2019
CHAPTER

ICMC SUMMER MEETING
ON DIFFERENTIAL EQUATIONS

4-6 FEBRUARY 2019 | SÃO CARLOS-SP, BRAZIL

summer.icmc.usp.br

SESSIONS:

- Boundary Perturbations of Domains for PDEs and Applications
- Computational Dynamics in the Context of Data
- Dispersive Equations
- Elliptic Equations
- Evolution Equations and Applications
- Fluid Dynamics
- Linear Equations
- Nonlinear Dynamical Systems
- Ordinary/Functional Differential Equations
- Poster Session

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Tomás Caraballo
(Universidad de Sevilla/Spain)
Alexandre Nolasco de Carvalho
(USP/Brazil)
Shui-Nee Chow
(GaTech/USA)
Djairo G. de Figueiredo
(UNICAMP/Brazil)
John Mallet-Paret
(Brown University/USA)
Hildebrando Munhoz Rodrigues
(USP/Brazil)
Yingfei Yi
(University of Alberta/Canada and JLU/China)

USP ICMC USP SÃO CARLOS CAPES CNPq FAPESP

Welcome

It is a pleasure to welcome you to the *ICMC Summer Meeting on Differential Equations - 2019 Chapter* and to São Carlos. We wish you a pleasant stay and that you enjoy the meeting.

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Anne Bronzi (UNICAMP/Brazil) & César J. Niche (UFRJ/Brazil) - Fluid Dynamics

Marcelo R. Ebert (USP/Brazil) & Marcello D'Abbicco (Università di Bari/Italy) - Linear Equations

Marcio Gameiro (USP/Brazil), Konstantin Mischaikow (Rutgers University/USA),

Marian Mrozek (Jagiellonian University/Poland) & Thomas Wanner (George Mason University/USA) - Computational Dynamics in the Context of Data

Alessia Kogoj (Università degli Studi di Urbino Carlo Bo/Italy) & Giulio Tralli (Sapienza University of Rome/Italy) - Evolution Equations and Applications

Jaqueline G. Mesquita (UnB/Brazil)- Ordinary/Functional Differential Equations

Marcone C. Pereira (USP/Brazil) & Ricardo Parreira da Silva (UnB/Brazil) - Boundary Perturbations of Domains for PDEs and Applications

Juliana Fernandes (UFRJ/Brazil) - Nonlinear Dynamical Systems

Márcia A. G. Scialom & Mahendra Panthee (UNICAMP/Brazil) - Dispersive Equations

Address

ICMC Summer Meeting on Differential Equations - 2019 Chapter
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo
Avenida Trabalhador São-carlense, 400
CEP: 13566-590 - São Carlos - SP
FAX: +55 (16) 3371-2238
E-mail: summer@icmc.usp.br

ICMC SUMMER MEETING on
DIFFERENTIAL EQUATIONS
CHAPTER 2019

Maps

Nome/Name: _____

Hosted in São Sarlos at the hotel: _____

Address: _____

Phone number: _____

Contact: _____

Tipo sanguíneo / Blood type: _____

Tomando medicação/Taking medication: _____

Alérgico a medicamentos / Alergic to medication: _____

Doença crônica / Chronic disease: _____

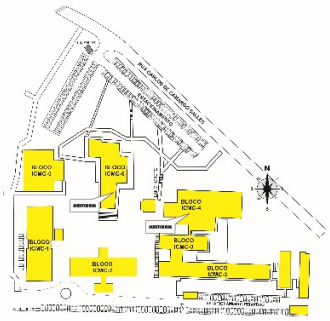
USEFUL NUMBERS

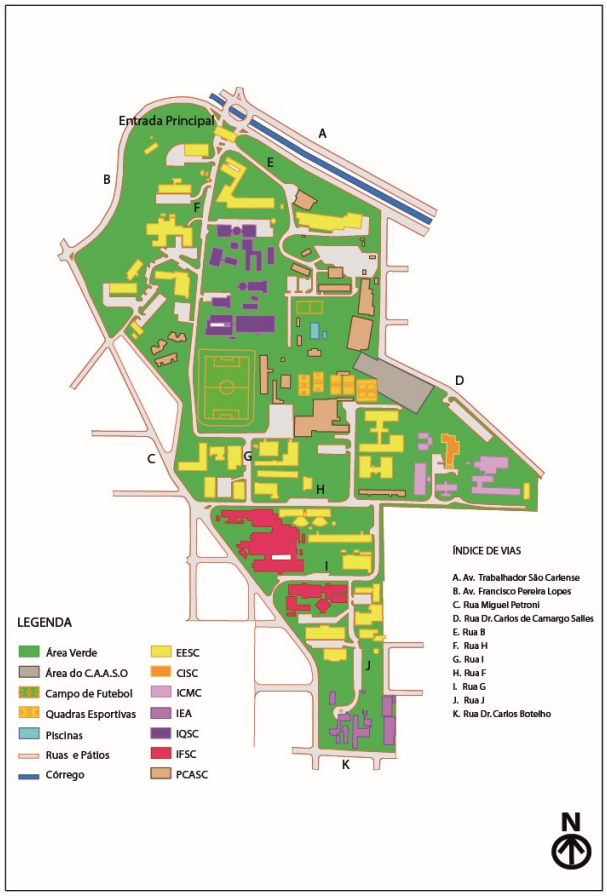
3373-9700	ICMC-USP
3373-9112	Campus Security Staff
190	Police Dept.
192	Medical Emergency
193	Fire Dept. and Rescue
0800-560159	National Courier Service
3371-2171	Local Post Office
3415-6005	Taxi-Cab Co.

www.visitesaocarlos.com.br/taxis.htm
(other Taxi-Cab Companies)

ENTRANCES AND EXITS OF THE CAMPUS

1. ICMC entrance
2. Main entrance
3. Observatory´s entrance
4. Physic´s institute entrance
5. Physic´s institute entrance





HOTÉIS - HOTELS

H1. Indaiá Hotel
R. Jacinto Favoreto, 351

H2. Atlantic Inn Residence
R. Salomão Dibbo, 321

H3. Indaiá Hotel Residence
R. Jacinto Favoretto, 782

H4. Parisi Hotel
Av. São Carlos, 3.163

H5. Hotel Anacã
Av. São Carlos, 2.690

H6. Hotel Ypê
Av. Dr. Carlos Botelho, 2.060

H7. Central Park Apart Hotel
Av. Francisco Pereira Lopes, 2.600

H8. Hotel Acaccio
Av. São Carlos, 1.981

H9. Atlantic Inn Royale
R. Conde do Pinhal, 1.630

H10. Hotel Caiçara
R. Conde do Pinhal, 2.217

H11. Hotel Othon Suítes
R. Cons João Alfredo, 77

H12. Ibis Hotel
R. Passeio dos Ipês, 140

H13. Sleep Inn
R. Cons João Alfredo, 455

BRAZILIAN SOUVENIRS

1. Papelaria CAASO (USP)
2. Praça XV de novembro (Sunday after 15h)
3. Duda Maria
   rua José Bonifácio, 1177 - 3307.2402

RECOMMENDED BAR, RESTAURANTS AND PIZZERIAS

R1. Restaurante Kallas
Av. São Carlos, 2784 - 3364.6362 - LUNCH/DINNER

R2. Quase 2 Restaurante e Hamburgueria
Av. São Carlos, 2796 - 3372.7240 - LUNCH and FAST FOOD at night

R3. Roda Chopp Restaurante
Av. São Carlos, 2603 - 3372.2343 - LUNCH ONLY

R4. Restaurante La Salute
R. Cons. João Alfredo, 47 - 3116.4939 - VEGETARIAN FOOD - LUNCH ONLY

R5. Casa do Café Restaurante
R. Riachuelo, 1201 - 3371.3033 - LUNCH/DINNER

R6. Cantina Ciao Bello
R. Riachuelo, 1.191 - 3371.4174 - LUNCH ONLY

R7. Restaurante Curinga
Av. Trabalhador Sãocarlense, 650 - 3413.3777 - LUNCH ONLY

R8. Barone Restaurante
Av. Dr. Carlos Botelho, 1.671 - 3412.8586 - LUNCH/DINNER

R9. Restaurante Mosaico
R. Aquidaban, 1.342 - 3371.4474 - LUNCH/DINNER

R10. Yo Konno
R. Aquidaban, 1368 - 3413.1666

R11. YouOki Culinária Asiática
R. Aquidaban, 1390 - 3415.2848 - ASIAN FOOD - LUNCH/DINNER

R12. Restaurante Paraíso
R. Dna. Maria Jacinta, 340 - 3376.6424 - LUNCH ONLY

R13. Habbib´s
Av. Trabalhador Sãocarlense, 620 - 3371.2223 - LUNCH/DINNER

R14. Bom Pedaço Pizza & Bar
Av. Trabalhador Sãocarlense, 584 - 3371.3696 - DINNER ONLY

R15. La Villa Restaurante e Lanchonete
R. Quinze de Novembro, 1330 - 3376.2843 - LUNCH/DINNER
Tip: Special brazilian food (Feijoada) and music at Saturdays

R16. Amici Pizza & Cibo
R. Quinze de Novembro, 1289 - 3371.6463 - DINNER ONLY

R17. West Brothers Chopp & Culinária
Av. Trabalhador Sãocarlense, 671 - 3415.5717 - DINNER ONLY

R18. Yasan Restaurante/Bar
Av. Dr. Carlos Botelho, 1768 - 3307.1165 - LUNCH/DINNER

R19. King Fish Restaurante
Av. Trabalhador Sãocarlense, 25 - 3412.7400 - DINNER ONLY

R20. Restaurante Picanha na Tábua
Av. Francisco Pereira Lopes, 2520 - 3361.1453 - LUNCH/DINNER

R21. McDonald´s
Av. São Carlos, 3134 - 3374.7402 - 24 hours

R22. Seo Gera
R. Episcopal, 2442 - 3372.1051 - DINNER ONLY

R23. Trembão Burger & Grill
Av. São Carlos, 3055 - 3307.6189 - DINNER ONLY

R24. Restaurante Mamãe Natureza
R. Cap. Adão Pereira Cabral, 457 - 3374.2653 - VEGETARIAN FOOD - LUNCH ONLY

R25. China in Box
Av. São Carlos, 3030 - 3376.1221 - LUNCH/DINNER

R26. Restaurante Panela
R. Dr. Orlando Damiano, 2157 - 3371.4157 - LUNCH/DINNER

R27. Casa do Filé Restaurante
R. Nove de Julho, 1688 - 3307.2412 - DINNER ONLY

R28. Cabanha Steakhouse & Bar
R. Episcopal, 1661 - 3364.3067 - LUNCH/DINNER

R29. Restaurante Frei Damião
R. Nove de Julho, 1606 - 3416.8133 - LUNCH ONLY

R30. Restaurante Frei Damião
R. Quinza de Novembro, 1447 - 3416-3276 - LUNCH ONLY

R31. Don Raffaele Pizze & Ristorante
R. Marechal Deodoro, 1758 - 3371.1478 - ONLY DINNER

R32. Subway
Av. São Carlos, 2911 - 3416.4000 - 24 hours

R33. Kalil Cozinha Árabe
R. Dr. Carlos Botelho, 1737 - 3372.1696
ARABIC AND VEGETARIAN FOOD - LUNCH/DINNER

R34. Café Sete - Café Gastrobar
R. Sete de Setembro, 1447 - 3415.1026 - DINNER ONLY

R35. Churrascaria Tabajara
Av. São Carlos, 3677 - 3361.4282 - STEAKHOUSE - LUNCH/DINNER

R36. Churrascaria Trevo
Av. Prof. Luis Augusto de Oliveira, 385
(extension of the Av. São Carlos) - 3361.3052 - STEAKHOUSE - LUNCH/DINNER

R37. Restaurante e Buffet Cantos e Contos
Rua Major José Inácio, 2211 - 3372.6661

R38. Restaurante Sushi Ya-San
Rua Tiradentes, 128 - 3307.1165 - JAPANESE FOOD - LUNCH/DINNER

R39. A Fábrica Cervejaria Artesanal
Rua 28 de Setembro, 2066 - 3372.6288 - DINNER ONLY

R40. Restaurante Niray
Rua Major José Inácio, 2273 - 3415-6505 - JAPENESE FOOD - LUNCH/DINNER

R41. Restaurante e Choperia Rola Papo
Rua Major José Inácio, 2270 - 3412-6757 - DINNER ONLY

R42. Água Doce Cachaçaria
Rua 9 de julho, 1625 - 3376.2077 - LUNCH/DINNER

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ICMC USP SÃO CARLOS


Rodovia Washington Luiz
UFSCAR
Churrascaria Trevo (R36)
Churrascaria Tabajara (R35)
USP (campus 2)
Café Tropical
Shopping Center
Ibis Hotel (H12)
Av. Comendador
Alfredo Maffei
USP - São Carlos
Campus 1
Al. dos Crisântemos
R. Alvarenga Peixoto
R. W. Cury
Av. Trabalhador Sãocarlense
Al. das Hortências
Al. das Crisandálias
Av. das Azaléias
Av. Prof. Luis A. Oliveira
R. Antonio Blanco
R. Dna. Alexandrina
R. São Joaquim
R. Salomão Dibbo
R. 9 de Julho
R. Achile Bassi
R. Cezar Ricomi
R. Jacinto Favoretto
Rodoviária
(Bus Station)
R. Episcopal
R. Adolfo Cattani
R. Alfredo Lopes
R. Orlando Damiano
R. Tiradentes
R. 28 de Setembro
R. Miguel Petrone
Rua Dr. Carlos de Camargo Salles
Bank of Brazil AG 6845-4
11h00m to 16h00m
MON to FRI
PAPELARIA
CAASO
BIBLIOTECA
PRAÇA
ALOGAMENTO
ICMC-USP
(ZOOM UP AT THE BACK OF THIS PAGE)
R. Conselheiro João Alfredo
R. Dna. Maria Jacinta
R. Dr. Serafim Vieira de Almeida
Santa Casa
(Hospital)
R. Princesa Isabel
R. Maestro João Seppe
R. Paulino Botelho de Abreu Sampaio
R. Paulino Botelho A. Sampaio
R. Cap. Adão Pereira de Souza Cabral
Av. Dr. Carlos Botelho
Av. São Carlos
Praça
XV de
Novembro
R. 15 de Novembro
R. São Sebastião
R. Visconde de Inhaúma
R. Aquidaban
R. José Bonifácio
Escola
Dr. Álvaro
Guião
R. Padre Teixeira
R. Riachuelo
R. Marechal Deodoro
R. 7 de Setembro
Praça
dos
Pombos
R. Major José Inácio
R. Conde do Pinhal
Catedral
Praça
da
Catedral
Av. Comendador Alfredo Maffei
Centro
Portaria "A"
Portaria "B"
Portaria "C"
Portaria "D"
Campus Entrances
Money Exchange
Hotel
ATM
Drugstore
Supermarket-Grocery
Post Office
Bus Station
Bakery-Coffee-Snack bar
Restaurant
Other restaurant or Pizzeria
Açaí (Brazilian Ice Cream)
Bar Restaurant

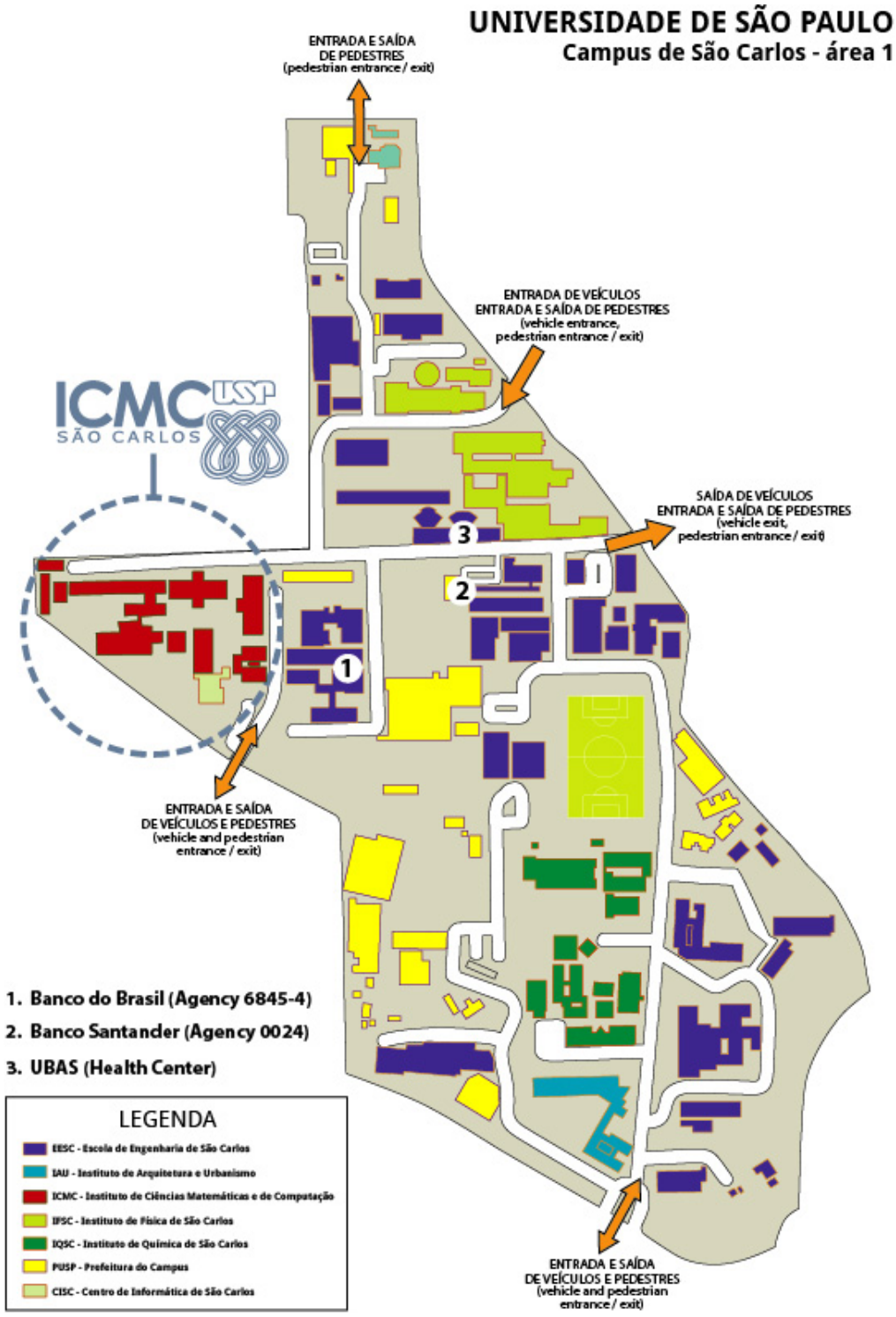

Figure 1: Campus map

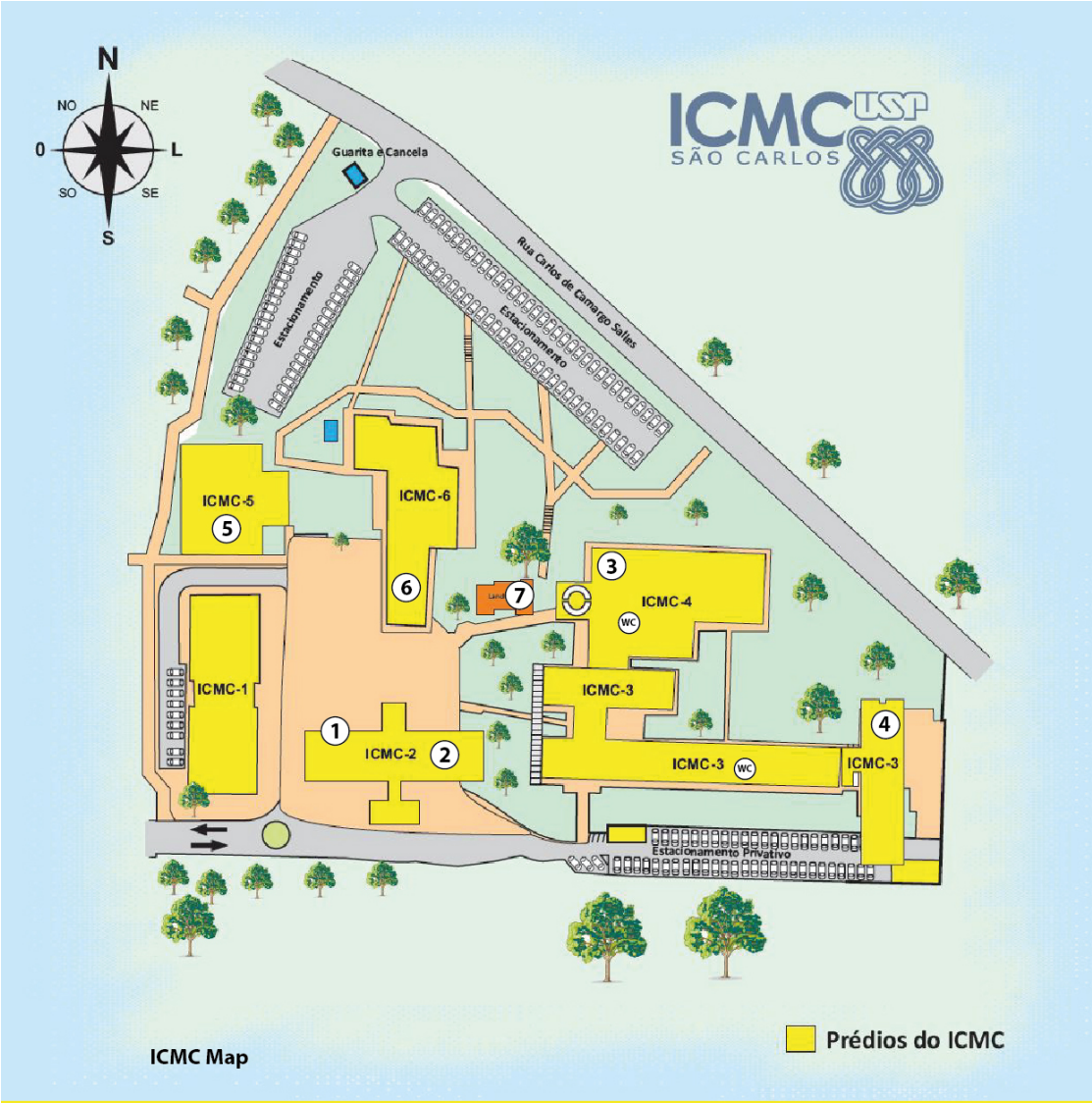

Figure 2: ICMC map

Contents

ICMC SUMMER MEETING on
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General Information

Conference site

The meeting will take place at Buildings 5 and 6, and also in the Coffee Area, of ICMC. All these buildings are indicated in the map on page 8.

Plenary talks will take place at the Auditorium (Building 6).

Special session on Boundary Perturbations of Domains for PDEs and Applications in Room 5104 (Building 5).

Special session on Computational Dynamics in the Context of Data in Room 31214 (Cetisc Building 6).

Special session on Dispersive Equations in Room 5102 (Building 5).

Special session on Elliptic Equations in Rooms 5001 and 5003 (Building 5).

Special session on Evolution Equations and Applications in Room 5101 (Building 5).

Special session on Fluid Dynamics in Room 5103 (Building 5).

Special session on Linear Equations in Room 5002 (Building 5).

Special session on Nonlinear Dynamical Systems at the Auditorium (Building 6).

Special session on Ordinary/Functional Differential Equations in Room 5004 (Building 5).

Poster session in Coffee Area (ground floor of the Library).

Registration

The registrations will be made in the following schedule:

Sunday, February 3^{th}: From 17:00hs to 19:00hs in the entrance of the ICMC Auditorium (Building 6).

Monday, February 4^{th}: From 8:00hs to 8:50hs in the entrance of the ICMC Auditorium (Building 6).

We will provide you a badge at registration. Please wear your badge at the event.

The Events Office (3, map at page 8) will be at your disposal for any questions and information, also the Events Office will set up a help desk with all the necessary information at the entrance of the Auditorium.

Registration fees

Student Fee: R$ 50,00 (reais).

Regular Fee: R$ 100,00 (reais).

Regular and student registration fees include access to sessions and lectures, conference materials and refreshments at coffee breaks.

