolev space, denoted as $W^{s,G}\left(\mathbb{R}^N\right)$, specifically focusing on radial functions. Our primary objective is to establish compact embeddings into target spaces of the Orlicz type. Additionally, we delve into the well-known concentration-compactness principle within the realm of fractional Orlicz-Sobolev spaces.

As a practical application of our findings, we investigate the existence of solutions to quasilinear elliptic problems in the entire space $\mathbb{R}^N$ that involve the fractional $g-$Laplacian operator.

Infinitely many sign changing radial solutions for a phi-Laplacian Dirichlet problem

Sigifredo Herrón, Emer Lopera, Diana Sánchez
Universidad Nacional de Colombia Sede Medellín

We establish the existence of a countably infinite family of radially symmetric solutions that exhibit sign variations. These solutions are obtained for a Dirichlet boundary value problem that incorporates the ϕ-Laplace operator. Our main tools are the shooting method, phase plane and energy analysis, which demand extensive use of a Pozohaev-type identity. More exactly, we consider

$$\begin{cases} -\Delta_\phi(u) = W(x)f(u), & x \in B_1(0), \\ u(x) = 0, & x \in \partial B_1(0), \end{cases} \tag{9}$$

where $B_1(0) \subset \mathbb{R}^N (N > 2)$ is the unit ball, Δ_ϕ denotes the ϕ-Laplace operator, which is defined as

$$\Delta_\phi(u) = \text{div}(\phi(|\nabla u|)\nabla u),$$

$W > 0$ is a C^1- weight and ϕ is given by

$$\phi(s) = \frac{|s|^{p-2}}{(1+s^2)^{m/2}},$$

with $m > 0$ small. We consider the non-linearity $f : \mathbb{R} \to \mathbb{R}$

$$f(s) := \begin{cases} s^{q_1}, & s \geq 0, \\ -|s|^{q_2}, & s < 0, \end{cases}$$

with

$$2 < p < q_1 + 1 < Np(q_1 + p - 1)/[Nq_1 + (p-1)(N-p)], \quad q_2 > p^* - 1 \tag{10}$$

and $p^* = \frac{Np}{N-p}$ $(N > p)$ is the well-known Sobolev critical exponent.

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Harmonic Analysis and Related Topics

Organizer: Guilherme da Silva (USP/Brazil), Lucas Oliveira (UFGRS/Brazil) & Tiago Picon (USP/Brazil)

A journey with positive definite functions to change dimension

Ana Paula Peron, Xavier Emery, Emilio Porcu
Universidade de São Paulo

In this lecture we will present the concept of positive definite functions and some of their elementary properties. We will take a journey with these functions to see how it is possible to walk through dimensions.

Sharp embeddings between weighted Paley-Wiener spaces.

Andrea Olivo
BCAM

The aim of this talk is to discuss a family of extremal problems that arise from estimating the operator norm of certain embeddings between weighted Paley-Wiener spaces. In general, we study the asymptotic behavior of this norm and, in some particular cases, we can determine the sharp constants through the theory of reproducing kernel Hilbert spaces. Also, we will present some connections with other extremal problems and a direct connection with Poincaré inequalities.

iOverdetermined Elliptic Boundary Value Problems in Uniformly Rectifiable Domains

Artur Andrade, Dorina Mitrea, Irina Mitrea, Marius Mitrea
Temple University

A number of physical phenomena are modeled by overdetermined boundary value problems, that is, boundary problems in which one imposes both Dirichlet and Neumann type boundary conditions.

The subject of this talk is the analysis of overdetermined boundary value problems (OBVP) for 2nd order, homogeneous, constant complex coefficient, weakly elliptic systems in non-smooth domains with boundary datum in Whitney–Lebesgue spaces with integrability index in the interval $(1, \infty)$. This analysis includes integral representation formulas, jump formulas, existence and uniqueness of solutions for the OBVP in uniformly rectifiable domains, and classical Hardy spaces associated with systems.

This is joint work with Irina Mitrea (Temple University), Dorina Mitrea and Marius Mitrea (Baylor University).

Hardy spaces in the unit disk and a density criterion for the Riemann hypothesis

Charles F. dos Santos
ICMC-USP, São Carlos, Brasil

Báez-Duarte's criterion asserts that the Riemann hypothesis (RH) is equivalent to the density of the linear span of a particular sequence in $L^2([0,1])$. This work builds on a unitarily equivalent version

in H^2. Here, H^p $(p > 0)$ are the Hardy of the unit disk, consisting of holomorphic functions with bounded integral p-means in circles centered at 0. Namely, the H^2-criterion reads: RH holds if and only if the linear span $\mathcal{N}$ of $\{h_k\}_{n=2}^{\infty}$ is dense in H^2, where

$$h_k(z) = \frac{1}{1-z}\log\left(\frac{1+z+\cdots+z^{k-1}}{k}\right), \qquad z \in \mathbb{D},\ k \geq 2\,.$$

This poses the following two questions: (A) *Density problem* How to weaken the H^2 topology assuring the density of $\mathcal{N}$? (B) *Orthogonality problem* What H^2 elements can be orthogonal to $\mathcal{N}$? For question (A), our answers are: $\mathcal{N}$ (unconditionally) is dense in H^p for $0 < p < 1$ and density in H^p for some $p > 1$ implies absence of zeros for the Riemann zeta function in the half-plane $\{s = \sigma + it \in \mathbb{C} : \sigma > 1/p\}$. For question (B), we can roughly say that any non-null function orthogonal to $\mathcal{N}$ must have a kind of irregular behaviour at the boundary. For example, they cannot have Hölder-continuous extension to the unit circle. This is a joint work with Waleed Noor (IMECC/Unicamp), A. Ghosh and K. Kremnizer (Oxford University, UK).

