

Universidade de São Paulo  
Instituto de Ciências Matemáticas e de Computação

*ICMC Summer Meeting on  
Differential Equations  
2012 Chapter*

Promoted by ICMC and INCTMat

Sponsored by FAPESP, CAPES, CNPq and USP

São Carlos - Brazil  
6-8 February 2012



## Welcome

It is a pleasure to welcome you to the *ICMC Summer Meeting on Differential Equations - 2012 Chapter* and to São Carlos. We wish you a pleasant stay and that you enjoy the meeting.

## Organizing committee

Alexandre Nolasco de Carvalho (ICMC / USP)

Ederson Moreira dos Santos (ICMC / USP)

Hildebrando Munhoz Rodrigues (ICMC / USP)

Sérgio Henrique Monari Soares (ICMC / USP)

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## Address

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*ICMC Summer Meeting on  
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2012 Chapter*

Maps

**Entrances and Exits of the Campus**

1. ICMC entrance
2. Main entrance
3. Observatório's entrance
4. Physics institute's entrance
5. Physics institute's exit

**Hotels**

1. Hotel Indaiá Residence
2. Parisi Hotel
3. Hotel Indaiá
4. Hotel Anacã
5. San Ciro Apart Hotel
6. Atlantic Inn Residence

**Bars**

1. Tio Joaquim
2. Boteco Santa Teresa
3. Vila Brasil Botequim
4. Donna Léo Choperia
5. Mosaico Bar e Restaurante
6. Cachaçaria Água Doce
7. Almanach Café e Restaurante
8. Pimentas Bar
9. Seo Gera

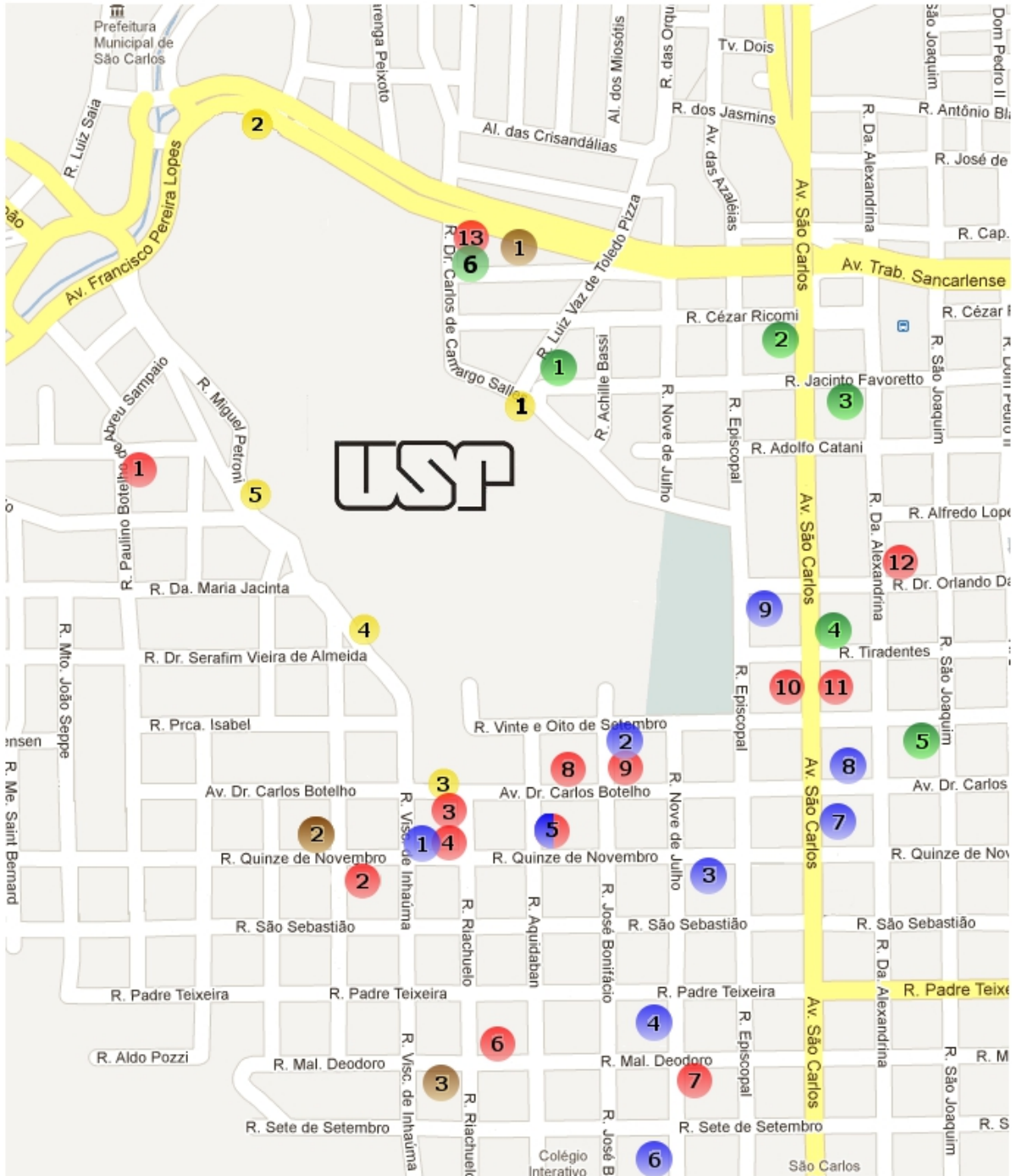
**Restaurants**

1. Restaurante La Salute (lunch only)
2. Restaurante La Villa
3. Casa do Café
4. Cantina Ciao Bello
5. Mosaico Bar e Restaurante
6. Sabor Oriental
7. Restaurante Mamãe Natureza (lunch only)

8. Restaurante Barone
9. Kalil Cozinha Árabe
10. Roda Chopp
11. Sushi-Ya San
12. Restaurante Panela
13. Restaurante Curinga