Financial support

The financial support from the local organizing committee will be available on Tuesday, February 5^{th}, at the help desk at the entrance of the Auditorium (Building 6, map of page 8). In order to receive your support, it is mandatory to completely fill out the on-line registration form available at `http://summer.icmc.usp.br/user_summer/index.php` .

Meals and refreshments

There are several restaurants near the campus. You can find them by looking at the city map located on pages 5 and 6. There are also choices of pizzerias. At night, there are many bars around the city.

There is also a canteen on the campus (7, map at page 8) where you can have either snacks or lunch.

Social events

Monday, February 4^{th}*:* Photo of the meeting at 12:10hs at ICMC.

Tuesday, February 5^{th}*:* Conference Banquet at 20:00hs at Café Sete at 1447, Sete de Setembro Street (R34, map at pages 5 and 6).

Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

Money exchanges

In case you need to exchange your money, we recommend:

• Confidence Câmbio at Shopping Center Iguatemi. The working hours are from 8:00 to 19:00 (Mon-Fri) and from 9:00 to 16:00 (Sat).

• JIS Câmbio at 1976, Episcopal Street. The working hours are from 9:30 to 17:30 (Mon-Fri).

Smoking

Smoking is prohibited inside any of the ICMC buildings also in the canteen and on the ground floor of the library.

Computer and wireless LAN use

There will be available computers and a printer for use at the lobby of the Auditorium.

The University provides access to wireless internet connection via **eduroam**. If you do not possess an eduroam account you can access another of our wireless connection through the following steps:

1. Enable wireless on your device.
2. Join the ICMC-GUEST wireless network.
3. Open a browser and try to visit any website.
4. You will be redirected to a login page. Enter the login and password as follows:

   User Name: *summer*

   Password: smode19@
5. You may freely browse the internet after logging in. You may occasionally need to re-authenticate using the above procedure.

ICMC SUMMER MEETING on
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Plenary Talks

Plenary Speaker

Topological decoupling and linearization of nonautonomous evolution equations

Christian Poetzsche, Evamaria Russ
Alpen-Adria University Klagenfurt, Austria

Topological linearization results typically require solution flows rather than merely semiflows. An exception occurs when the linearization fulfills spectral assumptions met e.g. for scalar reaction-diffusion equations. We employ tools from the geometric theory of nonautonomous dynamical systems in order to extend earlier work by Lu [1] to time-variant evolution equations under corresponding conditions on the Sacker-Sell spectrum of the linear part. Our abstract results are applied to nonautonomous reaction-diffusion and convection equations.

The talk is based on joint work with Evamaria Russ.

References:

[1] A Hartman-Grobman theorem for scalar reaction-diffusion equations. *J. Differ. Equations* **93**, 364–394, 1991

Existence and concentration phenomena for a class of indefinite variational problems with critical growth

Claudianor O. Alves, Geilson F. Germano
Universidade Federal de Campina Grande, Brazil

In this talk we will show some results results involving the existence and concentration of ground state solution for the following class of indefinite variational problems

$$-\Delta u + V(x)u = A(\epsilon x)f(u), \quad x \in \mathbb{R}^N, \tag{P}_\epsilon$$

where $N \geq 2$, $\epsilon > 0$, $A : \mathbb{R}^N \to \mathbb{R}$ is a continuous function that satisfies

$$0 < \inf_{x\in\mathbb{R}^N} A(x) \leq \lim_{|x|\to+\infty} A(x) < \sup_{x\in\mathbb{R}^N} A(x) = A(0), \tag{A}$$

$f : \mathbb{R} \to \mathbb{R}$ is a continuous function having critical growth, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous and $\mathbf{Z}^N$–periodic function with $0 \notin \sigma(\Delta + V)$. By using variational methods, we prove the existence of solution for ϵ small enough. After that, we show that the maximum points of the solutions concentrate around of a maximum point of A.

Ground state solutions of Hamiltonian systems in dimension two

Djairo Guedes de Figueiredo
Universidade Estadual de Campinas, Brazil

We prove the existence of ground state solutions for critical (in the sense of Moser) Hamiltonian systems in dimension two using a generalized Nehari manifold, due to Pankov.

Density of short solution segments by variable delay

Hans-Otto Walther
University of Giessen, Germany

Simple-looking autonomous delay differential equations

$$x'(t) = f(x(t-r))$$

with a real function f and single time lag $r > 0$ can generate complicated (chaotic) solution behaviour, depending on the shape of f. The same could be shown for equations with a variable, state-dependent delay $r = d(x_t)$, even for the case $f(\xi) = -\alpha\,\xi$ linear, with $\alpha > 0$ [1]. Here the argument x_t of the *delay functional* d is the segment, or history, of the solution x between $t-r$ and t defined as the function $x_t : [-r,0] \to \mathbb{R})$ given by $x_t(s) = x(t+s)$.

So the delay alone may be responsible for complicated solution behaviour. In both cases the complicated behaviour which could be established occurs in a thin dust-like invariant subset of the infinite-dimensional Banach space or *solution manifold* of functions $[-r,0] \to \mathbb{R}$ on which the delay equation defines a semiflow of differentiable solution operators [2,3].

The lecture presents results which grew out of an attempt to obtain complicated motion on a larger set with non-empty interior, as certain numerical experiments seem to suggest. In [4] a delay functional $d : Y \to (0,r)$ was constructed on an infinite-dimensional subset Y of the space $C^1([-r,0],\mathbb{R})$, with $r > 1$, so that the equation

$$x'(t) = -\alpha\, x(t - d(x_t)) \tag{1}$$

has a solution whose *short segments* $x_t|_{[-1,0]}$, $t \geq 0$, are dense in the space $C^1([-1,0],\mathbb{R})$. This implies a new kind of complicated behaviour of the flowline $[0,\infty) \ni t \mapsto x_t \in C^1_r$.

The set Y in [4] is small in the sense that it has infinite codimension, and it is not smooth like the said solution manifolds of finite codimension. Recent work [5] concerns the construction of a delay functional on an *open* subset of the space $C^1([-r,0],\mathbb{R})$ so that Eq. (1) defines a nice semiflow on the solution manifold, and has a solution whose short segments are dense in an open subset of the space $C^1([-1,0],\mathbb{R})$.

References:

[1] Lani-Wayda, B., and H. O. Walther, *A Shilnikov phenomenon due to state-dependent delay, by means of the fixed point index.* DOI 10.1007/s10884-014-9420-z, J. Dynamics Dif. Eqs. 28 (2016), 627-688.

[2] Walther, H. O., *The solution manifold and C^1-smoothness of solution operators for differential equations with state-dependent delay*. J. Dif. Eqs. 195 (2003), 46-65.

[3] Walther, H. O., *Smoothness properties of semiflows for differential equations with state-dependent delay*. J. Math. Sciences 124 (2004), 5193-5207.

[4] Walther, H. O., *A delay differential equation with a solution whose shortened segments are dense*. DOI 10.007/s10884-018-9655-1, J. Dynamics Dif Eqs., to appear.

[5] Walther, H. O., *Density of short solution segments by variable delay*. In progress.

Intricate structure of the analytic set for a class of integral equations

John Mallet-Paret
Brown University

We consider the integral operator $L : C_{2\pi} \to C_{2\pi}$ given by

$$(Lx)(t) = \int_{\eta(t)}^{t} x(s)\, ds,$$

where $C_{2\pi}$ is the space of 2π-periodic functions with the usual sup norm. Here $\eta : \mathbb{R} \to \mathbb{R}$ is continuous, satisfying $\eta(t) \leq t$ and $\eta(t + 2\pi) = \eta(t) + 2\pi$ for all $t \in \mathbb{R}$. We first give necessary and sufficient conditions for the spectral radius $r(L)$ of L to be positive; in this case, by the Krein-Rutman Theorem, there exists a nonnegative eigenfunction $x \in C_{2\pi}$ to the eigenproblem $Lx = \kappa x$, where $\kappa = r(L)$. If further the function η is analytic, and under certain dynamical conditions on the map η, we show that the eigenfunction x is analytic for $t \in \mathcal{A} \subseteq \mathbb{R}$ where $\mathcal{A}$ is an open dense subset of $\mathbb{R}$, and where its complement $\mathcal{N} = \mathbb{R} \setminus \mathcal{A}$ is a generalized Cantor set.

This is joint work with Roger Nussbaum.

A combinatorial/algebraic topological approach to nonlinear dynamics

Konstantin Mischaikow
Rutgers University, USA

Motivated by the increase in data driven science I will discuss a combinatorial/algebraic topological approach to characterizing nonlinear dynamics. In particular, I will describe how order theory can be used to efficiently and effectively organize the decomposition of dynamics and propose a definition of nonlinear dynamics based on these structures. I will then argue that algebraic topological tools such as the Conley index can be used to identify classical dynamical structures.

Uniform decay rate estimates for the semilinear wave equation in inhomogeneous media with locally distributed nonlinear damping

Marcelo M. Cavalcanti, Valeria N. Domingos Cavalcanti , Ryuichi Fukuoka, Ademir B. Pampu, Maria Astudillo
Universidade Estadual de Maringá, Brazil

We consider the semilinear wave equation posed in an inhomogeneous media Ω with smooth boundary $\partial\Omega$ subject to a non linear damping distributed around a neighborhood ω of the boundary according to the Geometric Control Condition. We show that the energy of the wave equation goes uniformly to zero for all initial data of finite energy phase-space.

Persistence in sampled dynamics

Marian Mrozek, T. Dey, M. Juda, T. Kapela, J. Kubica, M. Lipinski
Jagiellonian University, Poland

A challenging contemporary problem is the understanding of the information hidden in data together with the stability of the information with respect to perturbations. A particular problem concerns data gathered from dynamic processes. One of the methods in this case is to fit a differential equation to the data and use the differential equation to understand the dynamics and its

stability. One of the alternatives consists in adapting the methods of topological dynamics to study the data directly using the methods of topological persistence to investigate stability. This approach was opened with the seminal works of R. Forman on Morse theory for combinatorial vector fields and H. Edelsbrunner, D. Letscher and A. Zomorodian on homological persistence.

In this talk I will explain how dynamics known only from finite samples may be used to construct multivalued maps and combinatorial multivector fields on finite topological spaces. I will define Morse decompositions for such dynamical systems and I will discuss their homological persistence.

This talk is based on joint work with T. Dey, M. Juda, T. Kapela, J. Kubica and M. Lipinski.

Periodic functions on isolated time scales

Martin Bohner, Jaqueline Mesquita, Sabrina Streipert
Missouri University of Science and Technology, USA

In this talk, we formulate the definition of periodicity for isolated time scales. This provides the basis for future research regarding periodicity on time scales with a positive graininess. The introduced definition is consistent with the known formulations in the discrete and quantum calculus settings. Using the definition of periodicity, we discuss the existence and uniqueness of periodic solutions to a family of linear dynamic equations on isolated time scales. Examples in the quantum calculus case and on mixed isolated time scales are presented throughout.

Nonlocal issues

Michel M. Chipot
University of Zurich, Switzerland

The goal of this talk is to introduce some classes of nonlocal problems, to explain the advances achieved and to stress the challenging questions arising. This includes problems associated with nonlinear operators of the $p(x)$-Laplacian type.

A center manifold for second order semi-linear differential equations on the real line and application to the existence of wave trains for the Gurtin-McCamy equation

Pierre Magal
University of Bordeaux, France

This work is mainly motivated by the study of periodic wave train solutions for the so-called Gurtin-McCamy equation. To that aim we construct a smooth center manifold for a rather general class of abstract second order semi-linear differential equations involving non-densely defined operators. We revisit results on commutative sums of linear operators using the integrated semigroup theory. These results are used to reformulate the notion of the weak solutions of the problem. We also derive a suitable fixed point formulation for the graph of the local center manifold that allows us to conclude to the existence and smoothness of such a local invariant manifold. Then we derive a Hopf bifurcation theorem for second order semi-linear equations. This result is applied to study the existence of periodic wave trains for the Gurtin-McCamy problem, that is for a class of non-local age structured equations with diffusion.

Dynamics of a stochastic SIRI epidemic model with relapse and media coverage

Tomás Caraballo
Universidad de Sevilla, Spain

In this talk we will report on some recent results concerning the existence and uniqueness of global positive solutions for a stochastic SIRI epidemic model with relapse and media coverage. Our main emphasis will be on the study of the dynamical properties of the solution around both disease-free and endemic equilibria points of the deterministic model. Furthermore, we will also show the existence of a stationary distribution and will provide some numerical simulations to illustrate the theoretical results.

Prevalence of Haar null sets

Udayan B. Darji
University of Louisville, USA

Suppose we are in a function space or a Banach space. What does it mean to say the collection of objects in space satisfying certain property is small? Classically, one uses Baire category theorem and says that this collection is meager. In 1973 Christensen introduced analytic notion of smallness and proved its basic properties. His main motivation was to prove Rademacher in the setting of separable Banach space. This notion of smallness was rediscovered by Hunt, Sauer and York in 1992, in the Bulletin of the AMS, in the context of dynamical systems Since then the notion smallness has enjoyed attention from mathematicians working in analysis, dynamical systems and even model theory. We will give an introduction to this topic and in a variety of places is where it is useful.

Synchronization in random networks

Yingfei Yi
University of Alberta, Canada & Jilin University, China

We consider synchronization phenomena for discrete-time, discrete-state random dynamical systems, with random and probabilistic Boolean networks as particular examples. By studying multiplicative ergodic properties of the induced linear cocycle, we give a characterization of the synchronization in term of Lyapunov exponents.

ICMC SUMMER MEETING on DIFFERENTIAL EQUATIONS

CHAPTER 2019

Special Sessions

Boundary Perturbations of Domais for PDEs and Applications

Organizers: Marcone C. Pereira & Ricardo Parreira da Silva

Norm resolvent approximation of thin homogeneous tubes by heterogeneous ones

Alessandra A. Verri, Verri, César R. de Oliveira
Universidade Federal de São Carlos, Brazil

We study the operator $-\mathrm{div}(A(x)\nabla\psi(x))$ restricted to a waveguide $\Omega_\varepsilon \subset \mathbb{R}^3$, with heterogeneous function $A(x)$ constant in the longitudinal direction. The purpose is to obtain an effective operator, in the norm resolvent sense, when the diameter of Ω_ε tends to zero with heterogeneity approaching a homogeneous situation (i.e., a constant function A). The effective operator presents a potential that, besides the traditional dependence on waveguide geometric properties, there is also a contribution from $A(x)$ which results, when combined with the curvature, for example, in the possibility of a repulsive interaction.

Joint work with César R. de Oliveira.

Continuity of attractors for C^1 perturbations of a smooth domain

Antônio L. Pereira
Universidade de São Paulo, Brazil

We consider a family of semilinear parabolic problems with nonlinear boundary conditions in a family of C^2 domains converging to a fixed domain in the C^1-norm as a parameter ϵ goes to 0. Assuming suitable regularity and dissipative conditions for the nonlinearites, we show that the problem is well posed for $\epsilon > 0$ sufficiently small in a suitable scale of fractional spaces, the associated semigroup has a global attractor $\mathcal{A}_\epsilon$ and the family $\{\mathcal{A}_\epsilon\}$ is continuous at $\epsilon = 0$.

Recent advances in nonsmooth shape optimization

Antoine Laurain
Universidade de São Paulo, Brazil

We will see some recent results about distributed and boundary expressions of the Eulerian and Fréchet shape derivatives for several classes of nonsmooth domains such as open sets, Lipschitz domains, polygons and curvilinear polygons, semiconvex and convex domains. We will focus on the particular case of the Dirichlet energy, for which we compute first and second order distributed shape derivatives in tensor form. Depending on the type of nonsmooth domain, different boundary expressions can be derived from the distributed expressions. This requires a careful study of the regularity of the solution to the Dirichlet Laplacian in nonsmooth domains. These results are applied to obtain second order shape derivatives in matricial form for polygons.

Semilinear elliptic equations with terms concentrating on oscillatory boundaries

Ariadne Nogueira, Marcone C. Pereira, José M. Arrieta
Universidade Anhembi Morumbi

In this work we study concentrating integrals properties through two applications, both in Lipschitz domains of $\mathbb{R}^2$, where we analyze semilinear elliptic problems posed in a oscillating region with reaction terms concentrated in a neighborhood of an oscillatory boundary when a small parameter goes to zero: an oscillating boundary domain which tends to a fixed limit domain; and a thin region with a oscillatory boundary. In both cases under some conditions we prove the upper and lower semicontinuity of the family of solutions from these problems, showing that the solutions of our perturbed equation can be approximated with one limit equation defined in a fixed domain, which also captures the effects of all relevant physical processes that take place in the original problem.

Neumann Laplacian in nonuniformly collapsing strips

Cesar de Oliveira, Alessandra Verri
Universidade Federal de São Carlos, Brazil

We consider the Neumann Laplacian in the region below the graph of $\varepsilon g(x)$, for a (possibly unbounded) positive smooth function $g : [a, \infty) \to \mathbb{R}$. As $\varepsilon \to 0$ such region collapses to $[a, \infty)$ and (under suitable technical conditions) an effective operator is found, which has Robin boundary conditions at a. Then we recover, in the case of unbounded g, such effective operators through uniformly collapsing regions; in such approach, we have (roughly) got norm resolvent convergence for g diverging less than exponentially.

On the effects of small boundary perturbation on the fluid flow

Igor Pažanin, Eduard Marušić-Paloka, University of Zagreb, Croatia

It is well-known that only a limited number of the fluid flow problems can be solved (or approximated) by the solutions in the explicit form. To derive such solutions, we usually need to start with (over)simplified mathematical models and consider ideal geometries on the flow domains with no distortions introduced. However, in practice, the boundary of the fluid domain can contain various small irregularities (rugosities, dents, etc.) being far from the ideal one. Such problems are challenging from the mathematical point of view and, in most cases, can be treated only numerically. The analytical treatments are rare because introducing the small parameter as the perturbation quantity in the domain boundary forces us to perform tedious change of variables. Having this in mind, our main goal is to present recent analytical results on the effects of a slightly perturbed boundary on the fluid flow through a channel filled with a porous medium. We start from a rectangular domain and then perturb the upper part of its boundary by the product of the small parameter ε and arbitrary smooth function. The porous medium flow is described by the Darcy-Brinkman model which can handle the presence of a boundary on which the no-slip condition for the velocity is imposed. Using asymptotic analysis with respect to ε, we formally derive the effective model in the form of the explicit formulae for the velocity and pressure. The obtained asymptotic approximation clearly shows the nonlocal effects of the small boundary perturbation. The error analysis is also conducted providing the order of accuracy of the asymptotic solution. We will also comment on our recent results concerning the problem of a reactive solute transport, MHD flow and time-dependent setting.

The p-Laplacian equation in thin domains: the unfolding approach

Jean Carlos Nakasato, Marcone Corrêa Pereira, Jose M. Arrieta
Universidade de São Paulo, Brazil

In this work we apply the unfolding operator method to analyze the asymptotic behavior of the solutions of the p-Laplacian equation with Neumann boundary condition set in a bounded thin domain of the type $R^\varepsilon = \left\{(x,y) \in \mathbb{R}^2 : x \in (0,1) \text{ and } 0 < y < \varepsilon g\left(x/\varepsilon^\alpha\right)\right\}$ where g is a positive periodic function. We study the three cases $0 < \alpha < 1$, $\alpha = 1$ and $\alpha > 1$ representing respectively weak, resonant and high osillations at the top boundary. In the three cases we deduce the homogenized limit and obtain correctors.

Uniform stability of the ball with respect to the first Dirichlet and Neumann $\infty-$eigenvalues

João Vitor da Silva, Julio D. Rossi, Ariel Salort
Universidad de Buenos Aires, Argentina

In this Lecture we analyze how perturbations of a ball $\mathfrak{B}_r \subset \mathbb{R}^n$ behaves in terms of their first (non-trivial) Neumann and Dirichlet $\infty-$eigenvalues when a volume constraint $\mathcal{L}^n(\Omega) = \mathcal{L}^n(\mathfrak{B}_r)$ is imposed.

Our main result states that Ω is uniformly close to a ball when it has first Neumann and Dirichlet eigenvalues close to the ones for the ball of the same volume $\mathfrak{B}_r$. In fact, we show that, if

$$|\lambda^D_{1,\infty}(\Omega) - \lambda^D_{1,\infty}(\mathfrak{B}_r)| = \delta_1 \quad \text{and} \quad |\lambda^N_{1,\infty}(\Omega) - \lambda^N_{1,\infty}(\mathfrak{B}_r)| = \delta_2,$$

then there are two balls such that

$$\mathfrak{B}_{\frac{r}{\delta_1 r+1}} \subset \Omega \subset \mathfrak{B}_{\frac{r+\delta_2 r}{1-\delta_2 r}}.$$

In addition, we also obtain a result concerning stability of the Dirichlet $\infty-$eigen-functions.

This is joint work with Julio D. Rossi and Ariel M. Salort, Universidad de Buenos Aires, Argentina.

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Nonlocal equations in perforated domains

Marcone Pereira
Universidade de São Paulo, Brazil

In this talk, we analyze the asymptotic behavior of nonlocal problems widely used in the modeling of diffusion or dispersion processes. We consider an integral-differential equation, with nonsingular kernel, in a limited domain Ω from which we remove subsets that we call holes. We deal with Neumann and Dirichlet conditions in the holes setting Dirichlet outside of Ω. Assuming the weak convergence of the family of functions which represents such holes, we analyze the limit of the solutions of the equations obtaining the existence of a limit problem. In the case where the holes are removed periodically, we observe that the critical radius is of order of the typical cell size (which gives the period). Finally we study the behavior of these problems when we resize their kernel with the objective of approaching local partial differential equations discussing peculiarities.

Eigenvalues of the Neumann Laplacian in $\mathbb{S}^1$-invariant regions in $\mathbb{R}^3$

Marcus M. Marrocos, Antônio L. Pereira
Universidade Federal do ABC, Brazil

In this work we are concerned with the multiplicity of the eigenvalues of the Neumann Laplacian in regions of $\mathbb{R}^3$ which are invariant under the natural action of $\mathbb{S}^1$. We give a positive answer (in the Neumann case) to a conjecture by V. Arnold on the transversality of the transformation given by the Dirichlet integral to the stratification in the space of quadratic forms according to the multiplicities of the eigenvalues. We show that, generically in the set of $\mathbb{S}^1-$invariant, $\mathcal{C}^2$-regions, the action is irreducible in each eigenspace $Ker(\Delta + \lambda)$.

Locally periodic unfolding operator for highly oscillating rough domains

Ravi Prakash, S. Aiyappan, A. K. Nandakumaran
Universidad de Concepción, Chile

This presentation aims to explain the locally-periodic oscillating domain via unfolding operators. A three dimensional rough domain Ω_ε, $\varepsilon > 0$ a small parameter, has been considered for the study where the boundary is rapidly oscillating with high amplitude. There are few articles with locally-periodic boundary oscillations with small amplitude. But, we do not see any literature with high-amplitude $(O(1))$ locally-periodic oscillating domains. This is an attempt to study problem in locally periodic rough domains with an eye towards the general oscillating domains without periodicity. We develop locally-periodic unfolding operators to study our problems. M. Ptashnyk has developed an unfolding operator to study PDEs with locally-periodic coefficients. The development of these unfolding operators motivated us to develop unfolding operators for locally-periodic oscillating domains with high amplitude. We consider a non-linear inhomogeneous Robin boundary value problem posed on this domain to demonstrate the utility of this new operator.

Continuity of attractors of parabolic equations with nonlinear boundary conditions and rapidly varying boundaries. The case of a Lipschitz deformation

Simone Mazzini Bruschi, Gleiciane S. Aragão, José M. Arrieta
Universidade de Brasilia, Brazil

We analyze the asymptotic dynamics of nonlinear parabolic equations with nonlinear boundary conditions when the boundary of the domain varies very rapidly as a parameter ϵ goes to zero. Considering a uniformly Lipschitz deformation we prove the upper semicontinuity of attractors. Moreover, if every equilibrium of the limit problem is hyperbolic we also prove the continuity of local unstable manifolds and the lower semicontinuity of attractors $\epsilon = 0$.