The Ramanujan tau-function

Dimitar K. Dimitrov

Universidade Estadual Paulista

We repost some analytic, arithmetic and combinatorial aspects of the celebrated Ramanujan tau-function.

The Poly-Cauchy Operator in Uniformly Rectifiable Domains in the Complex Plane

Jeongsu Kyeong, Dorina Mitrea, Irina Mitrea, Marius Mitrea

Temple University

The classical Cauchy integral operator is one of the most famous and most studied singular integral operator in mathematics. In this talk, I will be presenting a higher-order analogue of the existing theory for the classical Cauchy operator, in which the salient role of the Cauchy-Riemann operator $\overline{\partial}$ is now played by natural powers of this. A central role will be played by integral representation formulas, jump relations and higher-order Fatou-type theorems.

This is joint work with Irina Mitrea (Temple University), Dorina Mitrea and Marius Mitrea (Baylor University).

Pseudodifferential operators over matrix algebras and groups

José Manuel Conde Alonso, Adrián Manuel González Pérez, Javier Parcet, Eduardo Tablate

Universidad Autónoma de Madrid

Pseudodifferential operators are generalizations of Fourier multipliers. Given a *symbol* $a : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{C}$, the associated pseudodifferential operator ψ_a has the form

$$\psi_a(f)(x) = \int_{\mathbb{R}^n} a(x,\xi)\widehat{f}(\xi)e^{2\pi i x\cdot\xi}d\xi.$$

In this talk, we study extensions of pseudodifferential operators to (unimodular) groups G with differential structures given by adequate cocycles. Our symbols are now functions $a : G \to \mathcal{L}(G)$, and we get column and row objects

$$\psi_a^c(f) = \int_G a(g)\widehat{f}(g)\lambda_g d\mu_G(g), \text{ and } \psi_a^r(f) = \int_G \widehat{f}(g)\lambda_g a(g) d\mu_G(g).$$

We study $L_p(\mathcal{L}G)$-boundedness results of ψ_a^c and ψ_a^r via transference to natural objects that act on $S_p(G)$, under smoothness assumptions on the cocycle derivatives of a. The column/row nature of the objects leads naturally to Hardy-type estimates when $p \neq 2$.

Based on work in progress with Adrián González Pérez, Javier Parcet and Eduardo Tablate.

Matrix valued orthogonal polynomials and Darboux factorizations

Lucia Morey, Erik Koelink, Pablo Roman

CONICET - UNC

Matrix-valued orthogonal polynomials (MVOPs) go back to the work of M.G. Krein around 1950. These are polynomials taking their values in the algebra of N $\times$ N-matrices satisfying a suitable matrix-valued orthogonality relation. Many properties of matrix-valued orthogonal polynomials are along the lines of corresponding well-known properties of the scalar-valued orthogonal polynomials. However, the explicit construction of families of arbitrary size is still a difficult problem.

In this talk, we will discuss Darboux factorizations of second order differential operators associated to a family of MVOPs. We will show that such a factorization can be used to construct new families. In the simplest case, this construction leads to lowering and rising shift operators for matrix valued orthogonal polynomials. More general factorizations lead to matrix valued analogues of exceptional polynomials. We will discuss the main properties and explicit examples.

A New Class of Herz-type Spaces and Boundedness Results

Pedro Takemura, Marius Mitrea

Baylor University

The theory of Herz spaces has seen numerous developments in the past two decades, resulting in a large family of function spaces that are relevant from the perspective of Harmonic Analysis. Their common nature invites the question of whether there is a larger functional analytic framework that unifies the existing body of work. In this talk we introduce a new class of Herz spaces, called Grand Herz spaces, which goes in the direction of answering such a question. This is done through the (recently) established groundwork of the theory of Generalized Banach Function Spaces. We provide several examples to illustrate how various Herz-type spaces fit in our theory and, as an application, we present a boundedness result for a broad class of sublinear operators. This is joint work with Marius Mitrea.

A new class of FBI transforms and applications

Renan Dantas Medrado

Universidade Federal de Alagoas

In this talk I will present a class of FBI transforms using weight functions (which includes the subclass of Sjostrand's FBI transforms used by M. Christ in 1997) that is well suited when dealing with

ultradifferentiable functions and ultradistributions defined by weight functions in the sense of Braun, Meise and Taylor (BMT). I will show how to characterize BMT local regularity of ultradistributions using this wider class of FBI transform. Also, I will present a characterization of Iterates of Partial Differential Operators in the sense of BMT classes (called BMT vectors) and, as an application, I will present a relation between BMT local regularity and BMT vectors. This is a joint work with G. Hoepfner.

Complex interpolation of matrix weighted L^p spaces and commutator estimates

Willian Corrêa, Félix Cabello Sánchez

Universidade de São Paulo

The celebrated Coifman-Rochberg-Weiss Theorem affirms that if $b \in BMO$ and T is a Calderón-Zygmund operator then

$$\|[b,T] : L^p \to L^p\| \leq C\|b\|_{BMO}$$

for $1 < p < \infty$, where C depends only on p and T, and $[b,T] = bT - Tb$. We explore a known proof of the Coifman-Rochberg-Weiss Theorem based on complex interpolation of weighted L^p spaces and extend it to the theory of Muckenhoupt matrix weights and matrix BMO functions.

Dynamical Systems via Ordinary Differential Equations

Organizer: Alex Carlucci Rezende (UFSCAr/Brazil) & Tiago Carvalho (USP/Brazil)

Limit cycles in refracted Hamiltonian systems with a straight switching line

Ali Bakhshalizadeh Badaki, Alex C. Rezende

Federal University of Sao Carlos

This paper presents a criterion that provides an easy sufficient condition for a collection of line integrals to have the Chebyshev property. The condition is based on the functions appearing in the line integrals. The criterion is used to study the number of limit cycles in refracted differential systems, which are formed by two Hamiltonian differential systems separated by a straight line. The paper concludes by presenting new results on such systems, which show the effectiveness of the criterion presented.