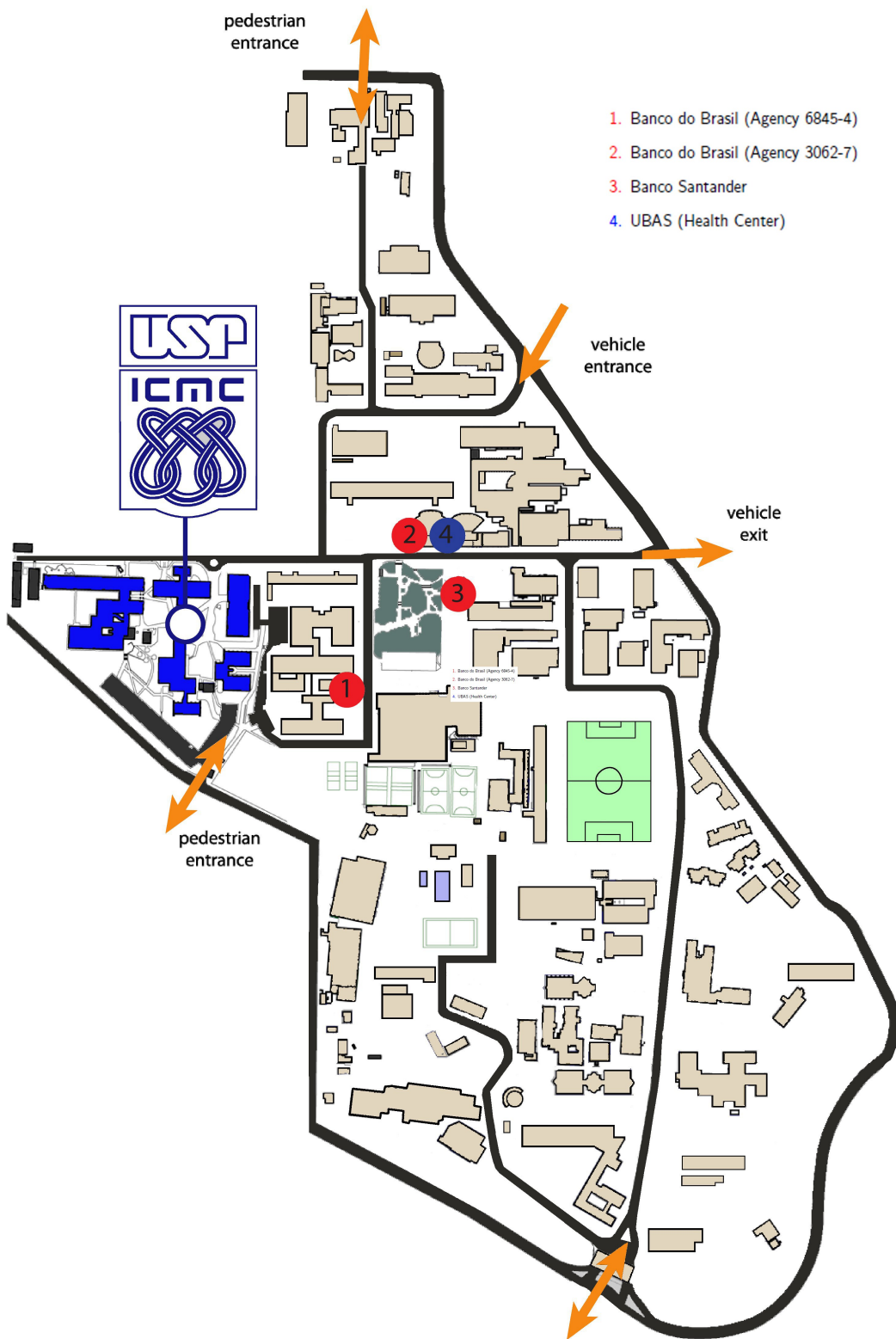
**Pizzerias (dinner only)**

1. Pizzaria Bom Pedaco
2. Pizzaria Amici
3. Pizzaria Don Raffaele















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General Information





## Conference site

The meeting will take place at Buildings 5 and 6, and also in the Coffee Area. All these buildings are indicated in the map on page 9.

Plenary talks will take place at the Auditorium (Building 6)

Special session on Elliptic Equation in Auditorium (Building 6)

Special session on Ordinary and Funct. Diff. Equations in room 5-001 (Building 5)

Special session on Nonlinear Dynamical Systems in room 5-003 (Building 5)

Special session on Computational Dynamics in room 5-004 (Building 5)

Special session on Fluid Dynamics in room 5-101 (Building 5)

Special session on Linear Partial Differential Equations in room 5-103 (Building 5)

Poster session in Coffee Room (ground floor of the Library)

## Registration

The registrations will be made in the following schedule:

*Sunday, February 5<sup>th</sup>* From 17:00hs to 18:00hs in the lobby of Anacã Hotel.

*Monday, February 6<sup>th</sup>* From 8:00hs to 8:40hs in the entrance of the Auditorium (Building 6).

We will provide you a badge at registration. Please wear your badge at the event.

The Event's Office (number 3, map at page 9) will be at your disposal for any questions and informations.

## Financial support

The financial support payment is planned to be made on Tuesday, February 7<sup>th</sup> in the Financial Office (number 4, map on page 9).

You are requested to fulfill your information at the on-line form available at [www.icmc.usp.br/~summer/user\\_summer/](http://www.icmc.usp.br/~summer/user_summer/), this is mandatory in order to receive the payment.

## Meals and refreshments

There are several restaurants near the campus. You can find them by looking at the city map located on page 5, the restaurants are in red. There are also three choices of pizzerias, which are in brown. At night, there are many bars around the city and they can be found in the map in the color blue.

There is also available a canteen on the campus (look for building (7) at the ICMC map located in page 9) where you can have either snacks or lunch.

## Social events

*Monday, February 6<sup>th</sup>*: Cocktail at 18:30hs in the Coffee Room.

*Tuesday, February 7<sup>th</sup>*: Photo of the meeting at 12:10hs at ICMC.

*Tuesday, February 7<sup>th</sup>*: Conference Banquet at 20:30hs in Anacã Hotel.

## Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

## Smoking

Smoking is prohibited inside any of the ICMC buildings also in the canteen and on the ground floor of the library.

## Computer and wireless LAN use

There will be available computers and a printer for use at room 5-002 (look for building (5) at the ICMC map located in page 9).

In order to access the wireless connection at the University you need to follow the steps:

1. Enable wireless on your device.
2. Join the ICMC-PORTAL wireless network.
3. Open a browser and attempt to visit a website (for example your home page).
4. Click on the button in the page to proceed.
5. You will be redirected to a login page. Enter the login and password as follows:

login: *summer12@icmc.usp.br*

password: verao2012

6. You may freely browse the internet after logging in. You may occasionally need to re-authenticate using the above procedure.

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Plenary talks



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## On finding solutions of boundary value problems for some classes of functional differential systems

**András Rontó**

Institute of Mathematics, AS CR, branch in Brno

For a wide class of boundary value problems for non-linear functional differential equations, we present an approach that allows one both to construct approximate solutions and to rigorously prove the solvability of the problem using properties of the approximate solutions. The approach is based upon a special type of successive approximations that are constructed analytically and, under suitable conditions, converge uniformly on the given interval.

## Density of the range of semilinear wave operator

**Alfonso Castro**

Harvey Mudd College

It is proven that a semilinear operator without resonance with discrete spectrum and an eigenvalue of infinite multiplicity has dense range. An extension of the saddle point principle is established and the variational characterization of the Fucik spectrum by the author and C. Chang is utilized.

## Populations dynamic under anomalous diffusion: predator dynamic

**Benedetta Pellacci**

Napoli "Parthenope"

Our topic will be the introduction of fractional elliptic problems in bounded domains subject to Neumann boundary conditions. This kind of problems well describes anomalous diffusion arising in biology. Fractional elliptic operator in bounded domains have been recently widely studied for Dirichlet boundary conditions while there is no literature concerning Neumann conditions which are more natural in the context of closed systems of populations.