Computational Dynamics in the Context of Data

Organizers: Marcio Gameiro, Konstantin Mischaikow, Marian Mrozek & Thomas Wanner

Rigorous computation of heteroclinic bifurcations

Bernardo Rivas, Jonathan Jaquette, Konstantin Mischaikow, Marcio Gameiro, Shane Kepley
Universidade de São Paulo

Rigorous numerical methods are used to prove the existence of heteroclinic bifurcations in systems of ordinary differential equations. The novelty of the method is to define an operator $F : X \to X$ in an appropriate Banach space such that its zero corresponds to the existence of a heteroclinic orbit. The next step consists of defining an Newton-like operator T whose fixed points are in 1-1 correspondence with the zeros of F. Finally, a contraction argument based on the Radii Polynomials Approach is used to validate the solutions.

References:

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Conley index approach to sampled dynamics

Bogdan Batko, Konstantin Mischaikow, Marian Mrozek, Mateusz Przybylski
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Motivation for our work comes from sampled dynamics. More precisely, the Conley index for multivalued maps may be useful in the reconstruction of the qualitative features of an unknown dynamical system on the basis of the available experimental data only. In general, the reconstruction problem is difficult, particularly in the case of chaotic dynamics. In this situation the coarseness of topological invariants turns out to be helpful. The potential benefits from applying topological tools to the reconstruction problem are demonstrated in [5,6].

In the talk we describe the construction of a multivalued upper semicontinuous map from experimental data. Although such a map need not be acyclic, it still may induce a map in homology. For this it suffices to construct its homologically consistent enlargement F which is homologically complete (cf. [3]). The problem with such an F is that it need not admit any continuous selector. Fortunately, we may use Conley theory for such maps developed recently in [1,2,4].

We focus on the structure of an isolated invariant set S with respect to F. Since isolating neighborhoods for multivalued maps do not necessarily admit index pairs (cf. [2]), we work with weak index pairs. We present accurate results to detect orbits passing through the disjoint components of S in a given fashion. Conditions sufficient for the existence of such orbits can be expressed in terms of the index map I_P associated with a weak index pair P for S. Applying the Lefschetz-type fixed

point theorem (cf. [7]) we provide conditions that imply the periodicity of such orbits. Moreover, we discuss the semiconjugacy with a shift dynamics of finite type.

At the end we present applications of our approach. We sample 2D Hénon map to detect 2-periodic orbit. Further examples concern chaotic dynamics. We consider time series coming from an actual physical experiment concerning the behavior of a magnetoelastic ribbon in a varying magnetic field, and sampled Hénon map.

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Applications of the software package DSGRN

Bree Cummins, Tomas Gedeon, Konstantin Mischaikow, Shaun Harker
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Dynamic Signatures Generated by Regulatory Networks (DSGRN) provides a queryable description of global network dynamics over all of parameter space. DSGRN is based on a new approach to dynamical systems, which moves the focus away from trajectories and invariant sets, and toward robust, scalable and computable description of dynamics in terms of lattices and posets. On the level of software, DSGRN takes as input a regulatory network and outputs a queryable SQL database that provides information about the structure of global dynamics over all of the associated parameter space.

The dynamical signatures stored in the DSGRN database can be queried to answer questions about regulatory network performance in the context of network design, discovery, and diagnosis. I will discuss the use of DSGRN in designing networks with desired dynamical behaviors, in diagnosing failures in desired synthetic network performance, and in discovering potential network models from time series data.

Transition matrix

Ewerton R Vieira
Universidade Federal de Goiás, Brazil

The Conley index theory has been a valuable topological technique for detecting global bifurcations in dynamical systems [1], [2], [3], [4] , [5], [6] and [7]. This index is a standard tool in the analysis of invariant sets in dynamical systems, and its significance owes partly to the fact that it is invariant under local perturbation of a flow (the continuation property).

In this setting, we present a new definition and applications of transition matrix as a Conley-index based algebraic transformation that tracks changes in index information and thereby identifies global bifurcations that could occur. Furthermore, we do not require that there exists a continuation for a Morse Decomposition related to the transition matrix, see [8], [9] and [10].

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A computational framework for connection matrix theory

Kelly Spendlove, Shaun Harker, Konstantin Mischakow, Robert Vandervorst
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Algebraic topology and dynamical systems are intimately related: the algebra may constrain or force the existence of certain dynamics. Morse homology is the prototypical theory grounded in this observation. Conley theory is a far-reaching topological generalization of Morse theory and the last few decades have seen the development of a computational version of the Conley theory. The computational Conley theory is a blend of combinatorics, order theory and algebraic topology and has proven effective in tackling problems within dynamical systems.

Within the Conley theory the connection matrix is the mathematical object which transforms the approach into a truly homological theory; it is the Conley-theoretic generalization of the Morse boundary operator. We'll discuss a new formulation of the connection matrix theory, which casts the connection matrix in categorical, homotopy-theoretic language. This enables the efficient computation of connection matrices via the technique of reductions in combination with algebraic-discrete Morse theory. We will also discuss a software package for such computations. Time permitting, we'll demonstrate our techniques with an application of the theory and software to the setting of transversality models. This application allows us to compute connection matrices for the classical examples of Franzosa and Reineck as well as high-dimensional examples from a Morse theory on spaces of braid diagrams introduced in.

A dynamical approach to spectral sequences of a Morse complex

Ketty de Rezende, Dahisy Lima, Mariana Silveira, Margarida Mello, Maria Alice Bertolim
Universidade Estadual de Campinas, Brazil

Conley's connection matrix theory and a spectral sequence analysis of a filtered Morse chain complex (C, Δ) are used to study global continuation results for flows on surfaces. The briefly described unfoldings of Lyapunov graphs have been proved a well suited combinatorial tool to keep track of continuations. The novelty herein is a global dynamical cancellation theorem inferred from the differentials of the spectral sequence (E^r, d^r). The local version of this theorem relates differentials d^r of the r-th page E^r to Smale's theorem on cancellation of critical points.

Dynamics of three-node gene regulatory networks

Marcio Gameiro
Universidade de São Paulo, Brazil

Understanding the dynamics of gene regulatory networks is a central problem in systems biology. We are studying the dynamics of all three-node networks using novel combinatorial and algebraic topological methods base on Conley theory. Our goal is to understand which networks most robustly support the phenotypic behavior of a hysteretic switch and to understand how the results from the three-node networks can be used to predict and understand this phenomenon in networks with more nodes where the combinatorial explosion of the number of networks prevents a similar exhaustive study.

Combinatorial topological dynamics II: multivector fields and connection matrices

Marian Mrozek, Thomas Wanner
Jagiellonian University, Poland

Forman's combinatorial vector fields found many applications in the study of the topological shape of data. So far they seem to be less successful in describing the dynamics encoded in data. This may be caused by the fact that when studying the dynamics the combinatorial vector field cannot be chosen but must be extracted from data.

To remedy this situation a more flexible concept of combinatorial multivector field has been proposed. In particular, we prove that every sublattice of the lattice of closed sets of a finite topological space induces a gradient combinatorial multivector field.

We show the difference between the vector and multivector fields by studying connection matrices. We prove that a gradient combinatorial vector field enforces a unique connection matrix of the associated Morse decomposition. However, non-uniqueness of connection matrices is an important feature in classical dynamics. We present examples that for gradient multivector fields connection matrices need not be unique.

This talk is based on joint work with Th. Wanner.

Helioseismic and magnetic imager data classification using combinatorial topological dynamics

Mateusz Juda
Jagiellonian University, Poland

In this note we present a method for topological features extraction of sampled vector fields. By a sampled vector field we mean a finite set of points in $\mathbb{R}^d$ with vectors attached. Such data arise in a natural way from sampling dynamics. As a real world example we study the data collected by the Helioseismic and Magnetic Imager (HMI) - an instrument designed to study the magnetic field on the surface of Sun [4]. We show that the proposed method significantly outperforms the presently available methods in the HMI solar flare classification task. Our method is general and can be applied to any sampled vector field data, however in this work we present results based only on HMI data.

This note is based on research projects with: Marian Mrozek, Bartosz Zielinski, Tomasz Kapela, Matthias Zeppelzauer.

1 HMI data. The goal of HMI project is to study the relationship between the behavior of the photospheric magnetic field and solar activity. In particular, space weather anomalies are linked to solar flares - a sudden explosion of energy. Solar flares can interfere with satellites and also with equipment such as power utility grids, electronics etc. Predicting solar flares is a challenging task. The recent prediction techniques are based on machine learning (ML) methods. Typically, ML methods for solar flares prediction use 25 numerical characteristics of the magnetic field, the so called data features: total unsigned current helicity, total magnitude of Lorentz force etc.

2 Methodology. We propose to extract features of a sampled vector field using a method based on combinatorial multivector fields [5], a generalization of Forman's combinatorial vector fields [9,8]. Namely, as a first step we reconstruct dynamics given by a cloud of vectors by building a simplicial complex $\mathcal{K}$ on the point cloud and a combinatorial multivector field $\mathcal{V}$ on $\mathcal{K}$. This way we obtain a graph on the set of all simplices with edges approximating the vector field. We analyze a collection of such graphs using DeepWalk [2] approach which transforms graphs into text documents. Next we use Fasttext [1] to learn embedding of words into $\mathbb{R}^d$, where d is a fixed parameter. Using that embedding we get a representation of the text documents in $\mathbb{R}^d$. The representation gives us a feature vector for each sampled vector field. In the following sections we present more details of the method.

2.1 Multivector fields. By a *combinatorial dynamical system* on a simplicial complex K (*cds* in short) we mean a multivalued map $F : K \multimap K$, that is a map which sends each simplex in K into a family of simplices in K. The cds F may be viewed as a digraph G_F whose vertices are simplices in K with a directed edge from simplex σ to simplex τ if and only if $\tau \in F(\sigma)$. However, F is more than just the digraph G_F because K, the set of vertices of G_F, is a finite topological space with Alexandrov topology given by the poset of face relation [11].

We construct a cds from a cloud of vectors in two steps. In the first step the cloud of vectors is transformed into a combinatorial multivector field [5]. In the second step, the combinatorial multivector field is transformed into a cds. In order to explain the steps, we introduce some definitions. We say that $A \subset K$ is *convex* if for any $\sigma_1, \sigma_2 \in A$ and $\tau \in K$ such that σ_1 is a face of τ and τ is a face of σ_2 we have $\tau \in A$. We note that convex subsets of K are precisely the locally closed sets of K (see [Sec. 2.7.1, pg 112, 6]) in the Alexandrov topology of K. We define a *multivector* as a convex subset of K and a *combinatorial multivector field* on K (*cmf* in short) as a partition $\mathcal{V}$ of K into multivectors. Given a cmf $\mathcal{V}$, we denote by $[\sigma]_{\mathcal{V}}$ the unique V in $\mathcal{V}$ such that $\sigma \in V$. We associate with $\mathcal{V}$ a cds $F_{\mathcal{V}} : K \multimap K$ given by $F_{\mathcal{V}}(\sigma) := \operatorname{cl}\sigma \cup [\sigma]_{\mathcal{V}}$.

We denote by $G_{\mathcal{V}}$ the graph obtained from G_F by contracting to a point the vertices in G_F sharing the same multivector.

2.2 DeepWalk. In order to analyze a collection of graphs $G_{\mathcal{V}}$ we use DeepWalk [2]. The method is used to analyze graphs as text documents with Natural Language Processing (NLP). Given $G_{\mathcal{V}}$ we generate a set of paths, that is random walks of length not exceeding a fixed k. We assume that for each vertex a word from a vocabulary is given as the vertex label. For a path p we generate a sentence by replacing each vertex on p by its label. A set of such sentences constitutes a text document associated with the set of paths. In this context the order of sentences is not important. For a given set of graphs we consider the documents as a text corpus. Using NLP techniques, in particular Fasttext [1], we learn the representation of words as vectors in $\mathbb{R}^d$ with a fixed d. Each document is represented as the average of its word vectors.

2.3 Topological vocabulary. The NLP procedure described above requires a vocabulary in order to assign labels to the vertices. We construct labels which graspe some local, topological properties of the vertex in the vector field. More precisely given a multivector $V \in \mathcal{V}$, that is a vertex in $G_{\mathcal{V}}$, we first define the *label of V at level 0*, denoted $l_0(V)$, as a tuple

$$l_0(V) := (\max_{\sigma \in V} \dim \sigma, |V|, \chi(V)),$$

where $\dim \sigma$ denotes the dimension of simplex σ, $|V|$ stands for the cardinality of V, and $\chi(V)$ is the Euler characteristic of V. We define *label of V at level d* , denoted $l_d(V)$, as a tuple

$$l_d(V) := (l_0(V), \mathrm{sorted}(\{l_0(u) \mid u \in N_d^+(V)\}), \mathrm{sorted}(\{l_0(u) \mid u \in N_d^-(V)\})),$$

where $N_d^+(V)$ (resp. $N_d^-(V)$) are sets of vertices in the forward (resp. backward) distance from V not bigger than d.

3 Results. To evaluate our method we use a data set proposed in [3]. The data set provides 823 HMI magnetograms. The state-of-the-art methods extract from each magnetogram 13 real number characteristics. Additionally, for each magnetogram we know a flare class (B, C, M, and X) according to the maximum magnitude of flares generated in the approaching 24 hours. Our goal is to find an ML model for the flare class prediction based on the megnetograms.

We use randomly selected 70% of the data as a training set, and the rest as a test set. We transform the magnetograms from the training set into text documents and create a model of the artificial language described above. Then, for each magnetogram (training and test), we create a new feature vector using the word embeddings. We present results obtained with the following parameters:

- level of labels is $k = 4$;
- for each label l we select randomly 50% of vertices v in $G_{\mathcal{V}}$, such that $l_k(v) = l$;
- for each selected vertex we generate a random walk which begins at v and a random walk which ends at v, both of length 20;
- the dimension of the word embeddings is 40.

To compare the state-of-the-art feature vector with the new one we compare classification metrics for LinearSVC [7] and AdaBoostClassifier [10] from sklearn python library. We provide classiffiers scores in Table 1. We observe that the features based on the proposed word embeddings always

	Classifier	test	training
proposed feature vector	LinearSVC	0.898	0.881
	AdaBoostClassifier	0.846	0.994
state-of-the-art feature vector	LinearSVC	0.417	0.392
	AdaBoostClassifier	0.663	0.918

Table 1: Classifiers scores on test and training data sets.

are significantly better than the state-of-the-art features. We emphesize that the proposed method outperforms state-of-the-art for the test set.

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The algebra of dynamical systems

Robert Vandervorst, K. Mischaikow, W. Kalies, D. Kasti, A. Goldstein
Vrije Universiteit Amsterdam, Netherlands

Since the work of Smale and Conley decompositions of dynamical systems have played an important role in the topological theory of dynamical systems. In this talk we discuss a systematic algebraic approach to dynamics which yields a unified theory for Morse decompositions and chain-recurrence as well as a framework for a computational theory.

Combinatorial topological dynamics I: linking combinatorial and classical dynamics

Thomas Wanner, Bogdan Batko, Tomasz Kaczynski, Marian Mrozek
George Mason University, USA

Forman's combinatorial vector fields on simplicial complexes are a discrete analogue of classical flows generated by dynamical systems. Over the last decade, many notions from dynamical systems theory have found interpretations in this combinatorial setting, such as for example discrete gradient flows and Forman's discrete Morse theory. In this talk, we survey recent results which aim to establish a formal tie between combinatorial and classical dynamics. After outlining a Conley-Morse theory in the combinatorial setting, we show that every combinatorial dynamical system in the sense of Forman defined on a finite simplicial complex $\mathcal{X}$ gives rise to a discrete-time multivalued dynamical system on its geometric realization $X = |\mathcal{X}|$. The constructed dynamical system can be chosen in such a way that the isolated invariant sets, Conley indices, Morse decompositions, and Conley-Morse graphs of the two systems are in one-to-one correspondence. Finally, we demonstrate that this correspondence can be extended to the case of continuous-time semiflows on the polytope X.

DSGRN: a bridge from networks to dynamics

Tomas Gedeon, Tomas Gedeon, Konstantin Mischaikow, Bree Cummins, Shaun Harker
Montana State University, USA

Dynamic Signatures Generated by Regulatory Networks (DSGRN) provides a queryable description of global network dynamics over the entire parameter space. DSGRN is based on a new approach to dynamical systems, which moves the focus away from trajectories and invariant sets, and toward robust, scalable and computable description of dynamics in terms of lattices and posets. On the level of software, DSGRN takes as input a regulatory network and outputs a queryable SQL database that provides information about the structure of global dynamics over all of the associated parameter space.

In this talk I will describe method, motivated by the switching systems of differential equations, that allows DSGRN to combinatorialize network dynamics in both phase space and parameter space. This leads naturally to construction of a lattice of attracting sets and via Birkhoff theorem, to a partially ordered Morse graph. These structures are naturally robust and provide conclusions for an entire class of ODE models of network dynamics.

Dispersive Equations

Organizer: Mahendra Panthee & Marcia A. G. Scialom

Solitary wave solutions and global well-posedness for a coupled system of gKdV equations

Andressa Gomes, Ademir Pastor, Mahendra Panthee
Universidade Estadual de Campinas, Brazil

We consider the initial value problem (IVP) associated with a coupled system of gKdV equations. We present a relationship between the best constant for a Gagliardo Nirenberg type interpolation inequality and a criterion for the existence of global solutions with initial data in the energy space. We prove that such a constant is directly related to the existence problem of solitary wave solutions with minimal mass, the so called ground states solutions. To guarantee the existence of ground states we use a variational method based on [1] and [2].

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On the propagation of regularity for solutions of the dispersion generalized Benjamin-Ono equation

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In this paper we study some properties of propagation of regularity of solutions of the dispersive generalized Benjamin-Ono (BO) equation. This model defines a family of dispersive equations, that can be seen as a dispersive interpolation between Benjamin-Ono equation and Korteweg-de Vries (KdV) equation.

Recently, it has been showed that solutions of the KdV equation and Benjamin-Ono equation, satisfy the following property: if the initial data has some prescribed regularity on the right hand side of the real line, then this regularity is propagated with infinite speed by the flow solution.

In this case the nonlocal term present in the dispersive generalized Benjamin-Ono equation is more challenging that the one in BO equation. To deal with this a new approach is needed. The new ingredient is to combine commutator expansions into the weighted energy estimate. This allow us to obtain the property of propagation and explicitly the smoothing effect.

On the Cauchy problem for Schrödinger-Bebye system in Besov space

Isnaldo Isaac Barbosa
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We study the Cauchy problem associated to the Schrödinger-Debye system for data in the Sobolev spaces with low regularity, which appears modeling problems in nonlinear optics, namely:

$$\begin{cases} i\partial_t u + \frac{1}{2}\partial_x^2 u &= u \cdot v \\ \sigma\partial_t v + v &= \varepsilon|u|^2, \end{cases} \tag{2}$$

where $u = u(x,t)$ is a complex valued function, $v = v(x,t)$ is a real valued function, $\sigma > 0$, $\varepsilon = \pm 1$ and $x, t \in \mathbb{R}$.

Well-posedness results for this system, in Sobolev space $H^{\kappa}(\mathbb{R}) \times H^{s}(\mathbb{R})$, were obtained by Matheus and Corcho in [2] using the method Bourgain.

In this work we develop a local theory for the system, where the regularity (κ, s), using the ideas of the work [1] in Besov space. Mereover, we obtain ill-posedness results based in the work [3].

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Existence of ground state solutions for a 2D generalization of the KdV equation

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In this paper, we are concerned with the existence of ground state solutions for the second order differential equation in $\mathbb{R}^N$

$$u_t + \beta \Delta u_x - \big(g(u)\big)_x = 0. \tag{3}$$

where $N = 2,3$, $t \in \mathbb{R}$, $\beta > 0$ and g is an operator with some special properties. We note that the nonlinear model (3) with nonlinearity $g(s) = as^p + bs^q$ is related with a 2D generalization of the KdV equation ($a = 0, q = 2$), a 2D generalization of the modified KdV equation ($b = 0, p = 3$), a 2D generalization of the quadratic-cubic KdV equation ($a \neq 0$, $b \neq 0$, $p = 3$, $q = 2$), and a 2D generalization of the Gardner equation ($a \neq 0$, $b \neq 0$, $p = 2q$). The travelling wave equation for the equation(3) is also related for example with the existence of standing wave solutions for the generalized Schrödinger equation in the case $g(u) = a|u|^p u$ and also with the existence of standing wave solutions for a class of systems of the Davey-Stewartson type in the case $g(u) = a|u|^p u + bE(|u|^2)u$, where E is a special linear operator defined via the Fourier transform and $b > 0$. The result is a consequence of the well-known principle of concentration-compactness due to P. Lions, although the non-linear term g is not necessarily a homogeneous function.

Global well-posedness and self-similarity for semilinear wave equations in a time-weighted framework of Besov type

Lucas C. F. Ferreira, Jhean E. Pérez-López
Universidade Estadual de Campinas, Brazil

We show global-in-time well-posedness and self-similarity for the semilinear wave equation with nonlinearity $f(u) = \pm u^p$ in a time-weighted framework based on the family of homogeneous Besov spaces $\dot{B}^{s_q}_{q,\infty}$ for $q > 2$. As a consequence, in some cases of the power p we obtain a larger initial-data class. Our approach relies on dispersive-type estimates and a suitable p-product estimate in Besov spaces.

On sharp global well-posedness and Ill-posedness for a fifth-order KdV-BBM type equation

Mahendra Panthee, Xavier Carvajal
Universidade Estadual de Campinas, Brazil

We consider the Cauchy problem associated to the recently derived higher order hamiltonian model for unidirectional water waves and prove global existence for given data in the Sobolev space H^s, $s \geq 1$. We also prove an ill-posedness result by showing that the flow-map is not C^2 if the given data has Sobolev regularity $s \lneq 1$. The results obtained in this work are sharp.

On the periodic Cauchy problem for a coupled system of third-order nonlinear Schrödinger equations

Marcia Scialom, Luciana Mendonça
Universidade Estadual De Campinas, Brazil

We investigate some well-posedness issues for the initial value problem (IVP) associated to the system

$$\begin{cases} 2i\partial_t u + q\partial_x^2 u + i\gamma\partial_x^3 u = F_1(u,w) \\ 2i\partial_t w + q\partial_x^2 w + i\gamma\partial_x^3 w = F_2(u,w), \end{cases}$$

where F_1 and F_2 are polynomials of degree 3 involving u, w and their derivatives. This system describes the dynamics of two nonlinear short-optical pulses envelopes $u(x,t)$ and $w(x,t)$ in fibers ([1] and [2]). We prove periodic local well-posedness for the IVP with data in Sobolev spaces $H^s(\mathbb{T})\times H^s(\mathbb{T})$, $s \geq 1/2$ and global well-posedness result in Sobolev spaces $H^1(\mathbb{T})\times H^1(\mathbb{T})$.

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On a higher dimensional version of the Benjamin-Ono equation

Oscar Guillermo Riano, Felipe Linares, Keith Rogers , James Wright
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We consider a higher dimensional version of the Benjamin–Ono equation

$$\partial_t u + \mathcal{R}_1 \Delta u + u\partial_{x_1} u = 0, \quad (x,t) \in \mathbb{R}^{d+1}$$

where $\mathcal{R}_1$ denotes the Riesz transform $\mathcal{R}_1 = -(-\Delta)^{-1/2}\partial_{x_1}$ with respect to the first coordinate. For the homogeneous equation we establish space–time estimates many of which are sharp. These estimates enable us to show that the initial value problem for the nonlinear equation is locally well-posed in the Sobolev space $H^s(\mathbb{R}^d)$, $s > 5/3$ if $d = 2$, and $s > d/2 + 1/2$ if $d \geq 3$. We also show some ill-posedness results for the associated Cauchy problem.

Properties of the biharmonic nonlinear Schrödinger equation

Roberto Capistrano Filho
Universidade Federal de Pernambuco, Brazil

In the first part of this talk we present some results about well-posedness of the biharmonic nonlinear Schrödinger equation on the torus $\mathbb{T}$ and initial boundary value problem in half-line $\mathbb{R}^+$ and star graphs.

The second part is devoted to presented some results of controllability and stabilizability of a class of distributed parameter control system described by the fourth order nonlinear Schrödinger on the torus $\mathbb{T}$ with internal control acting on a sub-domain ω of T. More precisely, by certain properties of propagation of compactness and regularity in Bourgain spaces for the solutions of the associated linear system, we will show that the system is globally exactly controllable and globally exponentially stabilizable.