Crossing limit cycles for discontinuous piecewise linear differential centers separated by two circles

Ana Mereu, Sonia Isabel Renteria Alva

Universidade Federal de São Carlos

In this work, we study the existence of crossing limit cycles in discontinuous planar piecewise differential systems formed by linear centers and separated by two concentric circles. We establish that the upper bound for the number of limit cycles for this class of systems is 2 and provide examples showing that the maximum number of limit cycles can be reached. We also provide examples of such systems with zero and one limit cycle.

Melnikov analysis for a class of nonsmooth differential systems

Claudio Aguinaldo Buzzi

IBILCE/UNESP

We develop the Melnikov functions for a class of nonsmooth differential systems, which generalizes, up to order 2, some previous results in the literature. Whereas the first order Melnikov function for the nonsmooth case remains the same as for the smooth one (i.e. the first order averaged function) the second order Melnikov function for the nonsmooth case is different from the smooth one (i.e. the second order averaged function). We show that, in this case, a new term depending on the jump of discontinuity and on the geometry of the switching manifold is added to the second order averaged function. We apply this result showing that 7 is a lower bound for the Hilbert number of the family of piecewise linear differential systems in the plane with two pieces separated by a cubic curve.

The Nollet-Xavier conjecture

Francisco Braun, Luis Renato Gonçalves Dias, Jean Venato-Santos
UFSCar

The claim "a local self-diffeomorphism of $\mathbb{R}^n$ is globally injective if the pre-image of every hyperplane is connected" has been known as the Nollet-Xavier conjecture after a paper of 2002 by Nollet and Xavier. The validity of this conjecture implies the validity of the Jacobian conjecture. In this lecture we discuss the problem of global inversion of local self-diffeomophisms of $\mathbb{R}^n$ and present a counterexamle to the Nollet-Xavier conjecture.

This is joint work with L.R.G. Dias and J. Venato-Santos.

The Algebra behind Dynamical Cancellations of Isolated Invariant Sets

Ketty de Rezende, Ketty Rezende, Mariana Silveira, Dahisy Lima, Margarida Melo
UNICAMP - Universidade Estadual de Campinas

In this talk we exhibit homotopical algebraic invariants, such as the Conley index, that capture some dynamical features of isolated invariant sets. We also present algebraic tools to comprehend how certain algebraic invariants indicate changes to the flow within its phase space under homotopical deformation. Changes such as cancellation phenomena between invariant sets, death and birth of connections of the flow within its phase space are presented.

Limit Cycles for a Class of Discontinuous Piecewise Differential Systems

Leonardo Pereira Serantola, Jaume Llibre, Márcio Ricardo Alves Gouveia
UNESP

These last decades a big interest has appeared for studying the discontinuous piecewise differential systems. This is mainly due to the fact that these differential systems allow to modelize many natural phenomena. In order to describe the dynamics of a differential system we need to control its periodic orbits, and in special its limit cycles, i.e. we need to solve the so-called extended 16th Hilbert problem. In general, this is an unsolved problem. Here for the class of discontinuous piecewise differential systems formed by a linear center and an isochronous center with cubic homogeneous nonlinearities separated by a non-regular line we solve the extended 16th Hilbert problem, i.e. we provide an upper bound for the maximum number of limit cycles that these differential systems can exhibit and additionally we prove that this upper bound is reached.

Phase portraits of a family of Hamiltonian cubic systems

Luci Any Roberto, Marcio Gouveia - IBILCE/UNESP, Jaume Llibre - UAB/Barcelona
IBILCE/UNESP

While all the phase portraits of the quadratic polynomial Hamiltonian systems in the Poincaré disc were classified in 1994 (see [1]), we are far from the classification of the phase portraits of the cubic polynomial Hamiltonian systems in the Poincaré disc. In this work, we deal with the one-parameter family of cubic polynomial Hamiltonian systems

$$\dot{x} = y - y(y^2 + 3x^2\mu), \qquad \dot{y} = x + x(x^2 + 3y^2\mu),$$

where $(x, y) \in \boldsymbol{R}^2$ are the variables and μ is a real parameter. We classify in the Poincaré disc the topological phase portraits of this family of systems when the parameter μ varies, describing the bifurcations which take place. This is a joint work with J. Llibre (UAB/Barcelona) and M. Gouveia (IBILCE/UNESP)

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Topological equivalence in the infinity of a planar vector field and its principal part defined through Newton polytope

Otavio Henrique Perez, Thaís Dalbelo, Regilene Oliveira

ICMC USP

The main goal of this study is to establish non-degeneracy conditions under which a real planar polynomial vector field X is topologically equivalent to its *upper principal part* X^U_Δ defined by the Newton polytope in a neighbourhood of the infinity. Such a result assures that the singularities of X and X^U_Δ positioned in the infinity have the same qualitative behavior (under topological equivalence), and it can be seen as a version of the main Theorem presented in [1, 2].

This work is supported by São Paulo Research Foundation (FAPESP) grant 2021/10198-9.

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Piecewise Holomorphic Systems

Paulo Ricardo da Silva, Armengol Gasull, Gabriel Rondon

Unesp

Holomorphic systems are well known in the literature due to their important properties: reversibility, integrability, non-existence of limit cycles and complete knowledge of the phase portraits around their non-essential singularities. In this talk we are concerned about studying the piecewise holomorphic systems (PWHS). Specifically, we study the sewing, sliding, and tangential regions of the PWHS and we classify the typical singularities of these systems. Also, we are interested in understanding how the trajectories of the regularized system associated with the PWHS transit through the region of regularization. In addition, we know that holomorphic systems have no limit cycles, but piecewise holomorphic systems do, so we provide conditions to ensure the existence of limit cycles of these systems. Additional conditions are provided to guarantee the stability and uniqueness of such limit cycles.