## Determining nodes for regulatory networks

**Bernold Fiedler, Atsushi Mochizuki**

Freie Universität Berlin

We consider systems of differential equations which model complex gene-regulatory networks like early Ascidian embryogenesis. We show that the concepts of informative nodes (Mochizuki) and of determining nodes (Foias, Temam) coincide with the notion of feedback vertex sets from graph theory.

As a result we can determine the long-time dynamics of the entire network from observations on the feedback vertex set, only. We present examples where that required observation set is much smaller than the entire regulatory network.

All results are joint work with Atsushi Mochizuki (RIKEN, Tokyo).

## Elliptic PDE with natural growth in the gradient

Boyan Sirakov

Université Paris Ouest

We report on some recent results on the Dirichlet problem for elliptic operators in divergence or non-divergence form involving powers of the gradient of the unknown function, with possibly unbounded coefficients and noncoercive zero-order terms.

## Yet another surprising structure underlying the Toda lattice

Carlos Tomei

PUC-Rio

The Toda lattice is a remarkable completely integrable system, with ties with numerical analysis and physics. But there is much more than complete integrability: recent work of Leite, Saldanha and myself introduce new bidiagonal coordinates, which extend to larger sets of matrices including limit points, evolve trivially under the evolutions and admit more robust inversion algorithms.

## Existence of a ground state solution for a nonlinear scalar field equation with critical growth

Claudianor O. Alves

Universidade Federal de Campina Grande

In this conference, we will show some recent results about the existence of Ground State Solutions for some class of Elliptic problems with Critical Growth in  $\mathbb{R}^N$  for  $N \geq 2$ . These results complete the study made by Berestycki and Lions [BL] and Berestycki, Gallouët and Kavian [BGK], in the sense that, in those papers only the subcritical growth was considered.

### References

[BL] Berestycki, H., Lions, P.-L., *Nonlinear scalar field equations, I - existence of a ground state*, Arch. Rat. Mech. Analysis, 82, (1983), 313–346.

[BGK] Berestycki, H., Gallouët, T. and Kavian, O., *Equations de Champs scalaires euclidiens non linéaires dans le plan*, C. R. Acad. Sci. Paris Ser. I Math, 297, (1984), 307–310.

( Joint work with M. Montenegro and M.A.S. Souto )

## On some reaction processes with saturating diffusion

Denis Bonheure

Université Libre de Bruxelles

I will discuss some simple stationary reaction-diffusion equations with a saturating diffusion flux. Basic questions such as existence, multiplicity, radial symmetry of ground states or one dimensionality of phase transitions will be tackled.

## On the Brezis-Nirenberg problem for the $p$ -Laplacian

Djairo G. de Figueiredo

IMECC - UNICAMP

We discuss the questions of existence and non-existence of critical problems for the  $p$ -Laplacian under both homogeneous and non-homogeneous boundary conditions.

## Some special properties of the solutions of nonlinear shallow shells models

Gustavo P. Menzala, Fabiana Travessini

Laboratório Nacional de Computação Científica

We shall describe the model called Vlasov/Malguerre for shallow shells. Then we will prove properties of the solutions. First, the "proximity" of some components of the model to a so called Timoshenko model and secondly we treat the asymptotic behavior of the Total Energy associated with the model.

## Higher order parabolic equations in $\mathbb{R}^N$

Jan Cholewa

Silesian University

Some fourth order linear and semilinear equations in  $\mathbb{R}^N$  are studied. For the linear equation we consider some weakly integrable potential terms, and for any  $1 < p < \infty$  prove that for a suitable family of Bessel potential spaces,  $H_p^\alpha(\mathbb{R}^N)$ , the linear equation defines a strongly continuous analytic semigroup. For the nonlinear equation we find the critical exponents, that is, the largest growth allowed for the nonlinear terms for the considered Bessel potentials classes of initial data.

We exhibit then the dissipative mechanism in the spaces of Bessel potentials and discuss some weak conditions that lead to the existence of a global attractor.

The results come from the joint works with A. Rodriguez-Bernal from Universidad Complutense de Madrid.

## New results for the PDEs of mathematical finance

Jerome A. Goldstein

University of Memphis

One of the concerns of mathematical finance is fair pricing of stocks and bonds. The problems involved are often very complicated, and the models often begin with Ito stochastic differential equations (SDE). Often the solution of the SDE does not contain enough information to solve the problem of interest. Further (stochastic) analysis in some cases leads to deterministic degenerate parabolic PDE for the fair price. The most notable examples are the Black-Merton-Scholes equation (BMS) and the Cox-Ingersoll-Ross equation (CIR). The former won a Nobel Prize in Economics; the latter is of considerable current interest.

With various coauthors, we prove new results about both equations, working in the context of the governing operator semigroups. With Gisele Goldstein and Hassan Emamirad, we show that the BMS semigroup is chaotic on a continuum of sup norm Banach spaces which arise "naturally". With Gisele

Goldstein, Rosa Maria Mininni and Silvia Romanelli, we prove various existence and regularity results for various versions of the CIR semigroup. Not all of these semigroups are strongly continuous. We also obtain Feynman-Kac type representations of the solutions.

## Critical nonlinearities and nonlinear Schrodinger equations

João M. do Ó, Kyril Tintarev, David Costa

Universidade Federal da Paraíba

We prove compactness of critical Sobolev embedding with applications to nonlinear singular Schrödinger equations.

## Morse decomposition theory for random dynamical systems

José A. Langa, T. Caraballo, Zhenxin Liu

Universidad de Sevilla

The random attractor is a suitable concept for the description of the asymptotic behavior of random dynamical systems associated to some stochastic differential equations. In this talk we will analyze the internal structure of random attractors via a Morse decomposition theory describing random isolated invariant sets and its connections, to which a continuous Lyapunov function is also associated. Some possible generalizations to non-autonomous dynamical systems will be also addressed.

## PDE's in thin domains with oscillatory boundaries

José M. Arrieta

Universidad Complutense de Madrid

We will consider several PDE's of elliptic and parabolic type in thin domains having oscillatory boundaries. We will obtain the limiting equations and study the relation between the solutions of the equation in the thin domain and the limit equation.

## On comparison principles for multivalued systems with applications to reaction-diffusion systems

José Valero

Universidad Miguel Hernández de Elche

In this talk we will consider comparison principles for multivalued dynamical systems.

Let us consider a differential equation for which uniqueness of the Cauchy problem fails (or just it is not known to hold), for example an abstract parabolic problem

$$\begin{cases} \frac{du}{dt} = A(t, u(t)), & \tau \leq t \leq T, \\ u(\tau) = u_\tau, \end{cases} \quad (1)$$

for which we can prove that for every initial data in the phase space  $X$  (with a partial order  $\leq$ ) there exists at least one solution.