Finally, we present works in progress about the controllability of the biharmonic nonlinear Schrödinger equation in half-line $\mathbb{R}^+$ and star graphs.

This is a join work with Márcio Cavalcante of Federal University of Alagoas (UFAL) and Fernando A. Gallego of Universidad Nacional de Colombia (UNAL).

Elliptic Equations

Organizer: Claudianor O. Alves

Global bifurcation results for a fractional equation in $\mathbb{R}^N$

Alânnio B. Nóbrega, Claudianor O. Alves, Romildo Nascimento de Lima
Universidade Federal de Campina Grande, Brazil

In this paper, we study the existence of positive solution for the following class of fractional equation

$$\begin{cases} (-\Delta)^s u = \lambda g(x) f(u), \text{ in } \mathbb{R}^N \\ 0 < u(x) < 1, \ x \in \mathbb{R}^N, \ \lim_{|x|\to+\infty} u(x) = 0, \end{cases} \tag{P}$$

where $N > 2s$, $s \in (0,1)$, $\lambda > 0$, $f : \mathbb{R} \to \mathbb{R}$ is a continuous function, $g : \mathbb{R}^N \to \mathbb{R}$ is a bounded continuous function that changes of signal and $(-\Delta)^s u$ is the fractional Laplacian. The main tools used are a theorem due to Crandall and Rabinowitz and the global bifurcation theorem due to Rabinowitz.

A class of critical Kirchhoff problem on the hyperbolic space $\mathbb{H}^n$

Augusto C. R. Costa, Paulo C. Carrião, Olimpio H. Miyagaki
Universidade Federal do Pará, Brazil

In this work we investigate questions on the existence of nontrivial solution for the following Kirchhoff-type equation

$$-\left(a + b\int_{\mathbf{B}^3} |\nabla_{\mathbf{B}^3} u|^2 dV_{\mathbf{B}^3}\right) \Delta_{\mathbf{B}^3} u \ = \ \lambda |u|^{q-2}u + |u|^4 u \quad \text{in} \quad u \in H^1(\mathbf{B}^3), \tag{4}$$

in Hyperbolic space $\mathbf{B}^3$, where a, b, λ are positive constants, $4 < q < 6$, $H^1(\mathbf{B}^3)$ is the usual Sobolev space on the disc model of the Hyperbolic space $\mathbf{B}^3$, and $\Delta_{\mathbf{B}^3}$ denotes the Laplace Beltrami operator on $\mathbf{B}^3$. By the use of the stereographic projection the problem becomes a singular problem on the boundary of the open ball $B_1(0) \subset \mathbb{R}^n$. The Hardy inequality, in a version due to the Brezis-Marcus, combined with the mountain pass theorem due to Ambrosetti-Rabinowitz are used to obtain the nontrivial solution. One of the difficulties is to find a range where the Palais Smale converges, because our equation involves a nonlocal term coming from the Kirchhoff term.

Multiplicity of solutions for elliptic strongly-singular problems

Carlos Alberto Santos, Lais Santos, Pawan Mishra
Universidade de Brasília, Brazil

In this talk, we show existence of *continuums* of positive solutions for non-local quasilinear problems with strongly-singular reaction term on a bounded domain in $\mathbb{R}^N$ with $N \geq 2$ both for autonomous and non-autonomous non-local terms. Besides this, by using a comparison principle for $W^{1,p}_{\text{loc}}(\Omega)$-sub and supersolutions, we obtain qualitative properties of the *continuum*. Moreover, this technique empowers us to study a strongly-singular and non-homogeneous Kirchhoff problem to get the existence of a *continuum* of positive solutions a well.

Existence of positive solutions for elliptic equations with local and nonlocal diffusion coefficient

Cristian Morales-Rodrigo, Tarcyana S. Figueiredo, Antonio Suarez, Giovany M. Figueiredo
Universidad de Sevilla, Spain

In this talk we consider a stationary problem arising from population dynamics with a local and nonlocal variable diffusion coefficient. We show the existence of an unbounded continuum of positive solutions that bifurcates from the trivial solution. The global structure of this continuum depends on the value of the nonlocal diffusion at infinity and the relative position of the refuge of the species and of the sets where it diffuses locally and not locally, respectively.

Sharp Regularity for the Inhomogenous Porous Medium Equation

Damião J. Araújo, José Miguel Urbano
Universidade Federal da Paraíba, Brazil

Here we show a refined parabolic geometric tangential analysis which provides sharp Hölder regularity exponent for weak solutions of the inhomogeneous porous medium equation.

Symmetry properties of positive solutions for fully nonlinear elliptic systems

Gabrielle Saller Nornberg
Universidade de São Paulo, Brazil

In this talk we will discuss some recent symmetry results for positive solutions of fully nonlinear uniformly elliptic systems up to quadratic growth in the gradient. We develop a unified treatment of the classical moving planes method in the spirit of Gidas-Ni-Nirenberg, which permits us to work without Lipschitz assumptions on the zero order term dependence. We also present different applications of our results, including uniqueness of positive solutions for Lane-Emden systems in the subcritical case in a ball, and symmetry for a class of equations with natural growth in the gradient.

Joint work with Ederson Moreira dos Santos.

Normalized solutions for a transmission problem

Gaetano Siciliano, Giovany M. Figueiredo
Universidade de São Paulo, Brazil

In this talk we consider the following problem:

$$\begin{cases} -\Delta u + \lambda u = |u|^{p-2}u & \text{in } \Omega, \\ -\Delta v + \mu v = a(x)|v|^{q-2}v & \text{in } \mathbb{R}^N \setminus \Omega, \\ u = v \ \text{ and } \ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded and smooth domain in $\mathbb{R}^N, N \geq 3$. The unknowns are $u : \Omega \to \mathbb{R}, v : \mathbb{R}^N \setminus \Omega \to \mathbb{R}$ and the real numbers λ, μ. Actually we are looking for solutions u, v with prescribed L^2-norm. The problem has then a natural variational formulation and indeed, under minimal assumptions, we are able to find infinitely many solutions by using the Krasnoselkii genus, whatever the L^2-norm of u and v *a priori* is.

Regularity theory for roughly degenerate diffusions

Giane Casari Rampasso, Edgard Pimentel, Makson Santos
Universidade Estadual de Campinas, Brazil

The purpose of this work is to study the regularity theory of a degenerate diffusion equation in the divergence form

$$\operatorname{div}(|Du|^{\theta(x)-2}Du) = f(x) \text{ in } B_1$$

for a variable exponent given by a function $\theta : B_1 \to \mathbb{R}$ merely mensurable and bounded, and $f : B_1 \to \mathbb{R}$ is in a suitable Lebesgue space with variable exponent. We resort to variational techniques to prove the solvability of this problem in $W^{1,\theta(\cdot)}(B_1)$. Our methods also produce regularity of the solutions in $\mathcal{C}^{1,\alpha}(B_1)$ with appropriate estimates. Under more regular exponents θ, we establish gains of regularity for the solutions. This is done through geometric and approximation methods. In particular, we prove that solutions are asymptotically of class $\mathcal{C}^{1,1}(B_1)$. This is a joint-work with E. Pimentel (PUC-Rio) and M. Santos (PUC-Rio).

On concentration of solution to a Schrödinger logarithmic equation with deepening potential well

Giovany M. Figueiredo, Claudianor O. Alves, Daniel C. de Morais Filho
Universidade de Brasília, Brazil

In this work we prove the existence of positive solution for the following class of problems

$$\begin{cases} -\Delta u + \lambda V(x)u = u\log u^2, & x \in \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases}$$

where $\lambda > 0$ and $V : \mathbb{R}^N \to \mathbb{R}$ is a potential satisfying some conditions. Using the variational method developed by Szulkin for functionals which are sum of a C^1 functional with a convex lower semicontinuous functional, we prove that for each $\lambda > 0$ large enough there exists a positive solutions and, as $\lambda \to +\infty$, such solutions converge to a positive solution of the limit problem in $\Omega = \operatorname{int}(V^{-1}(\{0\}))$.

Existence of positive solutions for a class of semipositone quasilinear problems through Orlicz-Sobolev space

Jefferson A. Santos, Claudianor O. Alves, Angelo R. F. de Holanda
Universidade Federal de Campina Grande, Brazil

In this work we show the existence of weak solution for a class of semipositone problem of the type

$$\begin{cases} -\Delta_\Phi u = f(u) - a & \text{in } \Omega, \\ u(x) > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a smooth bounded domain, $f : [0, +\infty) \to \mathbb{R}$ is a continuous function with subcritical growth, $a > 0$, $\Delta_\Phi u$ stands for the Φ-Laplacian operator. By using variational methods, we prove the existence of solution for a small enough.

Sharp regularity estimates for p-quasilinear elliptic models and their asymptotic limit as $p \to \infty$

João Vitor da Silva, Julio D. Rossi, Ariel M. Salort
Universidad de Buenos Aires, Argentina

In this Lecture, we discuss regularity issues and the limiting behavior as $p \to \infty$ of nonnegative solutions for elliptic equations of $p-$Laplacian type ($2 \leq p < \infty$) with a strong absorption as follows:

$$-\Delta_p u(x) + \lambda_0(x) u_+^q(x) = 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^N,$$

where $\lambda_0 > 0$ is bounded away from zero and infinity, Ω is a bounded domain and $0 \leq q < p-1$. When p is fixed, such a model is mathematically interesting since it permits the formation of dead core zones, i.e, *a priori* unknown regions where non-negative solutions vanish identically. First, we turn our attention to establishing sharp $C^{\frac{p}{p-1-q}}$ regularity estimates for solutions. Afterwards, assuming that $\ell := \lim_{p\to\infty} \frac{q(p)}{p} \in [0,1)$ exists, we establish existence for limit solutions as $p \to \infty$, as well as we characterize the corresponding limit operator governing the limit problem. We also establish sharp C^γ regularity estimates for limit solutions along free boundary points, that is, points on $\partial\{u > 0\} \cap \Omega$, where the sharp regularity exponent is given explicitly by $\gamma = \frac{1}{1-\ell}$.

The mathematical insights in order to prove such sharp regularity estimates are based on a refined compactness method, as well as a systematic iterative approximation procedure. These estimates can be found in [Theorem 1.2, 1.3 and 1.4, 1] and [Theorem 1.2, 2].

This is joint work with Julio D. Rossi and Ariel M. Salort from Universidad de Buenos Aires - Argentina.

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An elliptic system with logarithimic nonlinearity

Leandro S. Tavares, Claudianor O. Alves, Abdelkrim Moussaoui
Universidade Federal do Cariri, Brazil

In this work we are interested to prove the existence of solution for the following singular system

$$\begin{cases} -\Delta_{p(x)} u = -\gamma \log v + \theta v^{\alpha(x)} & \text{in } \Omega, \\ -\Delta_{q(x)} v = -\gamma \log u + \theta u^{\beta(x)} & \text{in } \Omega, \\ u, v > 0 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N (N \geq 3)$ is a bounded domain with smooth boundary, $-\Delta_{p(x)} u := -div(|\nabla u|^{p(x)-2}\nabla u)$ is the $p(x)-$ Laplacian operator, γ and θ are positive constants and $\alpha, \beta : \overline{\Omega} \to \mathbb{R}$ are positive exponents. Our study is based on Bifurcation Theory and on a new result involving sub-super solutions. This work was done in collaboration with Claudianor O. Alves (Federal University of Campina Grande/Brazil) and Abdelkrim Moussaoui (University of Bejaia/Algeria).

Comparison results for solutions of degenerate elliptic equations in exterior domains

Leonardo Prange Bonorino, Paulo Ricardo de Ávila Zingano, André Rodrigues da Silva
Universidade Federal do Rio Grande do Sul, Brazil

We investigate some Liouville's theorem for solutions of some degenerate elliptic equations in exterior domains $\mathbb{R}^n \backslash K$, provided the structure of this equation and the dimension n are related. We apply variational and nonvariational techniques to study these problems. For degenerate elliptic equations in the divergence form, a comparison result and the uniqueness of solution are obtained for such problems.

Neumann problems of the Ambrosetti-Prodi type for p-Laplacian

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Universidade Tecnológica Federal do Paraná, Brazil

In this work, we present a result existence and multiplicity of solutions for a Ambrosetti-Prodi type problem with Neumann condition at the border. Ambrosetti and Prodi [1], in 1972, considered the following problem:

$$(P_D)\begin{cases} -\Delta u = f(u) + v(x) \;\; ; x \in \Omega \\ u = 0 \;\; ; x \in \partial\Omega \end{cases}$$

were $v \in C^{0,\alpha}(\overline{\Omega})$, $0 < \alpha < 1$, e $f \in C^2(\mathbb{R})$ satisfying the conditions:

1. $f''(s) > 0 \quad \forall s \in \mathbb{R}$,
2. $0 < \lim\limits_{s\to-\infty} \dfrac{f(s)}{s} < \lambda_1 < \lim\limits_{s\to+\infty} \dfrac{f(s)}{s} < \lambda_2$.

Using inversion theorems for differentiable applications with singularities in Banach spaces, they proved the existence of a closed and closed Γ range, from classe C^1 to $C^{0,\alpha}(\Omega)$ dividing the space into two related components A_0 e A_1 so that the problem (PD), has exactly one solution if $v \in \Gamma$, no solution if $v \in A_0$ and exactly two solutions are $v \in A_1$.

Berger and Podolak, in 1975, made the decomposition of v in the form $v(x) = t\phi(x) + h(x)$, where $\phi(x)$ is the first positive self-function of $(-\Delta, W_0^{1,2}(\Omega))$ e $h(x) \in \left\{span(\phi)\right\}^{\perp}$. Using the Liapunov-Schmidt method, they showed precisely the same result as Ambrosetti and Prodi, presented as follows (P_D) has exactly two solutions, one or none of the solutions if $t < t_1$, $t = t_1$ ou $t > t_1$,respectively. The problem for the p -Laplaciano with condition of Neumann was considered by De Paiva and Montenegro [3]. More precisely, in [3], the authors consider the following problem

$$(P_{Nt})\begin{cases} -\Delta_p u = f(x,u) + t \;\; ; x \in \Omega \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} = 0 \;\; ; x \in \partial\Omega \end{cases}$$

with $f : \Omega \times \mathbb{R} \to \mathbb{R}$ a Carathéodory function satisfying conditions as in [2]. The authors prove that there exists $t_0 \in \mathbb{R}$ such that (P_{Nt}) has no solutions if $t > t_0$, and (P_{Nt}) has at least one minimal solution if $t < t_0$. If in addition f is locally Lipschitz continuous in s evenly qtp $x \in \Omega$, then there exists $t_1 \leq t_0$ such that for $t < t_1$ or problem (P_{Nt}) has at least two distinct solutions. In addition, the equality $t_1 = t_0$ occurs if $f \in C(\overline{\Omega} \times \mathbb{R})$.

The following problem of Neumann will be approached in this work:

$$(P_t)\begin{cases} -\Delta_p u = f(x,u) + t\phi(x) + h(x) \;\; ; x \in \Omega \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} = 0 \;\; ; x \in \partial\Omega \end{cases}$$

were $\phi(x) \geq 0$, $\phi(x) \not\equiv 0$, $\phi, h \in L^\infty(\Omega)$, $\Omega \subset \mathbb{R}^N$ an open, bounded and smooth $\partial\Omega$ and a Carathéodory function satisfying the following conditions:

$$\limsup_{s\to-\infty} \frac{f(x,s)}{|s|^{p-2}s} < 0 < \liminf_{s\to+\infty} \frac{f(x,s)}{|s|^{p-2}s} \quad \text{uniformly in } x \in \Omega.$$

We also assume that $\forall M > 0$, $\exists \lambda > 0$ such that,

$$g(x,u) = f(x,u) + \lambda|u|^{p-2}u \quad \text{is non-decreasing } \forall u \in [-M, M],$$

and the following growth condition

$$|f(x,s)| \leq c(1 + |s|^{p-1}); \quad \forall (x,s) \in \Omega \times \mathbb{R}.$$

We will give a brief demonstration of the following Ambrosetti-Prodi type result: there are $t_1 \leq t_0 \in \mathbb{R}$, such that

(i) If $t < t_1$, then (P_t) has at least two solutions.

(ii) If $t \leq t_0$, them the problem has (P_t) at least one solution.

(ii) If $t > t_0$, then the problem (P_t) has no solution.

References:

[1] AMBROSETTI, A.; PRODI, G. *On the inversion of some differentiable mappings with singularities between Banach spaces*. Ann.Mat.Pura Appl, v.4, n.93, p. 231-246, 1972.

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Discrepancy fields in systems modelled with partial differential equations

Nilson Costa Roberty
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Most of the engineering models can be represented as system of partial differential equations. Those models are mathematically elaborated with continuous thermomechanics and the constitutive theories of materials in order to formulate well posed problems. But frequently we have incomplete information about coefficients and/or sources. For a prescribed geometry and Cauchy data on the boundary, problems with constitutives equations using wrong parameters develops an internal field of discrepancy. We analyses this kind of boundary value problems in Lipschitz domains with many Cauchy data at the boundary. The main techniques are variational formulation, boundary integral equations and Calderon projector. To estimate those coefficients and sources we propose an optimization methodology based on the minimization of internal discrepancy. Numerical finite elements experiments are presented.

Remarks on a class of nonlocal elliptic systems with nonlinearities interacting with the spectrum

Olimpio H. Miyagaki, Fabio Rodrigues Pereira
Universidade Federal de Juiz de Fora, Brazil

In this talk we will discuss a class of nonlocal elliptic systems with nonlinearities intereacting with the spectrum. Existence and multplicity results are established by using Rabinowitz linking theorem, in resonant and non resonant cases.

Multiplicity of solutions for a Hénon-type system

Patrícia Leal da Cunha, Flávio de Almeida Lemos
Fundação Getúlio Vargas, Brazil

In this article we study the existence of infinitely many solutions for a semilinear elliptic system of Hénon-type in hyperbolic space. We prove a compactness result and together with the Clark's theorem we establish the existence of infinitely many solutions.

On fractional p-Laplacian problems with weight

Raquel Lehrer, Liliane de Almeida Maia, Marco Squassina
Universidade Estadual do Oeste do Paraná, Brazil

We investigate the existence of nonnegative solutions for a nonlinear problem involving the fractional p-Laplacian operator. The problem is set on a unbounded domain, and compactness issues have to be handled. In this paper, we are concerned with existence of solutions of

$$\begin{cases} (-\Delta)_p^s u = \varphi(x) f(u), & \text{in } \mathbb{R}^N, \\ u \geq 0, \ u \neq 0, \end{cases} \tag{5}$$

under suitable growth and sign assumptions on the functions φ and f. We will assume that $p > 1$, $\varphi \in L^\infty_{loc}(\mathbb{R}^N)$ and $f \in C(\mathbb{R}^+)$ satisfies the following conditions:

(f_1) $f(\tau) \geq 0$, for all $\tau \geq 0$;

(f_2) $\mu\tau^q \leq f(\tau) \leq c\tau^q$, for all $\tau \geq 0$, some $p-1 < q < p_s^* - 1$ and $c, \mu > 0$;

(f_3) if F denotes the function $F(u) = \int_0^u f(\tau)d\tau$, there exists $m < p$ such that

$$0 \leq (q+1)F(\tau) - f(\tau)\tau \leq C\tau^m, \quad \text{for all } \tau \geq 0 \text{ and some } C > 0;$$
$$0 \leq f(\tau)\tau - pF(\tau) \leq C\tau^{q+1}, \quad \text{for all } \tau \geq 0.$$

(W) $\sup_{\mathbb{R}^N \setminus \Omega} \varphi \leq 0 < \inf_{\omega} \varphi$ for some bounded domains $\omega, \Omega \subset \mathbb{R}^N$ with $\omega \subset \Omega$.

The main result of the paper is the following:

Theorem: Assume that (W) and (f_1)-(f_3) hold. Then problem (5) has a distributional solution, namely there exists a function $u \in L^{Np/(N-sp)}(\mathbb{R}^N) \setminus \{0\}$ with $u \geq 0$,

$$\int_{\mathbb{R}^{2N}} \frac{|u(x) - u(y)|^p}{|x-y|^{N+sp}} < \infty,$$

and

$$\int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))(\psi(x)-\psi(y))}{|x-y|^{N+sp}} = \int_{\mathbb{R}^N} \varphi(x) f(u)\psi,$$

for all $\psi \in C_c^\infty(\mathbb{R}^N)$. The same holds if (W) and (f_1) hold and (f_2) holds with $0 \leq q < p-1$.

Bifurcation properties for a class of fractional Laplacian equations in $\mathbb{R}^N$

Romildo Nascimento de Lima, Claudianor O. Alves, Romildo N. de Lima, Alânnio B. Nóbrega
Universidade Federal de Campina Grande, Brazil

This paper concerns with the study of some bifurcation properties for the following class of nonlocal problems

$$\begin{cases} (-\Delta)^s u = \lambda f(x)(u+h(u)), \text{ in } \mathbb{R}^N, \\ u(x) > 0, \quad \forall x \in \mathbb{R}^N, \\ \lim_{|x|\to\infty} u(x) = 0, \end{cases} \tag{P}$$

where $N > 2s$, $s \in (0,1)$, $\lambda > 0$, $f : \mathbb{R}^N \to \mathbb{R}$ is a positive continuous function, $h : \mathbb{R} \to \mathbb{R}$ is a bounded continuous function and $(-\Delta)^s u$ is the fractional Laplacian. The main tools used are the Leray-Shauder degree theory and the global bifurcation result due to Rabinowitz.

Nonradial solutions for the Hénon equation close to the threshold

Sérgio L. N. Neves, Pablo Figueroa
Universidade Estadual Paulista, Brazil

We consider the Hénon problem

$$\begin{cases} -\Delta u = |x|^\alpha u^{\frac{N+2+2\alpha}{N-2}-\varepsilon} & \text{in } B_1, \\ u > 0 & \text{in } B_1, \\ u = 0 & \text{on } \partial B_1, \end{cases}$$

where B_1 is the unit ball in $\mathbb{R}^N$ and $N \geqslant 3$. For $\varepsilon > 0$ small enough, we use α as a parameter and prove the existence of a branch of nonradial solutions that bifurcates from the radial one when α is close to an even positive integer.

A sub-super critical p-Laplacian problem

Sigifredo Herrón, Alfonso Castro, Rosa Pardo, Jorge Cossio, Carlos Vélez
Universidad Nacional de Colombia Sede Medellín, Colombia

We prove the existence of infinitely many sign changing radial solutions for a p-Laplacian Dirichlet problem which is p-sub-super critical at infinity. More exactly, we consider an equation defined by the p-Laplacian operator perturbed by a nonlinearity $g(u)$ that is p-subcritical at plus infinity and p-supercritical at minus infinity. Our results extend previous works in the literature for the corresponding semilinear case and those one where the subcritical case was studied.

References:

[1] A. Castro and A. Kurepa, Infinitely Many Radially Symmetric Solutions to a Superlinear Dirichlet Problem in a Ball, *Proc. Amer. Math. Soc.* 101 (1): 57–64, 1987.

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Singular Hamiltonian elliptic systems with critical exponential growth in dimension two

Yony R. Santaria Leuyacc, Sergio H. Monari Soares
Universidad Nacional Mayor de San Marcos, Peru

We will focus on the existence of nontrivial solutions to the following Hamiltonian elliptic system

$$\begin{cases} -\Delta u + V(x)u = \dfrac{g(v)}{|x|^a}, & x \in \mathbb{R}^2, \\ -\Delta v + V(x)v = \dfrac{f(u)}{|x|^b}, & x \in \mathbb{R}^2, \end{cases}$$

where a, b are numbers belong to the interval $[0, 2)$, V is a continuous potential bounded from below on $\mathbb{R}^2$ by a positive constant and the functions f and g possess maximal exponential growth range established by Trudinger-Moser inequalities in Lorentz-Sobolev spaces. The proof involves linking theorem and a finite-dimensional approximation.

Evolution Equations and Applications

Organizers: Alessia Kogoj & Giulio Tralli

On a Harnack inequality and an asymptotic mean-value property

Ahmed Mohammed
Ball State University, USA

The talk focuses on two themes related to viscosity solutions of equations associated with the normalized Finsler-infinity Laplace operators $\mathcal{L}$. A special case of these operators is the more familiar normalized infinity Laplacian. First, we discuss a Harnack inequality for non-negative viscosity solutions of $\mathcal{L}u \geq h$. As a consequence of the Harnack inequality, we obtain a Liouville property for nonnegative solutions of $\mathcal{L}u \geq 0$ in $\mathbb{R}^n$. The second objective is to give a characterization of viscosity solutions of $\mathcal{L}u = 0$ via an asymptotic mean-value property, in the viscosity sense.