Existence of crossing limit cycles in planar discontinuous fast-slow systems

Pedro Toniol Cardin

Universidade Estadual Paulista

In this lecture we will address a kind of global phenomenon known as *relaxation oscillation*, which typically occurs in fast-slow dynamical systems. Such phenomena are characterized by a special type of periodic motion which consists of long periods of quasi-static behavior interspersed with short periods of rapid transition. In the context of smooth fast-slow systems, the study on relaxation oscillations has a long history, which started with the mathematician Balthasar van der Pol in 1926. However, in the context of discontinuous (or piecewise smooth) fast-slow systems, the study on relaxation oscillations is still recent and has produced interesting results. In this presentation, we provide a geometric analysis of crossing limit cycles of the relaxation type in the context of planar discontinuous fast-slow systems.

The Orbit Space Approach for Piecewise Smooth Vector Fields and transitivity

Régis Varão

Unicamp

We propose the understanding of transitivity of a Piecewise Smooth Vector Fields in term of orbit space. We also provide that in some context transitivity for Piecewise Smooth Vector Fields implies chaos.

References

[1] Otávio M. L. Gomide and Pedro G. Mattos and Régis Varão, The Orbit Space Approach for Piecewise Smooth Vector Fields, arXiv:2310.13804, (2023).

The Ricci Flow on homogeneous manifolds: geometry meets dynamical systems

Ricardo M Martins

Unicamp

In this talk, we discuss how methods from the qualitative theory of differential equations are being used to study the existence of metrics on homogeneous manifolds. We will present some recent results for certain classes of manifolds. First we focus on Einstein metrics on homogeneous manifolds with isotropy 2 and also the case of SU(3)/T (in this case we also consider metrics with positive Ricci curvature that evolves into an invariant metric of mixed Ricci curvature). The talk covers how to calculate the basin of attraction in some of the cases, as well as discussing basic ideas about how to completely classify the dynamics in the space of metrics. In the case of the Wallach flag manifold SU(3)/T2, we give a complete dynamics description of positively curved metrics.

Non-smooth flows on the 2-sphere

Rodrigo Donizete Euzébio

Federal University of Goiás

In this talk we consider non-smooth vector fields $X : \mathbb{R}^3 \to \mathbb{R}^3$ having the 2-sphere $\mathbb{S}^2$ as an invariant set. We show that non-smooth flows generated by X may produce non-trivial dynamics

on $\mathbb{S}^2$. We investigate the topological transitivity of those vector fields and the their robustness and structurally stability. In addition, we investigate the occurrence of the sliding motion on $\mathbb{S}^2$ which is a fundamental ingredient for the occurrence of nontrivial recurrence in this context. This is a joint work with Joaby Jucá (UFG) and Régis Varão (UNICAMP).

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Poster Section

Poster

Organizer: Everaldo de Mello Bonotto

Normalized solutions for a supercritical Schrödinger problems with vanishing potencial

Carolina Santana Tomaz, Olimpio Hiroshi Miyagaki
Universidade Federal de São Carlos

In this work, we are interested in prove the existence of normalized solutions of nonlinear Schödinger equation

$$-\Delta u + V(x)u + \lambda u = |u|^{p-2}u, \quad \text{in } \boldsymbol{R}^N,$$

in the mass supercritical and Sobolev subcritical case $2 + \frac{4}{N} < p < 2^*$. The existence of a solution $(u, \lambda) \in H^1(\boldsymbol{R}^N)$, with norm prescribed $\|u\|_2 = \rho$, will be assured under various technical conditions on the potencial $V : \boldsymbol{R}^N \to \boldsymbol{R}$, which is positive and vanishing at infinity. The result will be proved using a variational methods. The solution u is obtained as a critical point of a functional constrained a sphere S_ρ of $L^2(\boldsymbol{R}^N)$ and λ will be a Lagrange multiplier. This result is contained in [1].

The author is a master degree student in mathematics, PPGM-UFSCar, and she is supported by CAPES. The work under guidance of Prof. Olimpio Hiroshi Miyagaki.

References:

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Dirichlet Laplacian in quantum waveguides

Diana Bello, Alessandra Verri
Universidade Federal de São Carlos

Let $-\Delta_D^{\Gamma_\theta}$ be the Dirichlet Laplacian in a rectangular V-shaped waveguide Γ_θ, with θ a parameter of opening. The goal is to prove that the discrete spectrum of $-\Delta_D^{\Gamma_\theta}$ is non- empty and that there are a finite number of discrete eigenvalues. This number depends on θ and tends to infinity as $\theta \to 0$, as it was done in [1].

References:

[1] M. Dauge, Y. Lafranche, and N. Raymond, Quantum waveguides with corners, ESAIM, Proc., 35, 14–45 (2012).

De Giorgi's conjecture in low dimensions and limitations of its proof

Gabriel Augusto Correia
IMPA

De Giorgi's 1978 conjecture regarding the one-dimensional symmetry of bounded, monotone in one direction, entire solutions of Allen-Cahn's equation $\Delta u = u^3 - u$ is explored with focus on Ambrosio-Cabré's (2000) proof in three dimensions and the difficulties related to its adaptation to higher dimensions. More specifically, we explore the impossibility of meaningfully generalizing a crucial Liouville-type theorem used in the proof.

Multiplicity of solutions to a nonlinear elliptic problem on a Riemannian orbifold

Gustavo de Paula Ramos
Universidade de São Paulo

An n-dimensional orbifold $\mathcal{O}$ is a space where each $x \in \mathcal{O}$ has an open neighborhood locally modeled on a quotient $\mathbb{R}^n/G_x$, where G_x (the *local group* of x) denotes a finite group of diffeomorphisms of $\mathbb{R}^n$.