If we try to compare solutions of (1) for two ordered initial data  $u_\tau^1 \leq u_\tau^2$ , then we can consider a strong comparison principle and a weak one.



The strong version would imply the existence of a solution  $u_1$  with  $u_1(\tau) = u_\tau^1$  such that

$$u_1(t) \leq u_2(t) \text{ for } t \in [\tau, T], \quad (2)$$

for any solution  $u_2$  with  $u_2(\tau) = u_\tau^2$ , and, viceversa, the existence of a solution  $u_2$  with  $u_2(\tau) = u_\tau^2$  such that (2) is satisfied for any solution  $u_1$  with  $u_1(\tau) = u_\tau^1$ .

The weak version of the comparison principle says that if  $u_\tau^1 \leq u_\tau^2$ , then there exist two solutions  $u_1, u_2$  of (1) such that  $u_1(\tau) = u_\tau^1$ ,  $u_2(\tau) = u_\tau^2$ , and (2) holds.

We prove a weak comparison principle for a system of reaction-diffusion equations without uniqueness of the Cauchy problem. Namely, we consider the system

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t} - a\Delta u + f(t, u) = h(t, x), (t, x) \in (\tau, T) \times \Omega, \\ u|_{x \in \partial\Omega} = 0, u|_{t=\tau} = u_\tau(x), \end{array} \right. \quad (3)$$

where  $\tau, T \in \mathbb{R}$ ,  $T > \tau$ ,  $x \in \Omega$ ,  $u = (u^1(t, x), \dots, u^d(t, x))$ ,  $f = (f^1, \dots, f^d)$ ,  $a$  is a real matrix of order  $d \times d$  with positive symmetric part  $\frac{a + a^t}{2} \geq \beta I$ ,  $\beta > 0$ ,  $h \in L^2(\tau, T; (L^2(\Omega))^d)$ . Moreover,  $f = (f^1(t, u), \dots, f^d(t, u))$  is continuous and

$$\sum_{i=1}^d |f^i(t, u)|^{\frac{p_i}{p_i-1}} \leq C_1 \left(1 + \sum_{i=1}^d |u^i|^{p_i}\right), \quad (4)$$

$$(f(t, u), u) \geq \alpha \sum_{i=1}^d |u^i|^{p_i} - C_2, \quad (5)$$

with  $p_i \geq 2$ ,  $\alpha, C_1, C_2 > 0$ .

We apply the abstract results to the Lotka-Volterra system with diffusion, a generalized logistic equation and to a model of fractional-order chemical autocatalysis with decay. Moreover, in the case of the Lotka-Volterra system a weak maximum principle is given, and a suitable estimate in the space of essentially bounded functions  $L^\infty$  is proved for at least one solution of the problem.

## Rigorous computations for nonlinear dynamics

Konstantin Mischaikow

Rutgers University

I will talk about ongoing work to develop rigorous, but efficient techniques for computing equilibria, periodic orbits and connecting orbits for parameterized families of differential equations. The essential idea is classical: construct a fixed point problem that can be solved using the contraction mapping theorem. However, our approach contains a few novelties including: (1) the introduction of radii polynomials which provide an efficient means of determining a domain on which the contraction mapping theorem is applicable; and (2) the use of rigorous high-order approximations of stable and unstable manifolds. We will give examples of these techniques in the context of PDEs and ODEs.

## Applications of averaging method

Marco A. Teixeira, Jaume Llibre, Douglas Novaes

UNICAMP

The main goal is to detect the existence of periodic orbits for some  $4D$  systems. The major tool for proving the results is the averaging theory.

## Dynamic equations on time scales and applications

**Martin Bohner**

Missouri University of Science and Technology

Time scales have been introduced in order to unify continuous and discrete analysis and in order to extend those theories to cases “in between”. We will offer a brief introduction into the calculus involved, including the so-called delta derivative of a function on a time scale. This delta derivative is equal to the usual derivative if the time scale is the set of all real numbers, and it is equal to the usual forward difference operator if the time scale is the set of all integers. However, in general, a time scale may be any closed subset of the reals.

We present some basic facts concerning dynamic equations on time scales (those are differential and difference equations, resp., in the above two mentioned cases) and initial value problems involving them. We introduce the exponential function on a general time scale and use it to solve initial value problems involving first order linear dynamic equation. We also present a unification of the Laplace and Z-transform, which serves to solve any higher order linear dynamic equations with constant coefficients. Throughout the talk, many examples of time scales will be offered. Among others, we will discuss the following examples:

1. The two standard examples (the reals and the integers).
2. The set of all integer multiples of a positive number (this time scale is interesting for numerical purposes).
3. The set of all integer powers of a number bigger than one (this time scale gives rise to so-called  $q$ -difference equations).
4. The union of closed intervals (this time scale is interesting in population dynamics; for example, it can model insect populations that are continuous while in season, die out in say winter, while their eggs are incubating or dormant, and then hatch in a new season, giving rise to a nonoverlapping population).

## Global optimization via intermittent diffusion and shortest path planning

**Shui-Nee Chow**

Georgia Institute of Technology

We introduce an intermittent diffusion (ID) method to find global minimizers of a given smooth scalar-valued function  $g$  on a compact connected manifold. The main idea is to add intermittent, instead of continuously diminishing, random perturbations to the gradient flow generated by the given function  $g$ , so that the trajectories can quickly escape from traps of minimizers and then approach others. As an application, we propose a new fast algorithm for finding a global shortest path connecting two points while avoiding given obstacles in a planar region. The idea is based on the fact that every shortest path possesses a simple geometric structure.

This enables us to restrict our search in a set of feasible paths that share the same structure. The resulting search is then reduced to solving initial value problems for ordinary differential equations with random perturbations as in the ID method. This is based on two recent works (1) with Tzi-Sheng Yang and Haomin Zhou; (2) with Jun Lu and Haomin Zhou.

## Large-amplitude periodic orbits for a delay differential equation

**Tibor Krisztin**

University of Szeged

We show that the structure of the global attractor — for equations with delayed monotone positive feedback — can be more complicated than the union of spindle-like structures. It is possible to have large-amplitude periodic orbits in the sense that they are not between two consecutive stable equilibria with respect to the pointwise ordering. For some nonlinearities there are exactly 2 large-amplitude periodic orbits, and a complete picture of the global attractor can be obtained outside the spindle-like structures.

## Random attractors for stochastic lattice systems without uniqueness of solutions

**Tomás Caraballo**

Universidad de Sevilla

In this talk we study the asymptotic behavior of solutions of a first-order stochastic lattice dynamical system perturbed by noise.

We do not assume any Lipschitz condition on the nonlinear term, just a continuity assumption together with growth and dissipative conditions, so that uniqueness of the Cauchy problem fails to be true.

Using the theory of multi-valued random dynamical systems we prove the existence of a random compact global attractor.