Linear and semilinear problems involving Δ_λ-laplacians

Alessia E. Kogoj, Ermanno Lanconelli, Stefanie Sonner
Università di Urbino Carlo Bo, Italy

We present a survey of recent results related to Δ_λ-laplacians, a class of degenerate elliptic operators containing for example Grushin-type operators. In particular we show Pohozaev identities, Liouville theorems, Hardy inequalities, existence and longtime behavior of solutions of the related semi-linear degenerate parabolic equations.

These results are obtained in collaboration with Ermanno Lanconelli and Stefanie Sonner.

Groups on Fréchet spaces and the heat equation solution for negative time

Alex Pereira da Silva, Éder Rítis Aragão Costa
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Evolution equations arises from biological, physical and economic phenomena and have been dealt by many mathematicians. The main approach consists in dealing with a linear operator $A\colon D(A) \subset X \to X$, which is closed and densely defined on a Banach space X, see [1]. However, many well-known topological vector spaces arising in PDEs analysis are not normable - such as $C\Big((-\infty,0]\Big)$ in equations with infinite delay. There is a natural trade-off in such a question: to formulate these problems in spaces with a weaker topology but to gain better properties on the solution operator.

We present a general method for generation of uniformly continuous groups on abstract Fréchet spaces. As a consequence, every pseudodifferential operator with constant coefficients can be defined on $\mathcal{F}L^2_{loc}$ - a Fréchet space of distributions defined in [2] - and its Cauchy problem has a unique solution on such a space. Besides, we conclude that the solution of the heat equation on $\mathcal{F}L^2_{loc}$ for all time extends the standard solution on Hilbert spaces for positive time.

References:

[1] *Treves, F.*: Study of a model in the theory of complexes of pseudodifferential operators. Annals of Mathematics (1976), 269-324.

[2] *Yosida, K.*: Functional Analysis. 6th ed. Springer Verlang, 1980.

Global solutions for a fractional reaction–diffusion equation

Arlúcio Viana
Universidade Federal de Sergipe, Brazil

We shall discuss the existence and nonexistence of global positive solutions for the semilinear fractional diffusion equation

$$u_t(t,x) = \int_0^t dg_\alpha(s)\Delta u(t-s,x) + |u(t,x)|^{\rho-1}u(t,x), \text{ in } (0,\infty)\times\mathbb{R}^N; \tag{6}$$

$$u(x,0) = u_0(x), \text{ in } \mathbb{R}^N, \tag{7}$$

where $\rho > 1$. Indeed, a combination of [Theorem 1, 1] and [Theorem 5, 2] provides the following result:

(i) If $1 < \rho < 1 + \frac{2}{\alpha N}$, there exists no nonnegative global solution of (**??**)-(7).

(ii) If $\rho \geq 1 + \frac{2}{\alpha N}$, there exists a global nonnegative solution of (**??**)-(7).

Eventually, we highlight some asymptotic properties of the existing global solution.

References:

[1] B. de Andrade and A. Viana, On a fractional reaction-diffusion equation, Z. Angew. Math. Phys. **68** (2017), no. 3, Art. 59, 11 pp.
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Exponential decay for waves with indefinite memory

Bianca Morelli Rodolfo Calsavara, Higidio Portillo Oquendo
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In the context of partial differential equations systems where the dissipative effects are given by memory terms and they change sign there are few studies about the energy decay rate. One of the earliest studies in this direction is due to Muñoz-Rivera and Naso. They considered the following functional equation with memory term

$$u_{tt} + Au - \int_0^t g(t-s)Au(s)ds = 0,$$

where the memory kernel g can change sign. They proved, in [1], the exponential decay of the solutions if $0 < g(0) < \lambda_1$, where λ_1 is the smaller eigenvalue of the self-adjoint, positive definite operator A in a Hilbert space. In this work, the memory dissipation is distributed on whole domain. Its a open study when this dissipation is distributed only in a part of its domain.

In this work we deal with the following wave equation with localized dissipation given by a memory term

$$u_{tt} - u_{xx} + \partial_x\left\{a(x)\int_0^t g(t-s)u_x(x,s)ds\right\} = 0.$$

Here we consider that the dissipation is indefinite due to sign changes of the coefficient a or by sign changes of the memory kernel g. With this condition, we prove the exponential decay of solutions when the average of coefficient a is positive and the memory kernel g is small.

References:

[1] J.E. Muñoz-Rivera, M.G. Naso. On the decay of the energy for systems with memory and indefinite dissipation, Asymptotic Analysis. 49, 2014, 189-204.

Entropic optimal transport and mean field planning

Carlo Orrieri, Alessio Porretta, Giuseppe Savaré
University of Trento, Italy

The mean field planning problem (MFPP) is formulated by a continuity equation and Hamilton-Jacobi equation with a nonlinear coupling. Firstly introduced by P.-L. Lions in the context of mean field games theory, MFPPs describe strategic interactions among large numbers of players when the initial and final distributions are prescribed. The aim of the presentation is to recast the PDE system as an optimality system of a suitable entropic regularization of the dynamic optimal transportation problem. We will discuss existence of weak solutions using some ideas of minmax duality and dynamic superposition principles.

Harnack inequality in doubling quasi-metric spaces and applications

Chiara Guidi
University of Bologna, Italy

In this talk we will discuss an axiomatic approach to Harnack inequality in the setting of doubling Hölder quasi-metric spaces. More precisely, we will adapt the abstract procedure developed by Di Fazio, Gutiérrez and Lanconelli, in order to take into account non-zero right-hand side of the equations. Then, we will use the axiomatic approach to prove Harnack inequality for a class of sub-elliptic operators in non-divergence form, with non-smooth coefficients, involving Grushin vector fields. This is a joint work with A. Montanari.

Dirichlet to Neumann characterization of fractional powers of non-symmetric operators and applications

Cristian Rios, Hugo Aimar
University of Calgary, Canada

We obtain a new general extension theorem in Banach spaces for operators which are not required to be symmetric, and apply it to obtain Harnack estimates and a priori regularity for solutions of fractional powers of several second order differential operators. We utilize the reflection extension technique introduced by Caffarelli and Silvestre.

Reaction-diffusion problems on time-dependent Riemannian manifolds: stability of periodic solutions

Dario Monticelli, C. Bandle, F. Punzo
Politecnico di Milano, Italy

In this talk we will briefly discuss some recent results concerning the stability of time-periodic solutions of semilinear parabolic problems posed on a bounded domain of a Riemannian manifold, with homogeneous Neumann boundary conditions when the boundary of the domain is not empty. On the domain we consider metrics that vary periodically in time. The discussion is based on the principal eigenvalue of periodic parabolic operators. The study is related to biological models on the effect of growth and curvature on pattern formation. Metric properties, for instance, the Ricci curvature, play a crucial role. These results are joint work with C. Bandle (University of Basel) and F. Punzo (Politecnico di Milano)

Moser's estimates for degenerate Kolmogorov equations with non-negative divergence lower order coefficients

Francesca Anceschi, Sergio Polidoro, Maria Alessandra Ragusa
Università degli Studi di Modena e Reggio Emilia, Italy

Degenerate Kolmogorov equations arise in the theory of stochastic processes (e.g. the simplest non-trivial Kolmogorov operator is the infinitesimal generator of the Langevin's stochastic equation), kinetic theory (e.g. linear Fokker-Planck equations, non-linear Boltzmann-Landau equations) and mathematical finance (e.g. problem of pricing Asian options). The study of the regularity theory for weak solutions to this kind of equations is carried out paralleling the weak theory for parabolic equations, such as Sobolev and Caccioppoli inequalities, Moser's iteration, Hölder regularity and Harnack inequality. In particular, we consider the following second order partial differential equation of Kolmogorov type

$$Łu = \sum_{i,j=1}^{m_0} \partial_{x_i} \left(a_{ij}(x,t) \partial_{x_j} u(x,t) \right) + \sum_{i,j=1}^{N} b_{ij} x_j \partial_{x_i} u(x,t) - \partial_t u(x,t) + \sum_{i=1}^{m_0} b_i(x,t) \partial_i u(x,t) = 0$$

where $(x,t) = (x_1, \dots, x_N, t) = z$ is a point of $\mathbb{R}^{N+1}$, and $1 \leq m_0 \leq N$. (a_{ij}) is an uniformly positive symmetric matrix with bounded measurable coefficients, (b_{ij}) is a constant matrix. We prove L^∞_{loc} estimates for positive solutions to $Łu = 0$. We apply the Moser's iteration method to prove the local boundedness of the solution u under minimal integrability assumption on the coefficients $b_1, \dots, b_{m_0}$.

Fractional operators of Kolmogorov-Fokker-Planck type

Giulio Tralli
University of Padova, Italy

In this talk we will discuss a class of nonlocal equations driven by the fractional powers of hypoelliptic operators of Kolmogorov-type. We will introduce an extension problem related to such operators and provide an explicit construction for its solution. This is a joint work with N. Garofalo.

Perturbation theory for the chemical principle PDE

Jayme de Luca
Universidade Federal de São Carlos, Brazil

In electrodynamics, the infinite-dimensional two-and-a-half body problem is a three-body problem where the third charge suffers no far-fields from the other two charges. We discuss the natural PDE of this two-and-a-half-body problem when two charges are in a globally-bounded orbit and the third charge is far away. We put the PDE in a Sobolev space W(2,2) and discuss a perturbation scheme to approximate its nonlinear terms using the stable manifold of a globally-bounded two-body orbit with an infinitesimal acceleration.

Inverse mean curvature flow via p-Laplace approximation under Ricci lower bounds

Luciano Mari, Marco Rigoli, Alberto G. Setti
Scuola Normale Superiore, Italy

In this talk, we consider the existence problem for weak solutions of the Inverse Mean Curvature Flow on a complete manifold with only a Ricci lower bound. Solutions either issue from a point or from the boundary of a relatively compact open set. To prove their existence in the sense of Huisken-Ilmanen, we follow the strategy pioneered by R. Moser using approximation by p-Laplacian kernels. In particular, we prove new and sharp gradient estimates for the kernel of the p-Laplacian on M via the study of the fake distance associated to it. We address the compactness of the flowing hypersurfaces, and time permitting some monotonicity formulas in the spirit of Geroch's one.

This is joint work with M. Rigoli and A.G. Setti.

Controllability of Grushin type equations

Roberto Guglielmi, Karine Beauchard, Piermarco Cannarsa
Gran Sasso Science Institute, Italy

We study the null controllability of the parabolic equation associated with the Grushin-type operator over a rectangle domain in dimension two. We show that there exists a threshold of the degeneracy parameter such that null controllability holds for values below such threshold, and it does not hold otherwise. Moreover, for the threshold value, a new behavior appears: a positive minimal time is required for null controllability. Our approach is based on the fact that, thanks to the particular geometric configuration of the problem, null controllability is equivalent to uniform with-respect-to-frequency observability for the Fourier components of the solution of the adjoint system.

p-Laplacians and Sobolev inequalities

Scott Rodney
Cape Breton University, Canada

In this talk I will discuss some improvements on joint work with D. Cruz-Uribe (University of Alabama) and E. Rosta (Cape Breton University). Using recent regularity results applied to a family of matrix weighted p-Laplacian equations, I will demonstrate a local-global result for matrix weighted Sobolev inequalities.

A geometric statement of the Harnack inequality for degenerate second order PDEs

Sergio Polidoro, Francesca Anceschi, Michela Eleuteri
Università degli Studi di Modena and Reggio Emilia, Italy

We consider second order, possibly degenerate, operators of the form

$$\mathcal{L}u(x,t) := \sum_{j=1}^{m} X_j^2 u(x,t) + X_0 u(x,t) - \partial_t u(x,t), \qquad (x,t) \in \Omega \subset \mathbb{R}^{N+1},$$

where Ω is a bounded open subset of $\mathbb{R}^{N+1}$ and $X_0 - \partial_t, X_1, \dots, X_m$ are smooth vector fields satisfying the Hörmander's condition. We give a geometric sufficient condition on the point $(x_0, t_0) \in \Omega$ and on the compact sets $K \subset \Omega$ for which the Harnack inequality

$$\sup_K u \leq C_K u(x_0, t_0),$$

holds for all non-negative solutions u to the equation $\mathcal{L}u = 0$ in Ω.

The above result has been recently extended to the following kinetic equation with rough coefficients

$$\partial_t u + \langle v, \nabla_x u \rangle = \operatorname{div}_v(A(v,x,t)\nabla_v u) + \langle b(v,x,t), \nabla_v u \rangle + f, \quad (v,x,t) \in \mathbb{R}^{2n+1}.$$

Here A is an uniformly positive symmetric matrix with bounded measurable coefficients, f and the components of the vector b are bounded and measurable functions, and this result extends the Harnack inequality proved by Golse, Imbert, Mouhot and Vasseur. As a direct consequence of our result we get a strong maximum principle, which extends the well known Bony's maximum principle for classical solutions of degenerate hypoelliptic partial differential equations with smooth coefficients.

Fluid Dynamics

Organizers: Anne Bronzi & César J. Niche

On the Euler equations with helical symmetry

Anne Bronzi
Universidade Estadual de Campinas, Brazil

In this talk we will survey some results regarding the global existence of weak solutions of the Euler equations in $\mathbb{R}^3$ with helical symmetry. This is joint work with H. Lopes and M. Lopes.

On the existence and regularity of solutions for a quimiotaxis-Navier-Stokes system

Élder J. Villamizar-Roa
Universidad Industrial de Santander, Colombia

Chemotaxis is the oriented movement of cells toward the concentration gradient of certain chemicals in their environment. In particular, when the movement of cells is directed toward the increasing concentration of a signal, the phenomena is known as chemotaxis by atracction. This kind of phenomena, which plays an outstanding role in a large range of biological applications, are modeled, in their simplest form, by the Keller-Segel system. However, some experimental studies reveal that the chemotactic motion in liquid environments affects substantially the migration of cells. This kind of interaction is modeled through a coupling between the Keller-Segel-System and the Navier-Stokes equations. In this talk we establish the existence of weak solutions and give a regularity criterion for a 3D Keller-Segel-Navier-Stokes equations. As consequence, we describe a possible application to an optimal control problem with state equation given by the 3D Keller-Segel-Navier-Stokes system.

Inviscid limits of solutions to the quasi-geostrophic equation with dispersion

César J. Niche, Leonardo Kosloff, Gabriela Planas
Universidade Federal do Rio de Janeiro, Brazil

The quasi-geostrophic equation with dispersion (QGED) has been recently studied because of its importance in the modelling of geophysical phenomena. In this work, we study the relation, when the viscosity tends to zero, between the solutions of the QGED with and without dissipation for different dispersive regimes. Joint work with Leonardo Kosloff (UNICAMP) and Gabriela Planas (UNICAMP).

Fluid dynamics on the logarithmic lattice and singularities of Euler flow

Ciro S. Campolina, Alexei A. Mailybaev
Instituto de Matemática Pura e Aplicada - IMPA, Brazil

We address the long-standing problem of a finite-time singularity (blowup) in the 3D incompressible Euler equations. We propose a new technique, which considers the Euler equations restricted to a 3D logarithmic lattice in Fourier space with specially designed calculus and algebraic operations,

giving rise to a simplified model structurally identical to the original one. With highly accurate simulations, we provide the unambiguous evidence of finite-time blowup, explained as a chaotic attractor in renormalized equations. The attractor has an anomalous multiscale structure, suggesting that it cannot be observed numerically with the current level of direct numerical simulations. This suggests an explanation for the existing controversy in literature and gives new insights for the original blowup problem.

References:

[1] C.S. Campolina and A.A. Mailybaev, Chaotic blowup in the 3D incompressible Euler equations on a logarithmic lattice, Physical Review Letters 121 (2018), 064501.

Three-dimensional phase-field model for solidification under a magnetic field effect

Gabriela Planas, André Ferreira e Pereira
Universidade Estadual de Campinas, Brazil

In this talk, we consider a mathematical analysis of a three-dimensional isothermal model of solidification for a binary alloy with melt convection and under a magnetic field effect. The model consists of a highly non-linear system of partial differential equations for the state variables: the velocity field, the pressure, the potential function of the electrical field, the phase-field which represents the solid/liquid phase of the alloy, and the concentration. The well-posedness of the model is discussed. Moreover, the existence of a solution when the diffusion coefficient of the concentration equation vanishes for some values of the phase-field is investigated.

Well-posedness for a non-isothermal flow of two viscous incompressible fluids

Juliana Honda Lopes, Gabriela Planas
Universidade Federal do Rio de Janeiro, Brazil

The dynamic interface of a mixture of two different fluids plays an important role in the hydrodynamic theory, due to the growing application in engineering. This work is concerned with a non-isothermal diffuse-interface model which describes the motion of a mixture of two viscous incompressible fluids. The model consists of modified Navier-Stokes equations coupled with a phase-field equation given by a convective Allen-Cahn equation, and energy transport equation for the temperature. More precisely, we consider the following system

$$\begin{aligned} u_t + u\cdot\nabla u - \nabla\cdot(\nu(\theta)Du) + \nabla p &= (-\epsilon\Delta\phi + F'(\phi))\nabla\phi - \alpha\Delta\theta\nabla\theta \\ \nabla\cdot u &= 0 \\ \phi_t + u\cdot\nabla\phi &= \gamma(\epsilon\Delta\phi - F'(\phi)) \\ \theta_t + u\cdot\nabla\theta &= k\Delta\theta \end{aligned}$$

in $\Omega\times(0,\infty)$, where Ω is a bounded domain in $\mathbb{R}^n$, $n=2,3$, with smooth boundary $\partial\Omega$.

This model admits a dissipative energy inequality. It is investigated the well-posedness of the problem in the two and three dimensional cases without any restriction on the size of the initial data. Moreover, regular and singular potentials for the phase-field equation are considered.

If the time permits, I will briefly talk about a more general problem, where the thermal conductivity is temperature dependent.

Long-time solvability for the 2D dispersive SQG equation with borderline regularity

Leonardo Kosloff, Vladimir Angulo-Castillo, Lucas C. F. Ferreira
Universidade Estadual de Campinas, Brazil

In this paper, we study the long-time existence and uniqueness (solvability) for the initial value problem of the 2D inviscid dispersive SQG equation. First we obtain the local solvability with existence-time independent of the amplitude parameter. Then, assuming more regularity and using a blow-up criterion of BKM type and a space-time estimate of Strichartz type, we prove long-time solvability of solutions in Besov spaces for large amplitude parameter and arbitrary initial data. In comparison with previous results, we are able to consider borderline cases of the regularity and larger initial data classes.

The onset of intermittency in stochastic Burgers hydrodynamics

Luca Moriconi, G. B. Apolinário, R. M. Pereira
Universidade Federal do Rio de Janeiro, Brazil

We study the onset of intermittency in stochastic Burgers hydrodynamics, as characterized by the statistical behavior of negative velocity gradient fluctuations. The analysis is based on the response functional formalism, where specific velocity configurations - the *viscous instantons* - are assumed to play a dominant role in modeling the left tails of velocity gradient probability distribution functions. We find, as it has been previously conjectured on purely empirical grounds, that the field theoretical approach becomes meaningful in practice only if the effects of fluctuations around instantons are taken into account. Working with a systematic cumulant expansion, it turns out that the integration of fluctuations yields, in leading perturbative order, to an effective description of the Burgers stochastic dynamics given by the renormalization of its associated heat kernel propagator and the external force-force correlation function.

The absence of classical profiles of turbulence over the Amazon forest

Nelson Luis da Costa Dias
Universidade Federal do Paraná, Brazil

On the basis of measurements over different surfaces, it is usually assumed that an inertial sublayer (ISL), where Monin-Obukhov Similarity Theory (MOST) applies, exists above $z = 3h$, where h is canopy height. The roughness sublayer (RSL) is within $h < z < 3h$. Most studies of the surface layer above forests, however, are able to probe only a narrow region above h. Therefore, direct verification of an ISL above tall forests is difficult. In this study we conducted a systematic analysis of unstable turbulence characteristics at heights from 40 to 325 m, measured at an 80-m and the recently built 325-m ATTO towers over the Amazon Forest. Our analyses have revealed no indication of the existence of an inertial sublayer; instead, the roughness sublayer directly merges with the convective mixed layer above. Implications for estimates of momentum and scalar fluxes in numerical models and observational studies can be significant.

Fractional 2D Stokes equation on bounded domains and Leibniz rule

Paulo M. Carvalho Neto, Renato Fehlberg Júnior
Universidade Federal de Santa Catarina, Brazil

This talk is dedicated to introduce a new inequality that involves an important case of Leibniz rule regarding Riemann-Liouville and Caputo fractional derivatives of order $\alpha \in (0,1)$. More specifically, we prove that for suitable functions f, it holds that

$$D^{\alpha}_{t_0,t}\Big[f(t)\Big]^2 \leq 2\Big[D^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \text{almost everywhere in } [t_0,t_1],$$

and

$$cD^{\alpha}_{t_0,t}\Big[f(t)\Big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \text{almost everywhere in } [t_0,t_1].$$

In the context of partial differential equations, the aforesaid inequality allows us to address the Faedo-Galerkin method to study the fractional version of the 2D Stokes equation on bounded domains Ω

$$\begin{aligned} cD^{\alpha}_t u - \nu\Delta u + \nabla p &= f && \text{in } \Omega,\ t>0,\\ \nabla\cdot u &= 0 && \text{in } \Omega,\ t>0,\\ u(x,t) &= 0 && \text{on } \partial\Omega,\ t>0,\\ u(x,0) &= u_0(x) && \text{in } \Omega. \end{aligned}$$

where cD^{α}_t is the Caputo fractional derivative of order $\alpha \in (0,1)$ and f a suitable function. This is a joint work with Prof. Renato Fehlberg Júnior.

Properties of the weak global attractor of the 3D incompressible Navier-Stokes equations

Ricardo Rosa
Universidade Federal do Rio de Janeiro, Brazil

The global well-posedness of the three-dimensional Navier-Stokes equations is one of the great open problems in Mathematics. It is directly connected with the regularity of the solutions of this system. Here, the plan is to discuss some results related to the asymptotic regularity of the solutions, i.e. the regularity of the solutions which belong to the weak global attractor of the system, which is a weakly-compact set attracting the orbits in the weak topology (the the weak topology in the space of square-integrable divergence-free vector fields). A conjecture of Prodi says that asymptotically the solutions are regular. The results presented are, somehow, related to this conjecture. More precisely, we discuss three results. The first one relates the strong converge to the weak global attractor with the strong continuity of the solutions within the global attractor. The second and third results show that there are regular parts of the weak global attractor which are large in both topological and measure-theoretic senses.

Linear Equations

Organizers: Marcelo R. Ebert & Marcello D'Abbicco

Analytic perturbations of global hypoelliptic operators

Alexandre Kirilov, Fernando de Ávila Silva
Universidade Federal do Paraná, Brazil

In this talk we consider the following class of the perturbed operators

$$L(\epsilon) = D_t + \omega D_x + \epsilon \mathcal{R}_x,$$

where $\epsilon, \omega \in \mathbb{C}$, $(t,x) \in \mathbb{T}^2$ and $\mathcal{R}_x : H^s(\mathbb{T}^1_x) \to H^{s-\delta}(\mathbb{T}^1_x)$ is a linear continuous operator of order $\delta < 1$ that commutes with the Laplacian.

Motivated by the perturbation theory of T. Kato and F. Rellich, we assume that the eigenvalues and eigenvectors of the perturbed operator $Q(\epsilon) = \omega D_x + \epsilon \mathcal{R}_x$ depend analytically on the eigenvalues and eigenvectors of $Q(0) = \omega D_x$, and construct nontrivial examples of perturbations that destroy the global hypoellipticity in the presence of diophantine phenomena.