Aiming to further the study of nonlinear partial differential equations on orbifolds, the present work considers the semilinear elliptic problem

$$-\epsilon^2\Delta u + u = u|u|^{p-2} \quad \text{in } \mathcal{O},$$

where $(\mathcal{O}, \mathfrak{g})$ denotes an n-dimensional compact connected Riemannian orbifold; $n \geq 3$ and $2 < p < 2n/(n-2)$.

In this context, we employ the photography method introduced in Benci and Cerami (ARMA, 1991) to estimate the number of positive weak solutions to our problem for sufficiently small $\epsilon > 0$ in function of the Lusternik–Schnirelmann category of $Z^{\mathcal{O}}$, the submanifold of $\mathcal{O}$ whose elements are the points in $\mathcal{O}$ whose local group is as large as possible.

During the proof of such a bound, we encounter a novel phenomenom of concentration of least energy solutions around points in $Z^{\mathcal{O}}$ and we have to adapt the extension of the Riemannian center of mass introduced in Petean (JFA, 2019) to the context of Riemannian orbifolds.

Critical Schrödinger-Bopp-Podolsky system: solution in the semiclassical limit

Heydy Melchora Santos Damian, Gaetano Siciliano
Universidade de São Paulo

We consider the following critical Schrödinger-Bopp-Podolsky system

$$\begin{cases} -\varepsilon^2\Delta u + V(x)u + Q(x)\phi u = h(x,u) + K(x)|u|^4 u & \text{in } \mathbb{R}^3 \\ -\Delta\phi + \Delta^2\phi = Q(x)u^2 & \text{in } \mathbb{R}^3 \end{cases}$$

in the unknowns $u, \phi : \mathbb{R}^3 \to \mathbb{R}$ and where $\varepsilon > 0$ are parameters. The functions V, K, Q satisfy suitable assumptions as well as the nonlinearity h which is subcritical. We show existence of "small" solutions in the semiclassical limit, namely whenever $\varepsilon \to 0$.

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Existence of solution for two classes of quasilinear systems defined on a non-reflexive Orlicz-Sobolev Spaces

Lucas da Silva, Marco Aurélio Soares Souto
Universidade Federal de Campina Grande

In this work we establish the existence of nontrivial solution for two classes of quasilinear systems of the type

$$\begin{cases} -\Delta_{\Phi_1} u = F_u(x,u,v) + \lambda R_u(x,u,v) \ \text{ in } \Omega \\ -\Delta_{\Phi_2} v = -F_v(x,u,v) - \lambda R_v(x,u,v) \ \text{ in } \Omega \\ \ \ u = v = 0 \ \text{ on } \partial\Omega \end{cases}$$

where $\lambda > 0$ is a parameter, Ω is a bounded domain in $\mathbb{R}^N (N \geq 2)$ with smooth boundary $\partial\Omega$. The first class we drop the Δ_2-condition of the functions $\tilde{\Phi}_i (i = 1, 2)$ and assume that F has a double criticality. For this class, we use a linking theorem without the Palais-Smale condition for locally Lipschitz functionals combined with a concentration–compactness lemma for nonreflexive Orlicz-Sobolev space. The second class, we relax the Δ_2-condition of the functions $\Phi_i (i = 1, 2)$. For this class, we consider $F = 0$ and $\lambda = 1$ and obtain the proof based on a saddle-point theorem of Rabinowitz without the Palais-Smale condition for functionals Fréchet differentiable combined with some properties of the weak* topology.

The classical obstacle problem: an approach via variational inequalities

Luis Urbiñes
UNIVERSIDADE ESTADUAL DE CAMPINAS

A classic problem in Mathematical Physics refers to the equilibrium position of a elastic membrane (whose boundary is maintained fixed) on top of a given body (an obstacle) under the action of contact forces, e.g. friction, tension, resistance of air and gravity. Actually, this archetypal model is often called *obstacle problem* (cf. [1] e [3]). From a variational point of view, if the membrane is over a defined obstacle, such as the graph of a function $\varphi \in C^2(\Omega)$, them, the problem reduces to minimize a functional

$$\mathcal{J}(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 dx + \int_\Omega f v dx,$$

with $v \in \mathcal{K}$, and

$$\mathcal{K} = \left\{ v \in H^1(\Omega) : v - g \in H_0^1(\Omega) \quad \text{e} \quad v \geq \varphi \quad \text{em} \quad \Omega \right\}.$$

Thus, we will denote such functions as *admissible*. Moreover, assume that $g > \varphi$ em $\partial\Omega$ so that $\mathcal{K} \neq \emptyset$.

We must emphasize that the existence of a minimizer is a well-known fact and occurs through the application of the direct method of calculating variations (see [2]). For this work, we will use Stampacchia, Lax-Milgram, and Lions-Stampachia's theorems to prove the existence/uniqueness and regularity of solutions (see [4, Corollary A1.1.1])

Therefore, this work is part of my Master's Thesis advised by Dr. João Vitor da Silva (Unicamp).

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Perturbation Theory of Linear Operators

Marco Antonio Dourado, Antônio Luiz Pereira (Advisor)
Universidade de São Paulo

The perturbation theory, initiated by the works of Rayleigh and Schrödinger, constitutes currently an important tool in many areas of Mathematics Analysis, including differential equations. Considering parabolic equations depending on a parameter, it can be applied to study the bifurcation and stability of solutions.

In the present work, we shall consider the concept of a relatively bounded operator in order to obtain stability theorems, such as the preservation of closedness and selfadjointness of operators under a perturbation by a relatively bounded operator with relative bound less than one. Considering the so called deficiency index of an operator, it can be proven the well-know Kato-Rellich Theorem and, using the previous results, we study the selfadjointness of the Schrödinger Operator with a static potential by means of the essential selfadjointness of the Laplacian operator in some function spaces.