## Degeneracy, complexity and robustness of bio-systems

**Yingfei Yi**

Georgia Institute of Technology & Jilin University

There has been recent emphasis on degeneracy as a feature of structural complexity due to the empirical observations of degenerate properties in known complex systems. The notion of degeneracy was first introduced for neural system as the ability of elements that are structurally different to perform the same function. Degeneracy is known to have close ties with structural complexity and robustness of a neural system. It is already observed for neural systems that high degeneracy not only yields high robustness, but also it is accompanied by an increase in the system complexity. In this talk, we will introduce the notions of degeneracy and structural complexity for a biosystem modelled by a differential equation. We will also discuss their connections with the robustness of the system.

This is a joint work with G. Dwivedi (Georgai Tech), W. Huang (USTC), M. Kemp (Georgia Tech) and Y. Li (Georgia Tech).



*ICMC Summer Meeting on  
Differential Equations  
2012 Chapter*

Special Sessions



## ELLIPTIC EQUATIONS

**Organizer:** Claudianor Oliveira Alves

### Existence of blow-up solutions for a class of elliptic systems

**Angelo R. F. de Holanda**

Universidade Federal de Campina Grande - UFCG

In this talk, the main goal is to prove the existence of blow-up solutions for a class of elliptic systems. Here, we combine variational methods with the existence of sub- and super-solution to get the blow-up solutions. This is joint work with Claudianor O. Alves (UFCG).

### About solutions of a quasilinear problem including sublinear, superlinear and convective term

**Carlos A. Santos**

Universidade de Brasília

In this talk, we present some results concerning to the existence of positive solutions for problem

$$\begin{cases} -\Delta_p u = a(x)f(u) + \lambda b(x)g(u) + \mu V(x, \nabla u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \quad u(x) = 0 & \text{on } \partial\Omega. \end{cases}$$

where  $\Delta_p$  is p-Laplacian,  $1 < p < \infty$ ;  $\lambda, \mu > 0$  are real parameters,  $f, g : (0, \infty) \rightarrow [0, \infty)$  and  $V : \mathbb{R}^N \times \mathbb{R}^N \rightarrow [0, \infty)$  are continuous functions and  $\Omega \subset \mathbb{R}^N$  is a smooth bounded domain or  $\Omega = \mathbb{R}^N$ . When  $\Omega = \mathbb{R}^N$ , the condition  $u = 0$  on  $\partial\Omega$  means that  $u(x) \rightarrow 0$  when  $|x| \rightarrow \infty$ .

No monotonicity conditions is required on the nonlinearities, but sublinear and superlinear terms are included in our results. The principal results are included in a work together with Goncalves, J. V. and Rezende, M.

### Positive solutions for fourth elliptic problems

**Edcarlos D. Silva**, J. V. Goncalves, M. L. Silva

UFG

In this talk we discuss existence of positive solutions for a fourth elliptic problem using bifurcation techniques.

### On a class of elliptic problems with indefinite nonlinearities

**Everaldo S. de Medeiros**, Elves A. de B. e Silva, Uberlandio B. Severo

UFPB

We present some results of existence and non existence of positive solutions for a class of elliptic equations involving an indefinite nonlinearity. In our approach, we invoke the Mountain Pass Theorem and an auxiliary problem in combination with sub-supersolution method.

## Radial solutions of Dirichlet problems with concave-convex nonlinearities

Francesca Dalbono, Walter Dambrosio

CMAF, Universidade de Lisboa

We prove the existence of a double infinite sequence of radial solutions for a Dirichlet concave-convex problem associated with an elliptic equation in a ball of  $\mathbb{R}^n$ . We are interested in relaxing the classical positivity condition on the weights, by allowing the weights to vanish. The idea is to develop a topological method and to use the concept of rotation number. The solutions are characterized by their nodal properties.

## On the number of solutions of NLS equations with magnetic fields in expanding domains

Giovany M. Figueiredo, Claudianor O. Alves, Marcelo F. Furtado

Universidade Federal do Pará

In this paper we look for multiple weak solutions  $u : \Omega_\lambda \rightarrow \mathbb{C}$  for the complex equation  $(-i\nabla - A(\frac{x}{\lambda}))^2 u + u = f(|u|^2)u$  in  $\Omega_\lambda = \lambda\Omega$ . The set  $\Omega \subset \mathbb{R}^N$  is a smooth bounded domain,  $\lambda > 0$  is a parameter,  $A$  is a regular magnetic field and  $f$  is a superlinear function with subcritical growth. Our main result relates, for large values of  $\lambda$ , the number of solutions with the topology of the set  $\Omega$ . In the proof we apply minimax methods and Ljusternick-Schnirelmann theory.

## Spectral and minimization properties for an indefinite linear elliptic eigenvalue problem

Gustavo F. Madeira

UFSCar

We consider a linear elliptic eigenvalue problem having a parameter and bounded indefinite weights in both the equation and the boundary condition. Such problem is interesting not only for being a generalization of the Neumann or Robin problems but it is also important in the study of parabolic problems under dynamical boundary conditions.

Our aim is to prove that the eigenvalues form two unbounded sequences and to study some properties of the positive principal eigenvalue for varying weights.

## An indefinite and non-homogeneous elliptic problem

Humberto R. Quoirin, Pedro Ubilla

Universidad de Santiago de Chile

We consider a class of elliptic problems for the operator  $-\Delta + V$ , acting on  $H_0^1(\Omega)$ , where  $\Omega$  is a bounded domain. The main feature here is the indefinite behaviour of both the operator and the nonlinearity. Based on the global geometry of the energy functional, we prove the existence of two local minimizers and a mountain-pass critical point for a concave-convex type nonlinearity. Subcritical and critical growth are allowed on the nonlinearity.



## Multivalued quasilinear equations

Jefferson A. dos Santos, Claudianor O. Alves, José V. A. Gonçalves  
Universidade Federal de Campina Grande

In this work we study the existence of nontrivial solution for the following class of multivalued quasilinear problems

$$-div(\phi(|\nabla u|)\nabla u) - b(u)u \in \partial_u F(x, u) \text{ in } \Omega,$$

where  $\Omega \subset \mathbb{R}^N$  is a domain limitado,  $N \geq 2$  and  $\partial_u F(x, u)$  is a generalized gradient of  $F(x, t)$  with respect to  $t$ . The main tools utilized are Variational Methods for Locally Lipschitz Functional and a Concentration Compactness Theorem for Orlicz space.

## Sign changing solutions for an asymptotically linear elliptic equation on $\mathbb{R}^N$

Liliane A. Maia, Ricardo Ruviaro  
UNB - Universidade de Brasília

We consider the nonlinear Schrödinger equation

$$-\Delta u + V(x)u = K(x)f(u) \text{ in } \mathbb{R}^N,$$

where  $\frac{f(s)}{s} \rightarrow a$  as  $|s| \rightarrow \infty$  and assume that  $V$  and  $K$  are invariant under an orthogonal involution. Moreover,  $V$  and  $K$  converge to positive constants  $V_\infty$  and  $K_\infty$ , as  $|x| \rightarrow \infty$ . We will present some results on the existence of a particular type of sign changing solution, which changes sign exactly once. The basic tool employed here is the Concentration-Compactness Principle and the interaction between translated solutions of the corresponding autonomous problem.