Li-Yorke chaos for composition operators on L^p spaces

Benito Pires, Udayan Darji, Nilson Bernardes Jr
Universidade de São Paulo

The dynamics of a continuous linear operator acting on an infinite dimensional space is as rich as the dynamics of a continuous non-linear transformation acting on a compact metric space. Given a σ-finite measure space $(X, \mathcal{B}, \mu)$ and a bimeasurable transformation $f : X \to X$, the linear operator

$$T_f : \varphi \in L^p(X, \mathcal{B}, \mu) \to \varphi \circ f \in L^p(X, \mathcal{B}, \mu)$$

is called *composition operator induced by* f. In this talk, we will provide necessary and sufficient conditions on f for T_f to be Li-Yorke chaotic. The notion of Li-Yorke chaos was introduced in [1] in the context of interval maps and studied in [2,3] in the context of linear dynamics. Joint work with N. C. Bernardes Jr. and U. B. Darji.

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Global Denjoy-Carleman hypoellipticity for a Class of systems of complex vector fields

Bruno de Lessa Victor, Alexandre Arias Jr.
Universidade Federal do Paraná, Brazil

The problem of global hypoellipticity for a class of systems of complex vector fields in $\mathbb{T}^{n+1}$ given by

$$L_j = \frac{\partial}{\partial t_j} + \Big(a_j(t_j) + ib_j(t_j)\Big)\frac{\partial}{\partial x}, \quad j = 1, 2, \ldots, n, \tag{8}$$

has been quite studied in the literature. For instance, the situation where (8) is given by a single vector field has been covered by [1](smooth case), [3] (Gevrey case, for any $s \geq 1$) and [5] (perturbations for the C^∞ case).

Meanwhile, the more general case when $n > 1$ has been characterized by [6] (smooth case), [2] (analytic case) and [4] (Gevrey case, for $s > 1$). The purpose of this work is to study global hypoellipticity in Denjoy-Carleman classes for vector fields as it follows:

$$L_{\lambda_j} = \frac{\partial}{\partial t_j} + \Big(a_j(t_j) + ib_j(t_j)\Big)\frac{\partial}{\partial x} + \lambda_j, \quad j = 1, 2, \ldots, n,$$

where a_j, b_j are real valued functions in Denjoy-Carleman classes and $\lambda_j \in \mathbb{C}$.

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Globally non-solvable involutive systems

Cleber de Medeira
Universidade Federal do Paraná, Brazil

We consider a class of involutive systems associated with a smooth and closed 1-form defined on the n-dimensional torus. We show the non-global solvability of the system by assuming a certain geometric condition on the global primitive of the imaginary part of this 1-form. We use this result to characterize the global solvability of certain partially coupled systems.

A class of globally hypoelliptic Cauchy operators on the torus and generalized Siegel conditions

Fernando de Ávila Silva
Universidade Federal do Paraná, Brazil

We present an investigation of global hypoellipticity problem for Cauchy operators of type $L = D_t^m + \sum_{j=1}^m Q_j(t, D_x) D_t^{m-j}$, defined on $\mathbb{T}^{n+1}$, where $Q_j(\cdot, D_x)$ is a pesudo-differential operator on $\mathbb{T}^n$, smoothly depending on $t \in \mathbb{T}$, satisfying suitable hypothesis. We propose a study by combining Hörmander and Siegel conditions on the symbol of L.

On the continuity of singular integral operators in distribution spaces

Lucas Oliveira, Lucas Chaffee, Jarod Hart
Universidade Federal do Rio Grande do Sul, Brazil

We will discuss some results obtained recently that permit the analysis of the continuity of singular integral operators with different degrees of singularity (fractional, singular and hypersingular) in Besov, Triebel-Lizorkin and other spaces in one strike. This results are part of a long term collaboration with Jarod Hart and Lucas Chaffee.

L1 estimates for the strongly damped plate equation

Marcello D'abbicco, Giovanni Girardi, Jinju Liang
Università di Bari, Brazil

We will derive L1 estimates for the solution to the strongly damped plate equation, showing how the presence of low frequencies oscillations gives a well-posedness constant which increases at long time.

Asymptotic profiles for a damped plate equation with time-dependent coefficients

Marcelo Rempel Ebert
Universidade de São Paulo, Brazil

In this talk we discuss the asymptotic profile of the solutions to the Cauchy problem for the damped plate equation

$$\begin{cases} u_{tt} + \Delta^2 u - \lambda(t)\Delta u + (-\Delta)^\alpha u_t = 0, & t \geq 0,\ x \in \mathbb{R}^n \\ u(0,x) = 0, & x \in \mathbb{R}^n, \\ u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \tag{9}$$

with $\alpha \in [0, 1/2]$ and

$$\lambda(t) = \frac{\mu}{(1+t)^\ell}, \qquad \mu > 0, \quad \ell \in (0,1). \tag{10}$$

The Cauchy problem for Schrödinger-type equations in Gelfand-Shilov spaces

Marco Cappiello, Alessia Ascanelli
University of Turin, Italy

We consider the initial value problem

$$\begin{cases} P(t,x,\partial_t,\partial_x)u(t,x) = f(t,x) \\ u(0,x) = u_0(x) \end{cases}, \qquad (t,x) \in [0,T] \times \mathbb{R}^n \tag{11}$$

where

$$P(t,x,\partial_t,\partial_x) = \partial_t - i\triangle_x + \sum_{j=1}^{n} a_j(t,x)\partial_{x_j} + b(t,x). \tag{12}$$

It is well-known that when the coefficients a_j, b and the Cauchy data f, u_0 are all real valued, smooth and uniformly bounded with respect to x the Cauchy problem (11) is L^2-well-posed, while if a_j are complex valued suitable decay conditions for $|x| \to \infty$ are needed on the imaginary part of the coefficients in order to obtain either H^∞ or Gevrey well posedness with a certain loss of derivatives. It is also known that a decay at infinity of the initial data has a smoothing effect on the regularity of the solutions of (11). Here we treat the case when the initial data belong to the Gelfand-Shilov space $\mathcal{S}^\theta_s(\mathbb{R}^n)$, (resp. $\Sigma^s_\theta(\mathbb{R}^n)$) defined as the space of the smooth functions f satisfying

$$\sup_{x\in\mathbb{R}^n} \sup_{\alpha\in\boldsymbol{N}^n} C^{-|\alpha|}\alpha!^{-\theta} e^{c|x|^{\frac{1}{s}}} |\partial^\alpha f(x)| < \infty,$$

for some (resp. for all) $C, c > 0$, with $s > 1, \theta > 1$, and prove a result of existence and uniqueness of the solution of (11) with precise information both on the regularity and on the behavior of the solution for $|x| \to \infty$.

Schrödinger operators with point interactions

Nataliia Goloshchapova
Universidade de São Paulo, Brazil

Let $L_{X,\alpha}$ be the operator associated in $L^2(\mathbb{R})$ with the differential expression

$$\mathfrak{L}_1 = -\frac{\mathrm{d}^2}{\mathrm{dx}^2} + \sum_{j=1}^{m_1} \alpha_j \delta(\cdot - x_j), \quad 1 \le m_1 \le \infty,$$

for any fixed sets $X = \{\alpha_j\}_{j=1}^{m_1} \subset \mathbb{R}, \alpha = \{\alpha_j\}_{j=1}^{m_1} \subset \mathbb{R}$. We will discuss some spectral properties of the operator $L_{X,\alpha}$. In particular, we intend to give a brief overview of the boundary triplets approach and its applications in our case.

Moreover, if we have time, we will mention some results about spectral properties of the operator associated in $L^2(\mathbb{R}^d)$, $d \in \{2,3\}$, with the differential expression

$$\mathfrak{L}_d = -\Delta + \sum_{j=1}^{m_d} \alpha_j \delta(\cdot - x_j), \quad \alpha_j \in \mathbb{R}, \quad x_j \in \mathbb{R}^d,$$

in the case of finite m_d.

Solvability and boundary value problems for a class of singular vector fields

Paulo L. Dattori da Silva
Universidade de São Paulo, Brazil

In this talk we will deal with the solvability of planar complex vector fields with homogeneous degeneracies. Hölder continuous solutions are obtained via a Cauchy type integral operator associated to the vector field. An associated boundary value problem of Riemann-Hilbert type is also considered.

This is a joint work with Camilo Campana (UFSCar) and Hamid Meziani (FIU).

Work supported in part by São Paulo Research Foundation (FAPESP) (grants 2012/03168-7 and 2015/20815-4) and CNPq (grant 306037/2015-7).

On the Cahn-Hilliard equations on manifolds with conical singularities

Pedro Tavares Paes Lopes
Universidade de São Paulo, Brazil

We recall some properties of the manifolds with conical singularities and the realizations of the Laplacian on them. Using results of maximal Lp regularity, we will show how to obtain regularity and how to prove the existence of global solutions of the Cahn-Hilliard Equations on these manifolds.

Inverse scattering with partial data on asymptotically hyperbolic manifolds

Raphael F. Hora
Universidade Federal de Santa Catarina, Brazil

I will present a continuation of my work with A. Sá Barreto on inverse scattering on asymptotically hyperbolic manifolds started. We consider the scattering operator acting only on functions supported on the source set $\mathbb{R} \times \overline{\mathcal{O}}$, where $\mathcal{O}$ is an open subset of the boundary and the resulting functions are then restricted to the observation set $\mathbb{R} \times \overline{\Gamma}$, where Γ is open. I consider the problem for arbitrary disjoint $\mathcal{O}$ and Γ. I will show that this question has very serious additional difficulties, and that if Γ is the complement of the closure of $\mathcal{O}$ and $\overline{\Gamma} \cap \overline{\mathcal{O}} \neq \emptyset$, the restriction of the scattering operator to $\mathcal{O}$ and Γ termine the manifold modulo isometries. I will also show that, differently from the Euclidean case, L^2 boundary controllability for the wave equation from radiation fields restricted to any open subset of the boundary at infinity does not hold.

Global hypoellipticity and global solvability for a first order differential operator of Vekua-type on a product of compact Lie groups

Wagner Augusto Almeida de Moraes, Alexandre Kirilov
Universidade Federal do Paraná, Brazil

Let $G = G_1 \times G_2$, where G_1 and G_2 are compact Lie groups. In this work we are interested of the study of global hypoellipticity and global solvability of the first-order differential operator of Vekua-type $P : \mathcal{D}'(G) \to \mathcal{D}'(G)$ given by

$$Pu = X_1 u + \alpha X_2 u + \beta u + \gamma \bar{u},$$

where $\alpha \in \mathbb{R}$, $\beta, \gamma \in \mathbb{C}$, $X_1 \in \mathfrak{g}_1$ and $X_2 \in \mathfrak{g}_2$.

Inspired by the work of Bergamasco, Datori and Meziani [1], in which the two-torus case is treated, we studied the constant coefficient case. At this point, we need to impose some conditions on the Weyl group of G_1 and G_2 to construct singular solutions.

References:

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Nonlinear Dynamical Systems

Organizer: Juliana Fernandes

About the exponential dichotomy in Fréchet spaces

Eder R. Aragão Costa
Universidade de São Paulo, Brazil

In this lecture we provide versions, in Fréchet spaces, of the classical theorems related to exponential dichotomy for a sequence of continuous linear operators on Banach spaces. To be more specific, We define a kind of exponential dichotomy in Fréchet spaces, which extends the former one in Banach spaces, establish necessary conditions for its existence and provide sufficient conditions for its stability under perturbation.

Banach fixed point theorem with parameters. Applications to linearization in infinite dimensions

Hildebrando Munhoz Rodrigues
Universidade de São Paulo, Brazil

- Some historic results. Some elementary motivations.
- On the relationship between norms of bounded linear operators and their spectral radius.
- A Hartman-Grobman theorem with parameters. Continuity of the conjugation with respect to parameters.
- The continuity of the fixed point with respect to parameters.
- The differentiability of the fixed point with respect to parameters.
- A Hartman-Grobman theorem with parameters. Differentiability of the conjugation with respect to parameters.

Effects of bounded random disturbances on chemostat models with several species and wall growth

Javier López-de-la-Cruz
University of Sevilla, Spain

Chemostat refers to a laboratory device used for growing microorganisms in a cultured environment and has been regarded as an idealization of nature to study microbial ecosystems at steady state, which is a really important and interesting problem due to the many applications that can be found in the real life. The simplest chemostat device consists of three interconnected tanks called *feed bottle*, *culture vessel* and *collection vessel*. The nutrient is pumped from the first tank to the culture vessel, where the interactions between the species and the nutrient take place, and there is also another flow being pumped from the culture vessel to the third tank such that the volume of the culture vessel remains constant. Nevertheless, very strong restrictions are supposed in the deterministic model and

the use of the white noise in the stochastic case produces drawbacks from the point of view of applications since it is unbounded. Because of this reason, we are really interested in considering random disturbances which are bounded, which have been proved to be very close to the ones in the laboratory.

In this talk, random disturbances in the chemostat model will be analyzed by making use of bounded stochastic processes. Particularly, the existence and uniqueness of global solution will be stated and the existence and uniqueness of an attracting set will be also proved. Thanks to a deeper analysis involving the internal structure of the attracting set, we will be able to provide conditions to ensure the persistence of the microbial biomass, which is also the main goal pursued by biologists. Finally, several simulations will be shown to support the previous results.

Hyperbolicity of Ginzburg-Landau vortex solutions

Jia-Yuan Dai, Phillipo Lappicy
National Taiwan University, Taiwan

We prove that each equilibrium of the Ginzburg-Landau equation restricted on the invariant subspace of vortex solutions is hyperbolic, that is, its associated linearization possesses nonzero eigenvalues. This result completely describes the global attractor of vortex solutions, and also yields the Ginzburg-Landau spiral waves of nodal type.

Semilinear parabolic equations with asymptotically linear growth

Juliana Fernandes, Liliane Maia
Universidade Federal do Rio de Janeiro, Brazil

We present some recent work on the existence and behaviour of solutions for a class of semilinear parabolic equations, defined on a bounded smooth n-dimensional domain, and we assume that the nonlinearity is asymptotically linear at infinity. We analyze the behavior of the solutions when the initial data varies in the phase space. We obtain global solutions which may be bounded or blow-up in infinite time (grow-up). Our main tools are the comparison principle and variational methods. Particular attention is paid to initial data at high energy level. We use the Nehari manifold to separate the phase space into regions of initial data where uniform boundedness or grow-up behavior of the semiflow may occur.

Long-time behavior for a class of semilinear viscoelastic Kirchhoff beams/plates

Marcio A. Jorge Silva
Universidade Estadual de Londrina, Brazil

In this talk we first present a breaf justification on the modeling for a nonlinear viscoelastic equation related to both beams (1D) and plates (2D). Then, some results on well-posedness, stability to equilibrium (trivial attractor) and long-time behavior of solutions (existence of non-trivial attractors) are presented.

On attractors for autonomous and non-autonomous impulsive dynamical systems

Matheus Cheque Bortolan
Universidade Federal de Santa Catarina, Brazil

In this talk we briefly present concepts of attractors for both autonomous and nonautonomous impulsive dynamical systems, as well as conditions to obtain their existence. We present some properties of these objects and a few examples to illustrate the theory.

This talk contains a result collection from several papers, which are joint work with Everaldo M. Bonotto (ICMC-USP), Tomás Caraballo (Universidad de Sevilla), Alexandre N. Nolasco (ICMC-USP), Rodolfo Collegari (UFU) and Radosław Czaja (Uniwersytet Śląski - University of Silesia, Poland). Such papers will be cited during the talk.

The gradient-like structure for the wave equations with localized damping on compact manifolds

Paulo N. Seminario Huertas
Universidade de São Paulo, Brazil

This paper is devoted to showing the different shapes in the localization in the damping for the wave equation defined in smooth connected compact Riemann manifolds with boundary. The gradient structure for the semigroup associated to this problem is proved. To this end, we need to use a version of the unique extension theorem proposed by Triggiani and Yao [1].

We would like to thanks CNPQ by financial support.

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Stability results for 2D Navier-Stokes equations with unbounded delay

Pedro Marín-Rubio, Tomas Caraballo, Linfang Liu
Universidad de Sevilla, Spain

Some results related to 2D Navier-Stokes equations when the external force contains hereditary characteristics involving unbounded delays are analyzed. First, the existence and uniqueness of solutions is proved by Galerkin approximations and the energy method. The existence of stationary solution is then established by means of the Lax-Milgram theorem and the Schauder fixed point theorem. The local stability analysis of stationary solutions is studied by several different methods: the classical Lyapunov function method, the Razumikhin-Lyapunov technique and by constructing appropriate Lyapunov functionals. Finally, we also verify the polynomial stability of the stationary solution in a particular case of unbounded variable delay. Exponential stability in this infinite delay setting remains as an open problem.

This work has been done in collaboration with Prof. Tomás Caraballo (Universidad de Sevilla, Spain) and Prof. Linfang Liu (Xi'an Jiaotong University, PR China).

References:

[1] L. Liu et al., Stability results for 2D Navier-Stokes equations with unbounded delay, *J. Differential Equations* (2018), https://doi.org/10.1016/j.jde.2018.07.008

Unbounded Sturm attractors for quasilinear equations

Phillipo Lappicy, Juliana Pimentel
Universidade de São Paulo, Brazil

We plan to construct explicitely the global attractors of quasilinear parabolic equations in one dimensional domain when solutions can grow-up, and hence there is a global attractor which is unbounded. In particular, we construct heteroclinic connections between bounded and/or unbounded hyperbolic equilibria.

Non-dissipative system as limit of a dissipative one

Ricardo Parreira da Silva
Universidade de Brasília, Brazil

Let $\Omega \subset \mathbb{R}^n$ be a bounded smooth domain in $\mathbb{R}^n$. Given $u_0 \in L^2(\Omega)$, $g \in L^\infty(\Omega)$ and $\lambda \in \mathbb{R}$, consider the family of problems parametrised by $p \searrow 2$,

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta_p u &= \lambda u + g, & \text{on} \quad (0,\infty)\times\Omega, \\ u &= 0, & \text{in} \quad (0,\infty)\times\partial\Omega, \\ u(0,\cdot) &= u_0, & \text{on} \quad \Omega, \end{cases}$$

where $\Delta_p u := \operatorname{div}\left(|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u}\right)$ denotes the p-Laplacian operator.

Our aim in this work is to describe the asymptotic behavior of this family of problems comparing compact attractors in the dissipative case $p > 2$, with non-compact attractors in the non-dissipative limiting case $p = 2$ with respect to the Hausdorff semi-distance between then.

Trajectory and global attractors for generalized processes

Rodrigo Antonio Samprogna, Cláudia Buttarello Gentile Moussa, Tomás Caraballo, Karina Schiabel
Universidade Federal de Alfenas, Brazil

In this work the theory of generalized processes is used to describe the dynamics of a nonautonomous multivalued problem and, through this approach, some conditions for the existence of trajectory attractors are proved. By projecting the trajectory attractor on the phase space, the uniform attractor for the multivalued process associated to the problem is obtained and some conditions to guarantee the invariance of the uniform attractor are given. Furthermore, the existence of the uniform attractor for a class of p-Laplacian non-autonomous problems with dynamical boundary conditions is established.

Singularly perturbed non-local diffusion systems applied to disease models

Sergio Oliva, Marcone Pereira
Universidade de São Paulo, Brazil

We study a model, suitable for modeling vector-borne diseases, where we assume that the human hostsâï¿ï¿ï¿ï¿¡ï¿ï¿ï¿ï¿¡ epidemiology acts on a much slower time scales than the one of the mosquitoes transmitting as a vector from human to human, due to their vastly different life cycles. This particular model also includes the spatial movement of both vectors and humans getting a couple system of non-local and local spatial dynamics. (Joined work with M. Pereira)

The model proposed takes the form, where i and j will model the density of infected human and vector population.

$$\begin{cases} \dfrac{\partial i}{\partial t} = \alpha_h(1-i)j - \beta_h i + d_1 K_J\, i, \\ \dfrac{\partial j}{\partial t} = \dfrac{\alpha_v}{\varepsilon}(1-j)i - \dfrac{\beta_v}{\varepsilon} j + d_2 \Delta j, \end{cases} \qquad x \in \Omega,\ t > 0. \tag{13}$$

We work in a regular bounded domain $\Omega \subset \mathbb{R}^N$ with exterior unit normal N. Also, we take the homogeneous Neumann boundary condition to the function j

$$\frac{\partial j}{\partial N} = 0, \quad x \in \partial\Omega. \tag{14}$$

The constants α_h, α_v, β_h, β_v, d_1 and d_2 are positive, Δ denotes the Laplacian differential operator and K_J is the following nonlocal operator

$$K_J\, i(x) = \int_\Omega J(x-y)(i(y) - i(x))dy, \quad x \in \Omega.$$

We assume that the kernel J satisfies the hypotheses

$(\mathbf{H_J})$ $J \in \mathcal{C}(\mathbb{R}^N, \mathbb{R})$ is non-negative with $J(0) > 0$, $J(-x) = J(x)$ for every $x \in \mathbb{R}^N$, and $\displaystyle\int_{\mathbb{R}^N} J(x)\,dx = 1.$

Under these conditions, the K_J is known as a nonlocal operator with non-singular kernel and Neumann condition.

Dynamics of coupled systems: reductions and emergence across scales

Tiago Pereira, Sebastian van Strien, Matteo Tanzi
Universidade de São Paulo, Brazil

We will talk about the dynamics of Heterogeneously Coupled Maps (HCM). Such systems are determined by a network with heterogeneous degrees. Some nodes, called hubs, are very well connected while most nodes interact with few others. The local dynamics on each node is chaotic, coupled with other nodes according to the network structure. Such high-dimensional systems are hard to understand in full, nevertheless we are able to describe the system over exponentially large time scales. In particular, we show that the dynamics of hub nodes can be very well approximated by a low-dimensional system. This allows us to establish the emergence of macroscopic behaviour such as coherence of dynamics among hubs of the same connectivity layer (i.e. with the same number of connections), and chaotic behaviour of the poorly connected nodes. This is a joint work with Matteo Tanzi and Sebastian van Strien.

Ordinary-Functional Differential Equations

Organizer: Jaqueline G. Mesquita

Almost automorphic solutions of second order equations on time scales

Aldo Pereira, Jaqueline G. Mesquita, Mario Choquehuanca
Universidade de Brasília, Brazil

In this work, we present a general formulation of the solution of linear and semilinear second order dynamic equation on time scales in their integral form. Also, we prove a result which ensures the existence of almost automorphic solution of the linear second order dynamic equation on time scales, and a result which ensures the existence and uniqueness of almost automorphic solution of the nonlinear second order dynamic equation on time scales.

A spectral dichotomy version of the nonautonomous Markus-Yamabe conjecture

Álvaro Castañeda, Gonzalo Robledo
Universidad de Chile, Chile

In this talk we introduce a nonautonomous version of the Markus- Yamabe conjecture from an exponential dichotomy spectrum point of view. We prove the validity of this conjecture for the scalar and triangular case. Additionally we show that the origin is a global attractor for an autonomous system by using nonautonomous dynamical systems tools.

Lyapunov stability for measure differential equations and dynamic equations on time scales

Eduard Toon, Márcia Federson, Rogélio Grau, Jaqueline G. Mesquita
Universidade Federal de Juiz de Fora, Brazil

In this lecture we present stability results for measure differential equations, considering more general conditions under the Lyapunov functionals and concerning the functions f and g. Moreover, we prove these stability results for the dynamic equations on time scales, using the correspondence between the solutions of these last equations and the solutions of the measure differential equations. In order to prove our main results, we use the fact that measure differential equations can be regarded as generalized ordinary differential equations.

Regular stability for generalized ODE

Fernanda Andrade da Silva, Márcia Federson, Eduard Toon
Universidade de São Paulo, Brazil

In this work, we prove regular stability results for a nonlinear generalized ODE considering more general conditions under the Lyapunov functionals. We also establish a inverse Lyapunov-like theorem for a nonlinear generalized ODE.

First integral for central forces

Gerard John Alva Morales
Universidade Federal do Maranhão, Brazil

In this work, we determine the normal form of the following system of second order ordinary differential equations

$$\ddot{x} = -f(x,y)x, \quad \ddot{y} = -f(x,y)y, \quad (x,y) \in \Omega \subset (\mathbb{R}^2, 0) \tag{15}$$

assuming that this system admits a first integral of the form

$$a(x,y)\dot{x}^2 + 2b(x,y)\dot{x}\dot{y} + c(x,y)\dot{y}^2 + d(x,y)\dot{x} + e(x,y)\dot{y} + \Pi(x,y)$$

at where $a, b, c, d, e, \Pi \in C^2(\Omega)$.