Nonlocal Diffusion Elliptic System Modelling The Behaviour Of a Bacteria And a Living Nutrient

Marcos Antonio Viana Costa, Antonio Suárez Fernández, Cristian Morales-Rodrigo
Universidade Estadual Paulista

In this work, we will present a result that provides the existence of a continuum of coexistence states for the following nonlocal and nonlinear elliptic system:

$$\begin{cases} -a\left(\int_\Omega v\right)\triangle u = \lambda u - u^2 + buv, & \text{in } \Omega \\ -\triangle v + \sigma v = \rho u, & \text{in } \Omega \\ u = v = 0, & \text{on } \partial\Omega \end{cases}, \tag{P}$$

where Ω is a bounded regular domain of $\mathbb{R}^N$, $N \geqslant 1$, $a : \mathbb{R} \to (0, +\infty)$ is a continuous increasing function, $b, \lambda \in \mathbb{R}$, $\rho \geqslant 0$ and $\sigma > 0$. In order to guarantee the existence of an unbounded continuum $\mathfrak{C}$ in $\mathbb{R} \times \left(C_0^1\left(\overline{\Omega}\right)\right)^2$ of coexistence states of (P) emanating from the point $(\lambda^*, 0, 0)$, where $\lambda^* := a(0)\,\lambda_1$ we will apply Theorem 4.1 of [3]. Furthermore, we prove results of non-existence and a priori bounds of coexistence states of (P) in cases where $b > 0$ and $b < 0$.

Joint work with Antonio Suárez Fernández (Universidad de Sevilla) and Cristian Morales-Rodrigo (Universidad de Sevilla).

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[1] M. Chipot. Remarks on some class of nonlocal elliptic problems. In Recent advances on elliptic and parabolic issues, pages 79-109. 2006.
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Stability and Bifurcations in the Hénon Map

Muriel Henrique Bueno, Tatiana Miguel Rodrigues de Souza
UNESP

The Hénon Map, introduced by Michael Hénon in 1976, is a two-dimensional discrete dynamical system defined by $H_{a,b}(x, y) = (1 - ax^2 + y, bx)$. In this study, the initial focus was on analyzing the fixed points of the map, determined by solving the equation $H_{a,b}(x, y) = (x, y)$. We explored the dynamical properties of the system with respect to the parameters a and b. This investigation involved the classification of fixed points, distinguishing between saddle points, attractors, and points of period-doubling for specific values of a. Classifying the fixed points provided a deeper understanding of the dynamic behavior of these points. The presence of attractors, which draw nearby solutions, and saddle points, characterized by instability in some directions and stability in others, was thoroughly examined. To visualize the changes in the system's solutions with variations in the parameter a, we generated a bifurcation diagram using MATLAB. This diagram facilitated the analysis of the transition from order to chaos, highlighting how the system's behavior transforms as a is modified.

Approximation Properties of Solutions for a Degenerate Elliptic Problem using Irregular Domains

Patrícia Neves de Araújo, Marcone Corrêa Pereira, Jean Carlos Nakasato
Universidade de São Paulo

In this work we study the existence and uniqueness of solutions of a degenerate elliptic problem given by

$$\begin{aligned} &-\frac{1}{a^n}[a^n u_x]_x + \beta u = f(u) \text{ in } (0,1) \\ &\lim_{x\to 0} a^n(x)u_x(x) = 0 \\ &u_x(1) = 0, \end{aligned} \tag{11}$$

where $a \in C^1([0,1])$ is such that $a(0) = 0$ and $a(x) > 0$ if $x \in (0,1]$, and $\beta > 0$ is a constant. We also investigate the properties of its approximation by a sequence of functions defined on a thin, irregular domain R^ε.

The non-autonomous nonlinear Schrödinger equation

Rodiak Nicolai Figueroa López, Marcelo J. D. Nascimento
Universidade Federal de São Carlos

We consider the non-autonomous non-linear Schrödinger equation with homogeneous Dirichlet boundary conditions:

$$\begin{cases} u_t - i\beta(t)\Delta u - i\gamma(t)g(|u|^2)u + i\delta(t)u + \eta(t)u = \epsilon(t)f(x,t), x \in \Omega, t > \tau, \\ u(x,t) = 0, x \in \partial\Omega, t > \tau, \\ u(x,\tau) = u_\tau(x), x \in \Omega, \end{cases} \tag{NNLS}$$

where $\Omega = (0,T)$ for $T < \infty$, when $N = 1$ and Ω is a bounded smooth domain if $N = 2$, $\tau \in \mathbb{R}$, $i = \sqrt{-1}$ with $u(x,t) : \Omega \times \mathbb{R} \to \mathbb{C}$. We impose some growth conditions for g and regularity conditions for f and assume that $\beta, \gamma, \delta, \eta$ and ϵ are continuous functions of real values of a variable real that satisfy some suitable conditions. This equation is a non-autonomous version of the equation studied in [3].

In this work (see [1]), we study the local and global existence of solutions to the problem (NNLS) using time rescaling and the theory of evolutionary processes (see [2]), and the existence of a pullback attractor in a suitable phase space. Furthermore, we will show that the pullback attractor has finite fractal dimension.

This work is together with Professor Phd. Marcelo J. D. Nascimento (UFSCar, Brasil).

References

[1] R. N. Figueroa-López and M. J. D. Nascimento, Long-time behavior for evolution processes associated with non-autonomous nonlinear Schrödinger equation, submitted in J. Differential Equations (2023).
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[3] J. M. Ghidaglia, Finite dimensional behavior for weakly damped driven Schrödinger equations. Ann. Inst. H. Poincaré Anal. Non Linéaire, 5 (4) (1988), 365-405.