This is joint work with Ricardo Ruviaro (UnB, Brazil).

## Multiplicity and concentration of solutions for elliptic systems with vanishing potentials

Marcelo F. Furtado, E.A.B. Silva, M. Xavier  
UnB

In this talk we use variational methods to study the strongly coupled elliptic system

$$\begin{cases} -\Delta u + \lambda a(x)u = \frac{p}{p+q}|u|^{p-2}u|v|^q, \\ -\Delta v + \lambda b(x)v = \frac{q}{p+q}|u|^p|v|^{q-2}v, \\ u, v \in \mathcal{D}^{1,2}(\mathbb{R}^N), \end{cases}$$

where  $N \geq 3$ ,  $\lambda > 0$  is a parameter,  $p, q > 1$  and  $p + q < 2^* := 2N/(N - 2)$ . We suppose that the potentials are nonnegative and the intersection of the sets where they vanish has positive measure. A technical condition, imposed on the product of the potentials, allows us to consider a setting where we do not assume any positive lower bound for the potentials. Considering the associated functional, defined on an appropriated subspace of  $\mathcal{D}^{1,2}(\mathbb{R}^N) \times \mathcal{D}^{1,2}(\mathbb{R}^N)$ , we are able to establish results on the existence and multiplicity of solutions for the system when the parameter  $\lambda$  is sufficiently large. We also study the asymptotic behavior of these solutions when  $\lambda \rightarrow \infty$ .

## Existence and non-existence of solutions for $p$ -Laplacian equations with decaying cylindrical

Olimpio H. Miyagaki, Paulo C. Carrião, Reginaldo Demarque

Universidade Federal de Juiz de Fora

In this paper we deal with the problem

$$-\Delta_p u + \frac{|u|^{p_*(s)-2}u}{|y|^s} = |u|^{m-2}u, u \in D^{1,p}(\mathbb{R}^N; R)$$

where  $1 < p < N$ ,  $x = (y, z) \in R^k \times R^{N-k}$ ,  $\Delta_p$  is  $p$ -Laplacian operator,  $p_*(s) = \frac{p(N-s)}{N-p}$  and  $p^* = \frac{pN}{N-p}$ . Combining a version of the concentration compactness result by Solimini, Hardy-Sobolev type inequality with the Mountain Pass Theorem, existence of nontrivial solutions are obtained. Decay properties of these solutions are showed by applying Vassilev results. Pohozaev type identities are established in order to get non-existence results.

## Ground state solutions for a quasilinear equation involving critical exponential growth

Uberlandio Severo

Universidade Federal da Paraíba

In this talk, we will present a result of existence of ground state solutions for a class of quasilinear equations involving critical growth of Trudinger-Moser type in the whole space. For our results, we mainly use minimax methods combined with a compactness result.

## LINEAR PARTIAL DIFFERENTIAL EQUATIONS

**Organizer:** Paulo L. Dattori da Silva

### **Solvability near the characteristic set for a special class of complex vector fields**

**Evandro R. da Silva**, Paulo L. Dattori da Silva

ICMC - USP

In this lecture we will present results about solvability near the characteristic set  $\Sigma = 0 \times S^1$  of operators of the form  $L = \partial/\partial t + (x^n a(x) + ix^m b(x))\partial/\partial x$ ,  $b \neq 0$  and  $a(0) \neq 0$ , defined on  $\Omega_\epsilon = (-\epsilon, \epsilon) \times S^1$ ,  $\epsilon > 0$ , where  $a$  and  $b$  are real-valued smooth functions in  $(-\epsilon, \epsilon)$  and  $m \geq 2n$ .

We will show that given  $f$  belonging to a subspace of finite codimension of  $C^\infty(\Omega_\epsilon)$  there is  $u \in L^\infty$  solution of the equation  $Lu = f$  in a neighborhood of  $\Sigma$ ; moreover, the  $L^\infty$  regularity is sharp.

The results were obtained in a joint work with Professor Paulo L. Dattori da Silva (ICMC/USP).

### **Injectivity of polynomial homogeneous perturbation of the identity in $\mathbb{R}^3$**

**Francico Braun**, José R. dos S. Filho

UFSCar

We consider an analogous to the Jacobian conjecture for the real Euclidean space: "Let  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$  of the form

$$F(x) = x + H(x),$$

with  $H = (h_1, \dots, h_n)$  where  $h_j$  is homogeneous polynomial of degree  $m$ , for every  $j$ . If  $\det DF = 1$ , is  $F$  injective?"

We have developed some techniques to deal with this question. In the talk, we will present these techniques and partial results regarding the above question, for  $n = 3$ .

This is a joint work with J. R. dos Santos Filho (DM-UFSCar).

### **The Borel property in locally integrable structures**

**Rafael F. Barostichi**, Paulo D. Cordaro, Gerson Petronilho

Universidade Federal de São Carlos

In this lecture we present the Borel property for smooth solutions to systems of complex vector fields associated to locally integrable structures. Inspired by the recent article of B. Lamel and G. Della Sala, in which the Borel property was studied for generic submanifolds of the complex space, we prove similar results in this more general set up. In particular we obtain, for the case of corank one structures, a necessary and sufficient condition for the validity of the Borel property.

## NONLINEAR DYNAMICAL SYSTEMS

**Organizer:** Marcelo José Dias Nascimento

### **A non-autonomous Chafee-Infante problem**

**Alexandre N. Carvalho**, José A. Langa, James C. Robinson

ICMC - USP

The autonomous Chafee-Infante problem is most likely the best understood infinite dimensional dynamical system. In this lecture we reveal some interesting features of its non-autonomous counterpart. In particular we show that the same sequence of bifurcations of the autonomous case is also present in the non-autonomous case.

### **Non-autonomous Morse decomposition and Lyapunov functions for gradient-like processes**

**Éder R. Aragão-Costa**

ICMC - USP

We define Morse Decompositions (time dependent) for non-autonomous evolution processes (non-autonomous dynamical systems) and prove that a non-autonomous gradient-like evolution process possesses a Morse decomposition on the associated pullback attractor. We also prove the existence of an associated Lyapunov function which describes the gradient behavior of the system. Finally, we apply these abstract results to non-autonomous perturbations of autonomous gradient-like evolution processes (semigroups or autonomous dynamical systems).