For this, it will be relevant to know the general solution of a partial differential equation of the type

$$N(x,y)u_x - M(x,y)u_y = (M_y - N_x)u, \quad M_y - N_x \neq 0 \tag{16}$$

which f satisfies.

The general solution of equation (16) will be given, and will also be observed, the implications in the study of stability of the equilibrium for the system (19).

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Smoothness of topological equivalence for non-autonomous systems

Gonzalo Robledo, Álvaro Castañeda, Pablo Monzon
Universidad de Chile, Chile

The topological conjugacy between the non-autonomous ODE system

$$x' = A(t)x \tag{17}$$

and a family of perturbations

$$y' = A(t)y + f(t,y) \tag{18}$$

has widely studied by several author when: i) the system (17) satisfies some properties of dicohotomy and ii) the nonlinearity of (18) verifies some mild assumptions. Nevertheless, the smoothness properties of the topological conjugacy have not been studied in depth and we present some preliminary results for the particular case when (17) is asymptotically stable (not necessarily uniform). We show also a converse result of stability via the preservation of Rantzer's density functions functions

An approach for integrating a class of multiplicative-noise stochastic differential equations under non-standard assumptions

Hugo de la Cruz
Fundação Getúlio Vargas, Brazil

Many important Stochastic Differential Equations (SDE) used for modelling noisy dynamical systems are driven by linear multiplicative-noise diffusion coefficients. For investigating the behavior of this kind of equations discrete-time approximations must be considered. Unfortunately, many of the available integrators in the literature assume global Lipschitz and/or uniform bounds of coefficient functions of the SDE. This is very restrictive since these assumptions are not satisfied by many SDEs in significant applications. The aim of this work is to propose an approach, based on the conjugacy between the underlying SDE and a suitable Random Differential Equation (driven by multidimensional Ornstein-Uhlhenbeck processes), which allows constructing new integrators that remarkably avoid the afore mentioned restrictive assumptions. The properties of the resulting methods are analyzed and the performance of the proposed approach is tested in a number of examples.

Discontinuous differential equations

Iguer Luis Domini dos Santos
Universdade Estadual Paulista, Brazil

A study on discontinuous differential equations will be presented. More specifically, a study will be presented on generalized solutions to discontinuous differential equations. Discontinuous differential equations are ordinary differential equations with a discontinuous right hand side. Both the autonomous ordinary differential equations as well as nonautonomous ordinary differential equations will be studied. This way, are studied and compared the Euler, Filippov, Hermes, Krasovskii, and Sentis generalized solutions to discontinuous differential equations. In particular, are studied relations between Euler and Hermes solutions. In addition, it is proved that Hermes solutions satisfy some properties that are analogous to the properties satisfied by Euler solutions.

Nonlocal problem with impulsive action: remark about solution growth estimate

Jaqueline da Costa Ferreira, Marcone Merreira
Universidade Federal do Espírito Santo, Brazil

This talk is concerned with formulate the nonlocal problem with impulsive action and to present the remark about growth estimates of their solutions. The theory of impulsive partial differential equations provide natural framework for mathematical model of processes which are subject to brief perturbations during their evolution. Many general work about impulsive parabolic equations are found in [1], [5] and [6]. Parallel, the nonlocal evolution equations of the form

$$u_t(x,t) = \int_{\mathbb{R}^n} J(x-y)u(y,t)dy - u(x,t)$$

and variations of it have been used to model diffusion processes [2], [3], [4], [7] and [8]. However, in the literature, is not established this nonlocal problem with impulsive action. We explored the properties of the operator $K_J(u)(x,t) := \int_\Omega J(x-y)u(y,t)dy$ in order to discuss about the growth estimates of impulsive solutions.

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Measure functional differential equations with infinite time-dependent delay

Jaqueline G. Mesquita, Claudio Gallegos, Hernan Henriquez
Universidade de Brasília, Brazil

In this work, we introduce measure functional differential equations (MFDEs) with infinite time-dependent delay, and we study the correspondence between the solutions of these equations and the solutions of the generalized ordinary differential equations (GODEs) in Banach spaces. Using the theory of GODEs, we obtain results concerning the existence and uniqueness of solutions and continuous dependence on parameters of measure functional differential equations with infinite time-dependent delay. We also establish a result of existence of solutions for a MFDE with infinite time-dependent delay in the presence of a perturbation independent of the state. We develop the theory in the context of phase spaces defined axiomatically. Our results in this paper generalize several previous works on MFDEs with infinite time-independent delay.

Existence of mild solutions to fractional evolution equations in Banach space

José Vanterler da Costa Sousa, Edmundo Capelas de Oliveira
Universidade Estadual de Campinas, Brazil

The theory of differential equations with non-instantaneous impulses and impulsive evolution equations in Banach spaces has been investigated by many researchers in the last decades. It is note that investigating the existence, uniqueness, stability of solutions of differential equations of evolution, has been object of study and applicability in the scientific community, since it describes processes that

experience a sudden change in their states at certain moments. The applicability of the obtained results related to the differential equations, especially with non-instantaneous impulses, can be found in several areas, such as: physics, engineering, economics, biology, medicine and mathematics itself, among other. Then, in this sense, we investigate the existence of mild solutions to Hilfer fractional differential equation of semi-linear evolution with non-instantaneous impulses in Banach space Ω, given by

$$\begin{cases} {}^{H}\mathbb{D}_{0+}^{\alpha,\beta}u(t)+\mathcal{A}(t) &= f\left(t,u(t)\right), \quad t\in \bigcup\limits_{k=0}^{m}(s_k,t_{k+1}] \\ u(t) &= \zeta_k\left(t,u(t)\right), \quad t\in \bigcup\limits_{k=1}^{m}(t_k,s_k] \\ I_{0+}^{1-\gamma}u(0) &= u_0 \end{cases} \tag{19}$$

where ${}^{H}\mathbb{D}_{0+}^{\alpha,\beta}(\cdot)$ is Hilfer fractional derivative, $I_{0+}^{1-\gamma}(\cdot)$ is Riemann-Liouville fractional integral with $0<\alpha\leq 1$, $0\leq\beta\leq 1$ and $0\leq\gamma\leq 1\left(\gamma=\alpha+\beta(1-\alpha)\right)$, $\mathcal{A}:\mathfrak{D}(\mathcal{A})\subset\Omega\to\Omega$ is a linear operator and is the infinitesimal generator of a strongly continuous semigroup $(C_0-\text{semigroup})$ $\left(\mathbb{P}(t)\right)_{t\geq 0}$ in Ω with $0<t_1<t_2<\cdots<t_m<t_{m+1}:=a$, $a>0$ is a constant, $s_0:=0$ and $s_k\in(t_k,t_{k+1})$ for each $k=1,2,...,m$. We also have $f:[0,a]\times\Omega\to\Omega$ a given nonlinear function satisfying some assumptions $\zeta_k:(t_k,s_k]\times\Omega\to\Omega$ is non-instantaneous impulsive function for all $k=1,2,...m$, and $u_0\in\Omega$.

Theory of well-posedness for delay differential equations via prolongations and C^1–prolongations

Junya Nishiguchi
Tohoku University, Japan

In this talk, we establish a theory of well-posedness for delay differential equations (DDEs) via notions of prolongations and C^1-prolongations, which are continuous and continuously differentiable extensions of histories to the right, respectively. In this sense, this talk treats a continuation and an extension of the previous paper by this speaker (JDE, 2107). The results are applicable to various DDEs, however, these results cannot be applied to general class of state-dependent DDEs, and its extendability is missing. We find this missing link by introducing notions of (C^1-) prolongabilities, regulation of topology by (C^1-) prolongations, and Lipschitz conditions about (C^1-) prolongations, etc. One of the main result claims that the continuity of the semiflow with a parameter generated by the trivial DDEs $\dot{x}=v$ plays an important role for the well-posedness. The results are applied to general class of state-dependent DDEs.

Oscillation theory on generalized ODEs

Marielle A. Silva, Marcia Federson, Everaldo Bonotto, Marta C. Gadotti
Universidade de São Paulo, Brazil

In this work we present the theory of oscillation for solutions of the following generalized ordinary differential equation

$$\frac{dx}{d\tau}=DF(x,t), \tag{20}$$

where the involved functions take values in a Banach space. In order to obtain an oscillation criterion, we define a generalized dynamical system associated to equation (20).

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Eigenvalue problems for Fredholm operators with set-valued perturbations

Pierluigi Benevieri, Antonio Iannizzotto
Universidade de São Paulo, Brazil

By means of a suitable degree theory, we prove persistence of eigenvalues and eigenvectors for set-valued perturbations of a Fredholm linear operator. As a consequence, we prove existence of a bifurcation point for a non-linear inclusion problem in abstract Banach spaces. Finally, we provide applications to differential inclusions.

Integral equations in the sense of Kurzweil integral and applications

Rafael S. Marques, John A. Burns, Márcia C. A. B. Federson, Terry L. Herdman
Universidade de São Paulo, Brazil

The purpose of this work is to develop the theory of integral equations, when the integrals involved are in the sense of Kurzweil-Henstock or Kurzweil-Henstock-Stieltjes, through the correspondence between solutions of integral equations and solutions of generalized ordinary differential equations (we write generalized ODEs, for short).

In order to obtain results for integral equations, we propose extensions of both the Kurzweil integral and the generalized ODEs . We develop the fundamental properties of this new generalized ODE, such as existence and uniqueness of solutions results. We, then, apply these results to a class of nonlinear Volterra integral equations of the second kind.

Finally, we consider a model of population growth that can be expressed as an integral equation that belongs to this class of nonlinear Volterra integral equations.

Variation-of-constants formula for generalized ODEs

Rodolfo Collegari, Márcia Federson, Miguel Frasson
Universidade Federal de Uberlãndia, Brazil

We present a variation-of-constants formula for functional differential equations of the form

$$\dot{y} = \mathcal{L}(t)y_t + f(y_t, t), \qquad y_{t_0} = \varphi. \tag{21}$$

where $\mathcal{L}$ is a bounded linear operator and φ is a regulated function where the application $t \mapsto f(y_t, t)$ is Kurzweil integrable, with t in an interval of $\mathbb{R}$, for each regulated function y. This means that $t \mapsto f(y_t, t)$ may admit not only many discontinuities, but it can also be highly oscillating. Our main goal is achieved via theory of generalized ordinary differential equations introduced by J. Kurzweil. As a matter of fact, we establish a variation-of-constants formula for general linear generalized ordinary differential equations in Banach spaces where the functions involved are Kurzweil integrable.

Lattice models in the biological sciences

Xiaoying Han, Peter Kloeden
Auburn University, USA

This talk focusses on dynamical behavior of lattice models arising in the biological sciences, in particular, the existence of attractors in such systems. Three types of lattice dynamical systems will be introduced; they are lattice reaction-diffusion systems, Hopfield neural lattice systems, and neural field lattice systems. For each model, the existence of a global, nonautonomous, or random attractor will be shown. The upper semi continuity of attractors for the Hopfield neural lattice model and the upper semi continuity of numerical attractors also discussed.

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CHAPTER 2019

Poster Section

Poster Session

On a nonlocal nonhomogeneous Neumann boundary problem with two critical exponents

Andréia Gomes Pinheiro, Augusto César dos Reis Costa
Universidade Federal do Pará, Brazil

In this work, we study the multiplicity of solutions, via a truncation argument, for a class of non-homogeneous problems, under Neumann boundary condition involving critical growth simultaneously in domain, and on the boundary of the domain, given by

$$\begin{cases} -\Delta_{p(x)}u + |u|^{p(x)-2}u + |u|^{h(x)-2}u = \lambda|u|^{v(x)-2}u + |u|^{r(x)-2}u\left[\displaystyle\int_\Omega \frac{1}{r(x)}|u|^{r(x)}dx\right]^{\alpha} \text{ in } \Omega, \\ |\nabla u|^{p(x)-2}\frac{\partial u}{\partial \nu} = |u|^{q(x)-2}u\left[\displaystyle\int_{\partial\Omega} \frac{1}{q(x)}|u|^{q(x)}dS\right]^{\beta} \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \in \mathbb{R}^N$ is a bounded smooth domain of $\mathbb{R}^N$, $N \geq 2$, and $p, h, v, r \in C(\overline{\Omega})$, $q \in C(\partial\Omega)$, $\frac{\partial u}{\partial \nu}$ is the outer unit normal derivative, λ, α, β are positive parameters, and $\Delta_{p(x)}$ is the $p(x)$-Laplace operator. We consider the following critical Sobolev exponents

$$p^*(x) = \frac{Np(x)}{N - p(x)} \text{ and } p_*(x) = \frac{(N-1)p(x)}{N - p(x)},$$

where p_* is the critical exponent from the point of view of the trace.

The main tools used are the Lions' Concentration-Compactness Principle for variable exponent spaces and Krasnoselskii's genus.

Qualitative properties of semigroups generated by variational formulation

Armando Santisteban Cárdenas
Universidade Federal do Rio de Janeiro, Brazil

We consider a class of partial differential equations defined from their variational formulation and from this formulation we look for hypothesis on the corresponding bilinear forms to be able to guarantee the existence of a C_0-semigroup that defines the solution of the problem, specifically we study a class of variational problems of second order in time

$$c(u_{tt}, v) + b(u_t, v) + a(u, v) = 0.$$

Imposing some conditions on the bilinear forms $c(.,.)$,$b(.,.)$, $a(.,.)$, is proved that the corresponding solution is defined by a semigroup that can be analytical, differentiable or asymptotically stable. This problem has several applications to evolutions equations with discontinuous coefficients.

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On mixing definitions

Brayan M. Rodríguez
Universidade Federal do Rio de Janeiro, Brazil

In this work we give a comprehensible interpretation of two definitions established in the literature to study the degree of mixing of a passive scalar when it is under the advection of a incompressible vector field u, we present some recent result about comparison and equivalence . We won't consider sources, sinks or diffusivity, the mathematical model for these assumptions is the continuity equation. The study of the evolution of the flow is performed on a bounded domain and the boundary conditions are periodic or no-slip ones.

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Diffusive radiative transfer in half space

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In this work we study the radiative transfer equation in the forward-peaked regime on the Half-Space. More precisely, it is shown that the equation is well-posed by proving instantaneous regularization of weak solutions for arbitrary initial datum in L^2. Classical techniques for hypo-elliptic operators, such as averaging lemma, are used in the argument. The (RTE) in the half-space reduces to

$$\begin{cases} \partial_t u + \theta \cdot \nabla_x u = \mathcal{I}(u), & \text{in} \quad (0,T)\times \mathbb{R}^d_+ \times \mathbb{S}^{d-1}, \\ u = u_0, \text{ on } \{t=0\}\times \mathbb{R}^d_+ \times \mathbb{S}^{d-1}, \\ u = g, \text{ on } (0,T)\times \partial\mathbb{R}^d_+ \times \mathbb{S}^{d-1} \text{ and } -v\cdot n(x) > 0. \end{cases} \tag{22}$$

where T is any arbitrary time, $0 \le u_o \in L_1(\mathbb{R}^d_+ \times \mathbb{S}^{d-1})$ and $0 \le g \in L^2(\Sigma^T_-)$.

Theorem [R.Alonso and E.Cuba] For any dimension $d \ge 3$ fixed and assume that $u \in \mathcal{C}([t_0,t_1]; L^2(\mathbb{R}^d_+ \times \mathbb{S}^{d-1})$ solves the RTE on the half-space (22) for $t \in (t_0,t_1)$ and $g \in L^2([t_0,t_1)\times \Gamma_-)$. Then for any $s \in (0,1)$, there exists a constant $C := C(d,s)$ independent of time such that

$$\begin{aligned} \|(-\triangle_x)^{s_0/2} u\|_{L^2((t_0,t_1)\times\mathbb{R}^d_+\times\mathbb{S}^{d-1})} \le C\Big(&\|u(t_0)\|_{L^2(\mathbb{R}^d_+\times\mathbb{S}^{d-1})} + \|u\|_{L^2([t_0,t_1)\times\mathbb{R}^d_+\times\mathbb{S}^{d-1})} \\ &+ \left\|(-\Delta_v)^{s/2} w_{\mathcal{J}}\right\|_{L^2([t_0,t_1)\times\mathbb{R}^d_+\times\mathcal{S}^{d-1})} + \|g\|_{L^2([t_0,t_1)\times\Gamma_-)}\Big), \qquad s_0 = \frac{s/4}{2s+1}. \end{aligned} \tag{23}$$

Together with other related results, this work in progress is part of my doctoral thesis.

Existence and concentration for a singularly perturbed problem with nonlinear Neumann boundary condition

Eduardo Hitomi
Universidade Estadual de Campinas, Brazil

In this poster we present some results concerning a min-max existence theory of capillary hypersurfaces in compact manifolds with boundary, based on the theory of phase transitions. We also provide the existence of a family of solutions of the Allen-Cahn equation with nonlinear Neumann boundary condition under some constraints, whose nodal sets concentrate asymptotically to a given volume nondegenerate capillary hypersurface.

Moving planes method

Fiorella María Rendón García
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As a first beautiful illustration of the power of the Maximum Principle, we will prove the following symmetry theorem, which plays an important role in the study of nonlinear elliptic and parabolic PDE.

Introduction. The method of moving planes is used in proving symmetry, in fact, x_1 direction for solutions of nonlinear elliptic equation $F(x, u, Du, D^2u) = 0$ in a bounded domain Ω in $\mathbf{R}^n$ which is convex in the x_1 direction.

The essential ingredient in their use is the Maximum principle. In this method one point is the reflection of the other in a hyperplane $\{x_1 = \lambda\}$, and then, the plane is moved up to a critical position.

We choose a very simple example to illustrate such a method. The following result was first proved by Gidas, Ni and Nirenberg.

Theorem.1. Suppose $u \in C(B_1) \cap C^2(B_1)$ is a positive solution of

$$\begin{aligned} \triangle u + f(u) = 0 \quad &\text{in} \quad B_1 \\ u = 0 \quad &\text{on} \quad \partial B_1 \end{aligned}$$

where f is locally Lipschitz in $\mathbf{R}$. Then u is radially symmetric in B_1 and $\frac{\partial u}{\partial r}(x) < 0$ for $x \neq 0$.

The original proof requires that solutions be C^2 up to the boundary. Here we give a method which does not depend on the smoothness of domains nor the smoothness of solutions up to the boundary.

Lemma.1. Suppose that Ω is a bounded domain which is convex in the x_1 direction and symmetric which respect to the plane $\{x_1 = 0\}$. Suppose $u \in C(B_1) \cap C^2(B_1)$ is a positive solution of

$$\begin{aligned} \triangle u + f(u) = 0 \quad &\text{in} \quad \Omega \\ u = 0 \quad &\text{on} \quad \partial\Omega \end{aligned}$$

where f is locally Lipschitz in $\mathbf{R}$. Then u is symmetric with respect ti x_1 and $D_{x_1}u(x) < 0$ for any $x \in \Omega$ with $x_1 > 0$.

Observation 1. This result is also valid if we replace the Laplacian by an operator invariant with respect to reflections in the x_1 direction.

Observation 2. Applying Lemma.1. in all directions we obtain Theorem 1.

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Existence of solutions for nonlinear systems

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In this work we study the following class of nonlinear elliptic system over a limited domain:

$$\begin{cases} -\Delta u = |v|^{p-1}v & \text{in } \Omega \\ -\Delta v = f(u) & \text{in } \Omega \\ u = 0,\ v = 0 & \text{on } \partial\Omega \end{cases}$$

where f is a superlinear and with no growth restriction.
We consider the following situations: for $p > 1$ we guarantee the existence of non-zero solution. For this it was necessary to make a variational reformulation, considering Sobolev spaces and the fractional Laplace operators. For $p\ leq1$ we also guarantee the existence of at least one non-zero solution. In this case it was also necessary to make a variational formulation of the main system.

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Existence and uniqueness of weak solution for the 2D Navier-Stokes equations and existence of the global attractor on some unlimited domains

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Let Ω be an open subset of $\mathbb{R}^2$ with boundary $\partial\Omega$, and assume that Poincaré inequality holds in Ω, i.e., there exists $\lambda_1 > 0$ such that

$$\int_\Omega \phi^2 \mathrm{dx} \leq \frac{1}{\lambda_1} \int_\Omega |\nabla \phi|^2 \mathrm{dx},$$

for every $\phi \in H_0^1(\Omega)$.

The variational formulation of the 2D Navier-Stokes equations of an incompressible fluid in Ω, can be expressed as follows: If $f \in V'$ and $u_0 \in H$, then there exists a unique function $u \in L^2(0,T;V) \cap L^\infty(\mathbb{R}^+;H)$, $\forall T > 0$, which satisfies

$$\frac{d}{dt}(u,v) + \nu\left((u,v)\right) + b(u,u,v) = \left\langle f,v\right\rangle, \quad \forall v \in V,$$
$$u(\cdot,0) = u_0(\cdot).$$

Since this solution also satisfies $u_t \in L^2(0,T;V')$, $\forall T > 0$, we are able to define a continuous semigroup $\{S(t)\}_{t\geq 0}$ in H, by $S(t)u_0 = u(t)$. By studying the associated energy equation, we obtain an absorbing set to the semigroup and we prove that the semigroup is asymptotically compact.

This allows us to conclude the existence of a global attractor which has finite Hausdorff and fractal dimensions.

On the weak solutions for the Navier-Stokes equations

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The idea of the poster is to present results that I explored during my undergraduate research. We will discuss the mathematical theory of the incompressible Navier-Stokes equations and some of the classical methods from the theory of partial differential equations such as the Galerkin method, which guarantees the existence of weak solution for the following equations

$$\begin{cases} \partial_t u + (u\cdot\nabla)u - \Delta u + \nabla p = 0 \\ \nabla \cdot u = 0 \end{cases}$$

with initial condition $u_0 \in H(\Omega)$, where Ω is a smooth bounded domain in $\mathbb{R}^3$ or the torus $\mathbb{T}^3$.

For this, we will need to state the weak formulation of the Navier-Stokes equations and the spectral decomposition of the Stokes operator. Furthermore we will show that such weak solution obtained by the Galerkin method satisfies the strong energy inequality.

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Hopf bifurcation for fish population model with delay

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The dispersion of a species is a well known phenomenon in nature and of great relevance, with an impact on the dynamics of the population, on their genetics and on the distribution of the species. In this work we present a delayed model in which we consider two environments, with a population of fish dispersing between these two areas and we present conditions that guarantee the asymptotic stability and the existence of Hopf bifurcation.

Introduction. We consider the following system:

$$\begin{aligned} \dot{x}_1(t) &= -dx_1(t) + ax_2(t) + \beta x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) &= -dx_2(t) + ax_1(t) + \beta x_2(t-\tau)e^{-x_2(t-\tau)}, \end{aligned} \tag{24}$$

where $d > 0$ is the mortality rate, $a > 0$ is the dispersion rate, $\tau \geq 0$ is the time it takes to reach maturity, this is, the necessary time for newborns to become mature for the reproduction.

We will then impose conditions on this model so that we have Hopf bifurcation. Considering the following hypotheses in our analysis. Be the RFDE of the form

$$\dot{x}(t) = F(\alpha, x_t), \tag{25}$$

then $F(\alpha,\phi)$ has first and second derivatives in α,ϕ for $\alpha \in \mathbb{R}$, $\phi \in C = C([-r,0],\mathbb{R}^n)$, and $F(\alpha,0) = 0$ for all α. Defining $L : \mathbb{R} \times C \to \mathbb{R}^n$ by

$$L(\alpha)\psi = D_\phi F(\alpha,0)\psi,$$

where $D_\phi F(\alpha,0)$ is the derivative of $F(\alpha,0)$ with respect to ϕ in $\phi = 0$. And also defining

$$f(\alpha,\phi) = F(\alpha,\phi) - L(\alpha)\phi,$$

we will have the following hypotheses:

1. Linear RFDE $(L(0))$ has a simple pure imaginary characteristic root $\lambda_0 = iv_0 \neq 0$ and all characteristic roots $\lambda_j \neq \lambda_0, \bar{\lambda_0}$, satisfy $\lambda_j \neq m\lambda_0$ for any integer m.
2. $Re\lambda'(0) \neq 0$.