Topological entropy and Markov chains for piecewise smooth vector fields

Tiago Carvalho
Universidade de São Paulo - Ribeirão Preto

In this poster we will present some recent results concerning topological entropy for piecewise smooth vector fields (even in the plane). Also, we present some Markov chains, corresponding to piecewise smooth vector fields, and we will be able to calculate the topological pressure of these piecewise smooth vector fields.

A Leray-Schauder continuation theorem and some applications to quasilinear elliptic problems

Vinicius Kobayashi Ramos
Universidade de Brasília

In this poster we will present a result that provides the existence of a continuum of positive solutions (λ, u) of $u = K(\lambda, u)$, emanating from a point (λ_0, u_0) with non zero Leray Schauder

Index, where K is a compact operator defined on $\overline{\mathcal{U}}$, $\mathcal{U}$ is an open subset of $\mathbb{R} \times E$ (E Banach space) and u_0 is an isolated solution of $u = K(\lambda_0, u)$. The result is an improvement of Theorem 2.2 of [1] which requires the set of solutions for $\lambda = \lambda_0$ to be unitary and $\mathcal{U} = \mathbb{R} \times E$. By applying the result for $\lambda_0 = 0$ and an appropriated $\mathcal{U}$, we prove that the problem

$$\begin{cases} -\Delta u - \lambda u \Delta(u^2) = \mu u - u^p & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with $\mu > \lambda_1$ and $p > 1$, admits a positive solution for each $\lambda > -1/\left(2\mu^{\frac{2}{p-1}}\right)$. Also we prove some existence and qualitative information about positive solutions of a Kirchhoff-Carrier-type problem.

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[2] Fernández-Rincón, S. & López-Gómez, J. The singular perturbation problem for a class of generalized logistic equations under non-classical mixed boundary conditions. Advanced Nonlinear Studies. **19**, 1-27 (2019)

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Crossing limit cycles of planar discontinuous piecewise differential systems formed by isochronous centres

Yagor Romano Carvalho, Claudio A. Buzzi, Jaume Llibre

ICMC/USP

These last years an increasing interest appeared in studying the planar discontinuous piecewise differential systems motivated by the rich applications in modelling real phenomena. The understanding of the dynamics of these systems has many difficulties. One of them is the study of their limit cycles. In this work, we study the maximum number of crossing limit cycles of some classes of planar discontinuous piecewise differential systems separated by a straight line and formed by of linear centres (consequently isochronous) and cubic isochronous centres with homogeneous nonlinearities. For these classes of planar discontinuous piecewise differential systems we solved the extension of the 16th Hilbert problem, i.e. we provide an upper bound for their maximum number of crossing limit cycles.

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Programme

(*) remote lecture

ICMC Summer Meeting on Differential Equations - 2024 Chapter

Schedule Overview

SUNDAY 28 from 16:00 to 18:00, Registration at the ICMC Auditorium (Building 6)

(*) Remote Lecture

	MONDAY 29	TUESDAY 30	WEDNESDAY 31
08:00-08:30	**Registration**		
08:30-09:00	**Opening**		
Auditorium	**PLENARY LECTURES**		
Chair	Irina Mitrea	Ederson Moreira dos Santos	Xiaoying Han
09:00-10:00	**Jerry Lloyd Bona**	**Liliane de Almeida Maia**	**Irina Mitrea**
10:00-10:30		Coffee Break and Poster Session	
10:30-11:30	**Denis Bonheure**	**Alberto Parmeggiani**	**Tomás Caraballo**
11:30-13:30		**Lunch**	
Auditorium	**PLENARY LECTURES**		
Chair	Gerardo Mendoza	Tomás Caraballo	Alberto Parmeggiani
13:30-14:30	**Xiaoying Han**	**Rosana Rodríguez-López***	**Gerardo Mendoza**
14:30-15:00		Coffee Break and Poster Session	
15:00-15:30	**Sessions 1-10**	**Sessions 1-10**	**Sessions 1-10**
	SOCIAL EVENTS		
11:30		**Photo**	
20:00		**Conference Banquet**	

Session 1 – Conservation Laws and Transport Equations
Session 2 – Domain Perturbation for PDEs and Applications
Session 3 – Free Boundaries Problems and Related Topics
Session 4 – Integral and Functional Differential Equations
Session 5 – Nonlinear Dynamical Systems
Session 6 – Dispersive Equations
Session 7 – Linear Equations
Session 8 – Elliptic Equations
Session 9 – Harmonic Analysis and Related Topics
Session 10 – Dynamical Systems via Ordinary Differential Equations

THEMATIC SESSIONS

Room 5-004	**Session 1 – Conservation Laws and Transport Equations**		
Chair	Wladimir Neves	Jean Silva	Grigori Chapiro
15:00-15:30	**Jean Silva**	**Wladimir Neves**	**Yulia Petrova**
15:30-16:00	**Gerardo Huaroto**	**Grigori Chapiro**	**Henrique Souza**
16:00-16:30	**João Nariyoshi**	**Claudia Espitia**	**Vernny Uriel Ccajma**
16:30-17:00	**Lauren Bonaldo***	**Luis Salvino**	**Marcelo Bongarti**
17:00-17:30		**Richard De la Cruz***	

Room 5-001	**Session 2 – Domain Perturbation for PDEs and Applications**		
Chair	Marcone Pereira		Alessandra Verri
15:00-15:30	**Alessandra Verri**		**Shuichi Jimbo**
15:30-16:00	**Leonardo Pires**		**Jean C. Nakasato**
16:00-16:30	**Gunvant A. Birajdar***		**Marcone C. Pereira**
16:30-17:00	**Carlos R. Mamani**		