To be more precise, we know that the Fundamental Theorem of Dynamical Systems (see [2]) describes the flow of autonomous reversible dynamical systems (groups) in compact metric spaces as a decomposition of an ordered family of isolated invariant sets and connections between them which respects their ordering. In the terminology of [2], this is called a Morse decomposition of a compact invariant set. The extension of this result to general autonomous dynamical systems (semigroups) is done in [1] (see also [3] for the Morse-Decomposition for semigroups in compact metric spaces). When trying to understand the structure of attractors or, more generally, of invariant sets for autonomous dynamical systems (or semigroups) the Morse-Decomposition plays a fundamental role. It allows us to decompose the dynamics into a gradient part and the dynamics in smaller isolated invariant sets reducing the study of the structure of the attractors to the study of the structure of these isolated invariant sets.

The aim of this presentation is show how to extend the notion of Morse-Decomposition to non-autonomous dynamical systems (or non-autonomous evolution processes) and to show the pullback attractors for these non-autonomous dynamical systems can be also decomposed as an ordered family of isolated invariant families and connections between them respecting their ordering. In particular, under some mild additional assumptions, we construct a non-autonomous Lyapunov function for the non-autonomous evolution process with a Morse-Decomposition. We show that this decomposition is observed for non-autonomous perturbations of gradient autonomous dynamical systems (or gradient semigroups). Some practical examples are also presented.

## References

- [1] E. R. Aragao-Costa, T. Caraballo, A. N. Carvalho and J.A. Langa, *Stability of gradient semigroups under perturbation*, Nonlinearity 24 (2011), 2099-2117.
- [2] C. Conley, *Isolated invariant sets and the Morse index*. CBMS Regional Conference Series in Mathematics, 38. American Mathematical Society, Providence, R.I. (1978).
- [3] K. P. Rybakowski, *The homotopy index and partial differential equations*, Springer-Verlag (1987).

## On a model for infiltrative tumour growth

Germán Lozada-Cruz, Jorge A. J. Avila  
UNESP / IBILCE / São José do Rio Preto

The dynamics of invasive growth tumor in one dimension is modeled by the following system of reaction-diffusion equations

$$\begin{cases} \frac{\partial a}{\partial t} = D_a \frac{\partial^2 a}{\partial x^2} - P(s)a + Ba, & -\infty < x < +\infty, t \geq 0 \\ \frac{\partial s}{\partial t} = D_s \frac{\partial^2 s}{\partial x^2} - qa, & -\infty < x < +\infty, t \geq 0 \end{cases} \quad (1)$$

subject to the boundary conditions

$$\begin{aligned} a = 0, \quad s = \sigma & \text{ when } x \rightarrow -\infty \\ a = 0, \quad s = 1 & \text{ when } x \rightarrow +\infty. \end{aligned} \quad (2)$$

Our goal in this work is to show that the problem (1) joint with boundary conditions (2) is well posed in a certain Banach space.

Joint work with Jorge A.J Avila (UFSJ, MG, Brazil).

## Approximate controllability of semilinear reaction diffusion equation

Hugo Leiva  
Universidad de Los Andes

In this paper we prove the approximate controllability of the following broad class of semilinear reaction diffusion equation in a Hilbert space  $Z$  given by

$$z' = -Az + B_1 u(t) + F(t, z, u(t)), \quad t \in [0, \tau],$$

where the control  $u \in L^2(0, \tau; U)$ , with  $U = Z$ ,  $B_1 : U \rightarrow Z$  is a linear and bounded operator (linear and continuous),  $A : D(A) \subset Z \rightarrow Z$  is an unbounded linear operator with the spectral decomposition  $Az = \sum_{j=1}^{\infty} \lambda_j \sum_{k=1}^{j_j} \langle z, \phi_{j,k} \rangle \phi_{j,k}$  and the nonlinear function  $F : [0, \tau] \times Z \times U \rightarrow Z$  is smooth enough and there are constants  $a, c \in \mathbb{R}$  such that

$$\sup_{(t,z,u) \in Z_\tau} \|F(t, z, u) - az - cB_2 u\|_Z < \infty,$$

where  $Z_\tau = [0, \tau] \times Z \times U$  and  $B_2 : U \rightarrow Z$  is a linear and bounded operator.

Under these conditions we prove the following statement: If the vectors  $(B_1^* + cB_2^*)\phi_{j,k}$  are linearly

independent in  $Z$ , then the nonlinear system is approximately controllable on  $[0, \tau]$ . Moreover, we exhibit a sequence of controls steering the system from an initial state to a  $\epsilon$ -neighborhood of final state in a prefixed time  $\tau$ . Our result can be apply to the semilinear  $nD$  heat equation, the Ornstein-Uhlenbeck equation, the Laguerre equation, the Jacobi equation, amount others.

## On $p(x)$ -Laplacian parabolic problems

Jacson Simsen

Universidade Federal de Itajubá- UNIFEI

In this talk, based at the papers "On  $p(x)$ -Laplacian parabolic problems authored by J. Simsen and M.S. Simsen" and "A global attractor for a  $p(x)$ -Laplacian parabolic problem authored by J. Simsen", I will show results on existence and uniqueness of a solution and existence of a global attractor for a  $p(x)$ -Laplacian parabolic problem.

## Vibrating beams models in atomic force microscopy

Julio R. Claeysen, Leticia Tonetto, Teresa Tsukazan

UFRGS

Elastic beams models that can include no-classical boundary conditions, multispan compatibility conditions and forcing are of importance in AFM-atomic force microscope. They are formulated in an unified matrix evolution equation which can be put in conservative or non-conservative form. The eigenanalysis of such beams is performed by using a basis generated by the initial value Green response of the corresponding modal equation which is very robust.

We discuss forced responses and frequency responses of the beam in terms of the distributed impulse response that is approximated by modal analysis.

## Perturbations in dynamical systems

Felipe Rivero

Universidad de Sevilla

In the framework of dynamical systems, the concept of attraction plays an important role in the study of their asymptotic behavior, defining a concrete object, that gives us relevant information about it, called the attractor. As any other mathematical object, the attractor needs to be robust under perturbations. In this talk we are going to see what kind of conditions we need to assume to obtain the continuity of the attractors, both in the autonomous and non-autonomous framework.

## Estimates on the fractal dimension of attractors for gradient-like semigroups

Matheus C. Bortolan, Alexandre N. Carvalho, José A. Langa, Tomás Caraballo

ICMC - USP

Is this talk our goal is to obtain an estimate on the fractal dimension of attractors for generalized gradient-like semigroups in general Banach spaces. We want to obtain these estimates in terms of the dimensions of the local unstable sets of isolated invariants, the Lipschitz constant of the semigroup and the exponential attraction rate of the attractor.

## Jensen's inequality and reaction-diffusion equations

Mikolaj Sierzega, James C. Robinson

Warwick, UK

I will present an application of the Jensen inequality to the problem of existence and regularity of solutions for a class of reaction-diffusion equations.