We then have the following theorem,

Theorem 0.0.1. *Suppose the previous assumptions are satisfied. Then there are constants $a_0 > 0$, $\alpha_0 > 0$, $\delta_0 > 0$, functions $\alpha(a) \in \mathbb{R}$, $\omega(a) \in \mathbb{R}$, and a function $\omega(a)$-periodic $x^*(a)$, with all functions being continuously differentiable in a for $|a| < a_0$, such that $x^*(a)$ is a solution of the equation (25) with*

$$x_0^*(a)^{P_\alpha} = \Phi_{\alpha(a)}y^*(a), \quad x_0^*(a)^{Q_\alpha} = z_0^*(a),$$

where $y^*(a) = (a,0)^T + o(|a|)$, $z_0^*(a) = o(|a|)$ when $|a| \to 0$. In addition, for $|\alpha| < \alpha_0$, $|\omega - (2\pi/v_0)| < \delta_0$, every solution ω-periodic of the equation (25) with $|x_t| < \delta_0$ must be of this type except for a translation in the phase.

Proof. Reference [2]. □

Main Results. Assuming that $\beta > d - a$, then the system (24) has a unique positive equilibrium solution given by

$$x^* = (x_1^*, x_2^*) = (\ln\frac{\beta}{d-a}, \ln\frac{\beta}{d-a}).$$

Replacing $y_1(t) = x_1(t) - x_1^*$ e $y_2(t) = x_2(t) - x_2^*$ in the system (24), introducing the function $h(\alpha) = \alpha e^{-\alpha}$ for $\alpha \in \mathbb{R}$ and linearizing the system about the origin, we obtain

$$\begin{aligned} \dot{y}_1(t) &= -dy_1(t) + ay_2(t) + \beta h'(x_1^*)y_1(t-\tau), \\ \dot{y}_2(t) &= -dy_2(t) + ay_1(t) + \beta h'(x_2^*)y_2(t-\tau). \end{aligned} \tag{26}$$

We denote $c = \beta h'(x_1^*)$, obtaining the characteristic equation associated with the system (26) given by

$$(\lambda + d - ce^{-\lambda\tau})^2 = a^2. \tag{27}$$

We have studied the existence of roots of this problem by replacing $\lambda = u + iv$ in (27) and separating the real and the imaginary part.

As the solutions (27) are invariant under complex conjugation, we only study the case $v \geq 0$. We verified the existence of a pure imaginary root and then show that $Re\frac{d\lambda}{d\tau} \mid_{\lambda=vi} \neq 0$. What led us to complete the expected result.

And as a consequence we were able to establish the following result.

Theorem. Suppose that in the system (24) we have $\frac{\beta}{d-a} > e^2$, then the equilibrium of (24) is asymptotically stable.

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On global attractor of the 3D Navier-Lamé equation

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In the present work we show the existence of an regular global attractor with finite fractal dimension for the Navier-Lamé equation in the three dimensional environment with constant density and the local body force depending on the displacement with critical exponent.

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On the Schrödinger-Debye system on compact manifolds

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In this work, we will consider the following Cauchy problem for the Schrödinger-Debye equation:

$$\begin{cases} i\partial_t u + \Delta_g u = uv, & \text{in } \mathbb{R} \times M \\ \kappa\ \partial_t v + v = \lambda |u|^2, & \\ u(0,x) = u_0, v(0,x) = v_0, & \end{cases} \tag{28}$$

where $M = (M^d, g)$ is a compact Riemannian manifold, without boundary, of dimension $d \geq 2$, Δ_g denotes the Laplace-Beltrami operator on M, and $u : \mathbb{R} \times M \to \mathbb{C}$, $v : \mathbb{R} \times M \to \mathbb{R}$ are unknown functions, and $\kappa > 0$, $\lambda = \pm 1$. The problem (28) has been studied by many authors in the Euclidean case [2,4,5], and the only result in compact sense $M = \mathbb{T}^d$, is due to [1]. We will present some partial results about local and global well posedness to (28) for initial data satisfying $u_0 \in H^s(M^d)$ $(d \geq 2)$ in that $s > \frac{d}{2} - \frac{1}{p}$ $(p > 2)$ and v_0 sufficiently regular. The principal ingredient in the proof of our local result is the Strichartz estimates with loss obtained in [3]; the global well-posedness is obtained for $d = 2$, and use a version of a Gagliardo-Niremberg inequality on compact manifolds obtained in [6].

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Study of piecewise smooth linear vector fields in $\mathbb{R}^3$ with sliding region

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A dynamic system deals, strictly speaking, with systems whose intrinsic properties vary over time being, therefore, an area of kwonledge with innumerable possibilities of application. One of the subareas of recent dynamical systems theory is the area known as piecewise-smooth vector fields, which we will denote by PSVF. The development of this subarea is mainly due to the fact that the study of such vector fields has shown that they provide a model much closer to reality in applications than those governed by smooth vector fields because they often appear in applications of electronic engineering, control theory (see [4], [5]). In the context of PSVF, it is assumed that a surface Σ of co-dimension 1, called the discontinuity surface, separates the phase portrait into two or more disjoint parts (except Σ over), with each vector field defining a vector fields of vectors and under the surface Σ are defined the adjacent vector fields. Therefore, on points of Σ we have defined two or more vector fields.

Filippov's convention. An important tool in the study of PSVF is the so-called Filippov convention, which determines the algorithm to induce a field of vectors on Σ (see [7]). Based on this convention, many authors contributed to the study of such systems. Despite this, the theory is pretty new and even now in ascendancy, it seeks to establish good definitions and translate results and concepts from classical theory to the piecewise-smooth (see [1], [2], [3], [8]).

In order to define a PSVF let $D \subset \mathbb{R}^n$ an open set. We denoted the points of the $\mathbb{R} \times D$ por (t, x) and we called the variable t of time. Let $h : D \longrightarrow \mathbb{R}$ a smooth function and let $\Sigma = h^{-1}(0)$.

Definition 0.0.2. *Let $X^+, X^- : \mathbb{R} \times D \longrightarrow \mathbb{R}^n$ two C^r-vector fields, with $r \geq 1$. We define a PSVF*

$$\dot{x} = X(t, x) = \begin{cases} X^+(t, x) & se \quad h(x) \geq 0, \\ X^-(t, x) & se \quad h(x) \leq 0. \end{cases}$$

We will denote $X = (X^+, X^-)$.

In Σ generally we distinguish three open regions:

- **Crossing regions:** $\Sigma^c = \{p \in \Sigma;\ X^+h(p) \cdot X^-h(p) > 0\}$,
- **Attractive sliding regions:** $\Sigma^s = \{p \in \Sigma;\ X^+h(p) < 0,\ X^-h(p) > 0\}$,
- **Repulsive sliding regions:** $\Sigma^e = \{p \in \Sigma;\ X^+h(p) > 0,\ X^-h(p) < 0\}$,

where $X^{\pm}h(p) = \langle X(p), \nabla h(p)\rangle$. When $p \in \Sigma^s$, according to Filippov's convention, the sliding vector fields associated with X is the vector fields X^s tangent to the Σ^s, it is define to

$$X^s(p) = \frac{-X^+h(p)X^-(p) + X^-h(p)X^+(p)}{X^-h(p) - X^+h(p)}.$$

Objective. The objective consists of the exploration of the linear vector fields in $\mathbb{R}^3$ separated by a plane (discontinuity surface) and we assume the existence of two parallel lines where the degree of contact of the fields is two (see [6]). This context, together with the Filippov's convention, allows us first to make the qualitative study of the sliding vector field associated with X, $X^(s)$ and thus reach our main goal, which is to determine the global dynamics of the vector field $X(t, x)$.

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Existence of solutions in Besov-Morrey spaces for the Keller-Segel system coupled with the Navier-Stokes fluid

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The goal of this project is to study the asymptotic behavior type to the Keller-Segel system coupled with Navier-Stokes fluid in $\mathbb{R}^N (N \geq 2)$. We will present the result about existence of global mild solutions to these equations by taking initial data belonging to homogeneus Besov-Morrey spaces. The mild solutions are obtained by means of a fixed point argument in a time-dependent space X, where the space X is constructed so that its norm is invariant by the intrinsic scaling of the equations in question. These results are part of my PhD thesis.

Global L^q-ultradifferentiable functions

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The ultradifferentiable classes of functions $\mathcal{E}^{\omega}(\Omega)$ is explored in works such as [2] and [3]. We will introduce a class of C^{∞} functions derivatives of which satisfy global estimates: Let $1 \leq q \leq \infty$ and set a weight function ω, a function $g \in W^{\infty,q}(\Omega)$ is said to satisfy global L^q-ultradifferentiable estimates if there exist constants $A, C > 0$ such that for every multiindex α

$$\|D^{\alpha} g\|_{L^q(\Omega)} \leq C \exp\left(\frac{\varphi^*(A|\alpha|)}{A}\right). \tag{29}$$

For a fixed $A > 0$

$$\mathcal{E}_A^{q,\omega}(\Omega) = \left\{ g \in W^{\infty,q}(\Omega) : \{\|D^{\alpha} g\|_{L^q} e^{-\frac{1}{A}\varphi^*(A|\alpha|)}\}_{|\alpha|\geq 0} \in \ell^q(\mathbb{Z}^d_{\geq 0}) \right\}$$

and

$$\mathcal{E}^{q,\omega}(\Omega) = \bigcup_{A>0} \mathcal{E}_A^{q,\omega}(\Omega).$$

Such a space is constructed based on the one made by Adwan, Hoepfner and Raich in [1] for the Gevrey class. In addition to presenting basic properties of the space $\mathcal{E}^{q,\omega}(\Omega)$ we will give a characterization of the same by FBI (Fourier-Bros-Iagolnitzer) transform.

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Instability results for a coupled system with delay term

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In this work, we consider a coupled system of two wave equations. One of these equations is conservative and the other has damping and delay terms. If the damping acts with more force than

the delay term, we show polynomial stability for strong solutions of this system. Explicit decay rates are found and their optimalities are shown. On the other hand, if the damping acts with the same or less force than the delay term, then we obtain a result of instability by constructing a sequence of time delays and initial data such that the solutions are not asymptotically stable.

Propagation of regularity for the dispersive 2d generalized Benjamin-Ono-Zakharov-Kuznetsov equation

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We study some special regularity properties of solutions to the IVP associated to the dispersive generalized Benjamin-Ono-Zakharov-Kuznetsov equation

$$\partial_t u - D_x^{\alpha+1}\partial_x u + u_{xyy} + uu_x = 0, \quad (x,y,t) \in \mathbb{R}^3,\ 0 < \alpha < 1.$$

Mainly, we establish that for initial datum $u_0 \in H^s(\mathbb{R}^2)$ $(s > 2)$ whose restriction belongs to $H^m((x_0,\infty)\times\mathbb{R})$ for some $m \in \mathbb{Z}^+$, $m \geq 3$ and $x_0 \in \mathbb{R}$, we prove that the restriction of the corresponding solution $u(t)$ belongs to $L^2_y H^m_x(\alpha,\infty)$, for any $\alpha \geq 0$ and any $t \in (0,T)$. It is also deduced from our analysis that the solutions corresponding to this equation gain extra regularity on the certain regions of the plane. In addition, we obtain that this kind of regularity travels with infinite speed.

Global well-posedness of the dissipative surface quasi-geostrophic equation

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The main objective of this work is to study the article [1], where Constantin and Vicol demonstrated the well-posedness of the dissipative 2D SQG equation in the critical case. Their paper introduces new estimates for the fractional Laplacian, which have since then become very useful for several problems involving non-local dissipative operators. During my master's degree I did a detailed study of the article [1].

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Some examples of topological structural stability

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A semilinear parabolic problem with Neumann conditions in a Dumbbell domain Ω_ϵ and with Dirichlet conditions in a domain Ω discretized by a family of subdivisions $\{\mathcal{T}^h\}_{h\in(0,1]}$ has nonlinear semigroups associated $T_\epsilon(\cdot)$ in $\mathcal{U}^p_\epsilon$ and $T_h(\cdot)$ in $X^{1/2}_h$, respectively, where $\mathcal{U}^p_\epsilon := L^p(\Omega_\epsilon)$ y $X^{1/2}_h := \{\mathcal{I}^h v : v \in C(\overline{\Omega}), v|_{\partial\Omega} = 0\}$.

In this work, we will analyze the topological structural stability for families of nonlinear semigroups $T_\epsilon(\cdot)$ and $T_h(\cdot)$ on Banach spaces $\mathcal{U}^p_\epsilon$ and $X^{1/2}_h$, respectively. In addition, we will study the robustness

of the internal dynamics of the families of global attractors $\mathcal{A}_\epsilon \subset U_\epsilon^p$ and $\mathcal{A}_h \subset X_h^{1/2}$ on small perturbations of ϵ and h, respectively.

This work is together with the teachers Dr. German Lozada Cruz (IBILCE-UNESP, Brasil), José Langa Rosado (EDAN-US,España) and Éder Aragão Costa (ICMC-USP, Brasil).

This work is supported by FAPESP (Process 2014/19915-1).

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Some fractional models for damping in harmonic motion

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The Fractional Calculus, although it is not a new topic, has been drawing some attention in recent years due to its wide range of interesting applications. In this work we discuss some fractional versions of the Harmonic Oscillator Equation, in which the presence of the Fractional Derivative plays a role related to the damping of the system. Since this kind of problem requires an initial condition, our approach is related to the Caputo definition and we will use the Laplace Transform Method in order to solve these equations. We conclude this work by comparing these models and its results to those well-known from the ordinary Harmonic Oscillator Equation.

Atomic decompositions and operators on Hardy spaces

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The present work aims to present an example of linear a functional defined on a dense subspace of the Hardy space $H^1(\mathbb{R}^n)$ to be built, with the intention of showing that despite the fact that this functional is uniformly bounded on all atoms, it does not extend to a bounded functional on the whole $H^1(\mathbb{R}^n)$. This example was published by Bownik, M.B [2].

Therefore, this shows that in general is not enough to verify that an operator or a functional is bounded on atoms to conclude that it extends boundedly to the whole space. The construction is based on the fact due to Y. Meyer [1] which states that quasi-norms corresponding to finite and infinite atomic decomposition in $H^p(\mathbb{R}^n)$, $0 < p \leqslant 1$ are not equivalent.

On the other hand it will be given a necessary and sufficient condition for when and operator T defined in a dense Hardy subspace $H^p(\mathbb{R}^n)$ for $0 < p \leqslant 1$ is bounded extended. Such conditions were published by D. Yang and Y. Zhou [3].

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ICMC SUMMER MEETING on
DIFFERENTIAL EQUATIONS
CHAPTER 2019

Programme

	SUNDAY 3 from 17:00 to 19:00, Registration at the ICMC Auditorium (Building 6)		
Auditorium	**MONDAY 4**	**TUESDAY 5**	**WEDNESDAY 6**
08:00-08:50	**Registration**		
08:50-09:00	**Opening**		
Auditorium	**Plenary Talks**		
Chair	Hildebrando M. Rodrigues	Michel M. Chipot	Pierre Magal
09:00-09:40	**John Mallet-Paret**	**Djairo G. de Figueiredo**	**Michel M. Chipot**
09:40-10:20	**Hans-Otto Walther**	**Claudianor O. Alves**	**Marcelo M. Cavalcanti**
10:20-10:50	**Coffee Break**	**Coffee Break and Posters**	**Coffee Break and Posters**
Chair	Yingfei Yi	Konstantin Mischaikow	Tomás Caraballo
10:50-11:30	**Tomás Caraballo**	**Pierre Magal**	**Yingfei Yi**
11:30-12:10	**Konstantin Mischaikow**	**Marian Mrozek**	**Christian Poetzsche**
12:10-12:50		**Martin Bohner**	**Udayan Darji**
	Lunch	**Lunch**	**Lunch**
Auditorium	**Special Session on Nonlinear Dynamical Systems**		
Chair	Juliana Fernandes	Pedro Marín Rubio	Sergio Muniz Oliva Filho
14:30-15:00	**Tiago Pereira**	**Sergio Muniz Oliva Filho**	**Hildebrando M. Rodrigues**
15:00-15:30	**Eder Ritis Aragão Costa**	**Jia-Yuan Dai**	
15:30-16:00	**Matheus Cheque Bortolan**	**Phillipo Lappicy**	**Pedro Marín Rubio**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	**Coffee Break**
16:30-17:00	**Rodrigo Antonio Samprogna**	**Marcio A. Jorge da Silva**	**Javier López-de-la-Cruz**
17:00-17:30	**Ricardo Parreira da Silva**	**Paulo N. Seminario Huertas**	**Juliana Fernandes**
Room 5104	**Special Session on Boundary Perturbations of for PDEs Domains and Applications**		
Chair	Marcone Pereira	Alessandra A. Verri	
14:00-14:30	**Cesar Rogerio de Oliveira**	**Igor Pazanin**	
14:30-15:00	**Alessandra A. Verri**	**Ravi Prakash**	
15:00-15:30	**Marcus M. Marrocos**	**Ariadne Nogueira**	
15:30-16:00	**João Vitor da Silva**		
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	
Chair	Marcus M. Marrocos	Antônio L. Pereira	
16:30-17:00	**Antônio L. Pereira**	**Antoine Laurain**	
17:00-17:30	**Simone M. Bruschi**	**Marcone C. Pereira**	
17:30-18:00		**Jean Carlos Nakasato**	
Room 31214	**Special Session on Computational Dynamics in the Context of Data**		
Chair	Thomas Wanner	Marcio Gameiro	
14:30-15:00	**Tomas Gedeon**	**Thomas Wanner**	
15:00-15:30	**Breschine Cummins**	**Marian Mrozek**	
15:30-16:00	**Robert Vandervorst**	**Mateusz Juda**	
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	
16:30-17:00	**Kelly Spendlove**	**Ketty Abaroa de Rezende**	
17:00-17:30	**Ewerton Rocha Vieira**	**Bogdan Batko**	
17:30-18:00	**Marcio Gameiro**	**Bernardo Rivas**	

Room 5102	Special Session on Dispersive Equations	
Chair	Marcia Scialom	Mahendra Panthe
14:30-15:00	**Jose Raul Quintero**	**Roberto Capistrano Filho**
15:00-15:30		**Oscar G. Riano Castaneda**
15:30-16:00	**Argenis Jose Mendez**	**Andressa Gomes**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**
16:30-17:00	**Isnaldo Isaac Barbosa**	**Lucas C. F. Ferreira**
17:00-17:30	**Mahendra Panthe**	**Marcia Scialom**

Room 5001	Special Session on Elliptic Equations I	
Chair	Claudianor O. Alves	Giovany M. Figueiredo
14:30-15:00	**Giovany M. Figueiredo**	**Olimpio H. Miyagaki**
15:00-15:30	**Jefferson Abrantes Santos**	**Sérgio L. N. Neves**
15:30-16:00	**Sigifredo Herrón**	**Raquel Lehrer**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**
16:30-17:00	**Gaetano Siciliano**	**Augusto César dos Reis Costa**
17:00-17:30	**Leandro da Silva Tavares**	**Moisés Aparecido do Nascimento**
17:30-18:00		**Nilson Costa Roberty**

Room 5003	Special Session on Elliptic Equations II	
Chair	Leonardo Prange Bonorino	Cristian Morales-Rodrigo
14:30-15:00		**Damião J. Araújo**
15:00-15:30	**Alânnio Barbosa Nóbrega**	
15:30-16:00	**Romildo Nascimento de Lima**	**João Vitor da Silva**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**
16:30-17:00	**Cristian Morales-Rodrigo**	**Gabrielle Saller Nornberg**
17:00-17:30	**Patrícia Leal da Cunha**	**Leonardo Prange Bonorino**
17:30-18:00	**Yony Santaria Leuyacc**	**Giane Casari Rampasso**

Room 5101	Special Session on Evolution Equations and Applications		
Chair	Giulio Tralli	Alessia Kogoj	Sergio Polidoro
14:30-15:00	**Sergio Polidoro**	**Luciano Mari**	**Alessia Kogoj**
15:00-15:30	**Chiara Guidi**	**Dario Daniele Monticelli**	**Giulio Tralli**
15:30-16:00	**Roberto Guglielmi**	**Alex Pereira**	**Arlúcio Viana**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	**Coffee Break**
16:30-17:00	**Cristian Rios**	**Bianca Morelli Rodolfo Calsavara**	
17:00-17:30	**Francesca Anceschi**	**Scott Rodney**	**Carlo Orrieri**
17:30-18:00	**Ahmed Mohammed**		**Jayme Vicente de Luca**

Room 5103	Special Session on Fluid Dynamics	
Chair	Anne C. Bronzi	Cesar J. Niche
14:30-15:00	**Luca Moriconi**	**Ricardo Rosa**
15:00-15:30	**Ciro Sobrinho Campolina**	**Elder J. Villamizar-Roa**
15:30-16:00	**Nelson Luis da Costa Dias**	**Gabriela Planas**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**
16:30-17:00	**Juliana Honda Lopes**	**Paulo Mendes Carvalho**
17:00-17:30	**Cesar J. Niche**	**Anne C. Bronzi**
17:30-18:00	**Leonardo Kosloff**	

Room 5002	**Special Session on Linear Partial Differential Equations**		
Chair	Marcelo Rempel Ebert	Marcello D'Abbicco	Paulo Leandro Dattori da Silva
14:30-15:00	**Paulo Leandro Dattori da Silva**	**Alexandre Kirilov**	**Marcello D'Abbicco**
15:00-15:30	**Wagner A. Almeida de Moraes**	**Fernando de Ávila Silva**	**Nataliia Goloshchapova**
15:30-16:00	**Bruno de Lessa Victor**	**Pedro Tavares Paes Lopes**	**Marco Cappiello**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	**Coffee Break**
16:30-17:00	**Cleber de Medeira**	**Lucas da Silva Oliveira**	**Marcelo Rempel Ebert**
17:00-17:30		**Raphael F. Hora**	**Benito Frazao Pires**

Room 5004	**Special Session on Ordinary/Functional Diferential Equations**		
Chair	Pierluigi Benevieri	Jaqueline G. Mesquita	Gonzalo Robledo Veloso
14:30-15:00	**Jaqueline G. Mesquita**	**Gonzalo Robledo Veloso**	**José Vanterler da Costa Sousa**
15:00-15:30	**Eduard Toon**	**Álvaro Castañeda**	**Iguer Luis Domini dos Santos**
15:30-16:00	**Fernanda Andrade da Silva**	**Xiaoying Han**	**Jaqueline da Costa Ferreira**
16:00-16:30	**Coffee Break and Posters**	**Coffee Break and Posters**	**Coffee Break**
16:30-17:00	**Aldo Pereira**	**Hugo de la Cruz**	**Marielle Aparecida Silva**
17:00-17:30	**Rafael Marques**	**Junya Nishiguchi**	
17:30-18:00	**Rodolfo Collegari**	**Pierluigi Benevieri**	**Gerard John Alva Morales**

Coffee Area	**Poster Session**	
10:20-10:50	**Heydy Melchora Santos Damian**	**Ricardo Carlos Freire**
10:20-10:50	**José Guilherme Simion Antunes**	**Rodiak Nicolai Figueroa López**
10:20-10:50	**Juliane Carolina Baiochi Dalben**	**Stefania Jarosz**
10:20-10:50	**Kleber Santana Souza**	**Victor Hugo Falcão Francheto**
10:20-10:50	**Ricardo Martins Guimarães**	

16:00-16:30	**Andréia Gomes Pinheiro**	**Lito Edinson Bocanegra**
16:00-16:30	**Armando Santisteban Cárdenas**	**Marcelo A. Cabral Nogueira**
16:00-16:30	**Brayan M. Rodríguez Garzón**	**Mariana Queiroz Velter**
16:00-16:30	**Edison Fausto Cuba Huamani**	**Monisse Postigo Alves**
16:00-16:30	**Eduardo Eizo Aramaki Hitomi**	**Patrícia Yukari Sato Rampazo**
16:00-16:30	**Fiorella María Rendón García**	**Rafael Lima Oliveira**

	Social Events	
12:10	**Photo**	
20:00		**Conference Banquet**

ICMC SUMMER MEETING on
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CHAPTER 2019

List of Speakers

List of speakers **(ordered by first name)**

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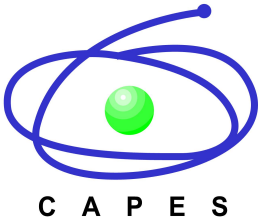

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