Room 5-003	**Session 3 – Free Boundaries Problems and Related Topics**		
Chair	João Vitor da Silva	Claudia Lederman	Hernán Vivas
15:00-15:30	**Makson Santos**	**Mariana Smit Vega Garcia***	**Claudia Lederman**
15:30-16:00	**Aelson Sobral**	**Giane Rampasso**	**Jefferson Santos**
16:00-16:30	**Juan Pablo Cabeza**	**Elzon Bezerra Jr**	**João Vitor da Silva**
16:30-17:00	**Hernán Vivas**	**Romário Frias**	

Room 5-101	**Session 4 – Integral and Functional Differential Equations**		
Chair	Pierluigi Benevieri	Jaqueline Mesquita	
15:00-15:30	**Jaqueline Mesquita**	**Pierluigi Benevieri**	
15:30-16:00	**José Manuel Uzal**	**Manuel Jimenez**	
16:00-16:30	**Eduard Toon**	**Aldo Pereira**	
16:30-17:00	**Fernanda Andrade**	**Adriano Peixoto**	
17:00-17:30	**Pablo Amster***	**Antonio Veloso**	

Auditorium	**Session 5 – Nonlinear Dynamical Systems**		
Chair	Estefani Moreira	Maykel Boldrin Belluzi	Javier Lopez de la Cruz
15:00-15:30	**José Arrieta**	**Alexandre Nolasco**	**Denis Fernandes**
15:30-16:00	**Alexandre Oliveira Sousa**	**Arthur Cunha**	**Heráclio Lopez**
16:00-16:30	**Maykel Boldrin Belluzi**	**Jacson Simsen**	**Yessica Yulieth Julio**
16:30-17:00	**Javier Lopez de la Cruz**	**Ma To Fu**	
17:00-17:30	**Juan Garcia Fuentes**	**Estefani Moreira**	

Room 5-104	**Session 6 – Dispersive Equations**		
Chair	Marcia Scialom	Fábio Matheus Natali	
15:00-15:30	**Fábio Matheus Natali**	**Argenis Mendez**	
15:30-16:00	**Renata Figueira**	**Francisco Javier Leal**	
16:00-16:30	**Andressa Gomes**	**Raphael Antunes Santos**	
16:30-17:00	**Xavier Carvajal**	**Ricardo Carlos Freire**	
17:00-17:30	**Mahendra Panthee**		

Room 5-103	Session 7 – Linear Equations		
Chair	Nicholas Braun Rodrigues	Luis Fernando Ragognette	Gabriel Araújo
15:00-15:30	Marcelo Rempel Ebert	Rafael Borro Gonzalez	Luiz Fernando Ragognette
15:30-16:00	Ricardo Paleari Silva	Vinícius Novelli	Renato Laguna
16:00-16:30	Igor Ambo Ferra	Antonio Lagioia	Catarina Barbosa Machado
16:30-17:30	Dmitri Zaitsev	Dmitri Zaitsev	Dmitri Zaitsev

Room 5-002	Session 8 – Elliptic Equations		
Chair	Ederson M. Santos	Sergio Monari	Eugenio Massa
15:00-15:30	Maurizio Garrione	Edcarlos Silva	Makson Sales Santos
15:30-16:00	Giane Rampasso	Cláudia Santana	Eduardo Böer
16:00-16:30	Romildo Lima	Eudes Barboza	Gustavo Madeira
16:30-17:00	Sigifredo Herrón	João Rodrigues Santos	Alânnio Nóbrega
17:00-17:30	Flávio Lemos	Sabri Bahrouni	Raul Horta

Room 5-102	Session 9 – Harmonic Analysis and Related Topics		
Chair	Guilherme Silva	Tiago Picon	Lucas Oliveira
15:00-15:30	José Manuel C Alonso	Dimitar Dimitrov	Willian Corrêa
15:30-16:00	Andrea Olivo	Lucía Morey	Artur Henrique Andrade
16:00-16:30	Ana Paula Peron	Jeongsu Kyeong	Renan Medrado
16:30-17:00	Pedro Henrique T F Silva	Charles Ferreira Santos	

Room 4-001	Session 10 – Dynamical Systems via Ordinary Differential Equations		
Chair	Tiago Carvalho	Claudio Buzzi	Alex C Rezende
15:00-15:30	Ketty Rezende	Ana Cristina Mereu	Régis Varão
15:30-16:00	Paulo Ricardo Silva	Francisco Braun	Otávio Perez
16:00-16:30	Ricardo M Martins	Luci Any Roberto	Ali Bakhshalizadeh
16:30-17:00	Pedro Cardin	Leonardo Serantola	Rodrigo Euzébio
17:00-17:30	Claudio Buzzi		

Ground floor of the Library 10:00-10:30 and 14:30-15:00	Poster Session	
	Diana Carolina Bello	Carolina Santana Tomaz
	Gabriel Augusto Correia	Gustavo de Paula Ramos
	Lucas da Silva	Heydy Melchora Damian
	Luiz Carlos Urbiñes Suares	Marco Antonio Dourado
	Marco Antonio Viana Costa	Muriel Henrique Bueno
	Tiago Carvalho	Patricia Neves de Araújo
	Yagor Romano Carvalho	Rodiak Nicolai López
		Vinicius Kobayashi Ramos

	SOCIAL EVENTS
11:30	Photo
20:00	Conference Banquet

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

List of Speakers

List of speakers **(ordered by first name)**

ICMC SUMMER MEETING ON DIFFERENTIAL EQUATIONS 2024 CHAPTER

Sponsors

The ICMC Summer Meeting on Differential Equations - 2024 Chapter are promoted by Instituto de Ciências Matemáticas e de Computação (ICMC), Pró-Reitoria de Pesquisa e Inovação (PRPI), Instituto Nacional de Ciência e Tecnologia de Matemática (INCTMat), and Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro.

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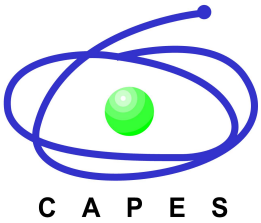

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