## Pullback attractors for three dimensional non-autonomous Navier-Stokes-Voight equations

Pedro Marín-Rubio, Julia García-Luengo, José Real

Universidad de Sevilla

In this talk we consider a non-autonomous Navier-Stokes-Voight model, to which a continuous process can be associated. We study the existence and relationship between minimal pullback attractors for this process in two different frameworks, namely, for the universe of fixed bounded sets, and also for another universe given by a tempered condition.

Since the model does not have a regularizing effect, to obtaining asymptotic compactness for the process is a more involved task. We prove this in a relatively simple way just by using an energy method. Our results simplify –and in some aspects generalize– some of those obtained previously for the autonomous and non-autonomous cases, since for example regularity is not required for the boundary of the domain and the force may take values in  $V'$ . Under additional suitable assumptions, regularity results for these families of attractors are also obtained, via bootstrapping arguments. Finally, we also conclude some results concerning the attraction in  $D(A)$  norm.

Work done in collaboration with Julia García-Luengo and José Real.

## Inflammatory phase dynamics in diabetic wounds: a PDE approach

Sergio M. Oliva Filho

IME - USP

The objective of the present paper is the modeling and analysis of the dynamics of macrophages and certain growth factors in the inflammatory phase, the first one of the wound healing process. It is the phase where there exists a major difference between diabetic and nondiabetic wound healing, an effect that we will consider in this paper. We will propose and analyze a partial differential equation as a model for the interaction of some of the crucial elements involved in this phase of the wound healing. This model will generalize a previous existing ODE model. The final model is a system of 3 PDE equations with nonlinear boundary conditions and we will show that it is a well posed problem both from a mathematical and biological point of view. More concretely, we will prove there exists a bounded invariant set where all the solutions are global and positive.

## **Bifurcation Analysis for the Lugiato-Lefever equation in two space dimensions**

**Tomoyuki Miyaji**, Isamu Ohnishi, Yoshio Tsutsumi

Kyoto University

We consider a nonlinear Schrödinger equation with cubic nonlinearity, damping, detuning and external force in two space dimensions. It is a model equation for pattern formation in nonlinear optics. Because of the damping term, it defines a weak dissipative system. We study the steady-state bifurcation of spatially homogeneous equilibrium point for the equation on square and hexagonal lattices within the space of periodic functions with respect to the lattice.



## ORDINARY AND FUNCTIONAL DIFFERENTIAL EQUATIONS

**Organizers:** Márcia Federson and Ma To Fu

### Stabilizability of linear distributed hereditary control systems

**Andréa Prokopczyk**, Hernán Henríquez, Eduardo Hernández

UNESP

In this work we study the stabilizability of the control system described by the equation

$$\begin{cases} x'(t) = Ax(t) + L(t)(x_t) + Bu(t), & t \geq 0, \\ x_0 = \varphi \in C([-r, 0]; X), \end{cases}$$

where  $r > 0$ ,  $x(t) \in X$ ,  $X$  is a Banach space,  $U(t) \in \mathbb{C}^n$ ,  $x_t : [-r, 0] \rightarrow X$  is defined by  $x_t(\theta) = x(t + \theta)$ ,  $A$  is the infinitesimal generator of a strongly continuous semigroup  $(T(t))_{t \geq 0}$  of bounded linear operators on  $X$ ,  $B : U \rightarrow X$  is a bounded linear map that represents the control action and, for each  $t \geq 0$ ,  $L(t) : C([-r, 0]; X) \rightarrow X$  is a bounded linear map such that  $t \mapsto L(t)\varphi$  is continuous for each  $\varphi \in C([-r, 0]; X)$ ,  $L(\cdot)$  is  $\omega$ -periodic, with  $\omega > r$ , and

$$L(t)\varphi = A_1(t)\varphi(-r) + \int_{-r}^0 \eta(t, \theta)\varphi(\theta)d\theta,$$

with  $\eta : [0, \infty) \times [-r, 0] \rightarrow \mathcal{L}(X)$  and  $A_1 : [0, \infty) \rightarrow \mathcal{L}(X)$  continuous functions for the norm of operators and  $\eta(t, \theta)$  of bounded variation on  $\theta$ .

### Vector fields whose linearization is Hurwitz almost everywhere

**Benito Pires**, Roland Rabanal

USP

A real matrix is Hurwitz if its eigenvalues have negative real parts. The following generalization of the Bidimensional Global Asymptotic Stability Problem (BGAS) is provided:

Let  $X : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a  $C^1$  vector field whose derivative  $DX(p)$  is Hurwitz for almost all  $p \in \mathbb{R}^2$ . Then the singularity set of  $X$ ,  $\text{Sing}(X)$ , is either an empty set, a one-point set or a non-discrete set. Moreover, if  $\text{Sing}(X)$  contains a hyperbolic singularity then  $X$  is topologically equivalent to the radial vector field  $(x, y) \mapsto (-x, -y)$ . This generalizes BGAS to the case in which the vector field is not necessarily a local diffeomorphism.

### Radially regulated functions of multiple variables

**Dana Frankova**

Institute of Mathematics, Academy of Sciences of Czech Republic

Radially regulated function is a function which has radial limits at each point. A function which has continuous inverse and has continuous radial limits is called radial shrink function (as an example, an increasing function in  $\mathbb{R}$ ). It is proven that a function in locally compact Banach space is radially regulated and its radial limits are continuous if and only if it is composition of a continuous function with a radial shrink function.

## Continuous dependence for a class of autonomous generalized ODEs

Eduard Toon

ICMC-USP

We shall discuss some results concerning continuous dependence for a class of autonomous generalized ODEs. To this end, we have to define the concept of an autonomous generalized ODE and the class we are interested in. Some other results will be discussed. This is a joint work with Márcia Federson. FAPESP support: 2009/06259-0

## A continuous dependence result for generalized linear differential equation: application on FDEs

Giselle A. Monteiro, Márcia Federson, Milan Tvrdý

Universidade de São Paulo

In [1], two continuous dependence results for integral equations in a Banach space  $X$  of the form

$$x(t) = \tilde{x} + \int_a^t d[A]x + f(t) - f(a), \quad t \in [a, b], \quad (\text{NH})$$

the so-called generalized linear differential equations, where  $-\infty < a < b < \infty$ ,  $\tilde{x} \in X$ ,  $A : [a, b] \rightarrow L(X)$  has a bounded variation on  $[a, b]$  and  $f : [a, b] \rightarrow X$  is regulated.

In 1966 of F. Oliva and Z. Vorel investigated the correspondence between functional differential equations and a class of generalized differential equations. Later, this subject also called the attention of M. Federson and P. Táboas (2003) and M. Federson and Š. Schwabik (2006). Using this relation, we apply our continuous dependence results to linear functional differential equations.

## References

[1] G. Monteiro, M. Tvrdý: *Generalized linear differential equations in a Banach space: Continuous dependence on a parameter*. Discrete and Continuous Dynamical Systems, to appear (2012).

## Nonperiodic and periodic averaging principles for various types of equations

Jaqueline G. Mesquita, Márcia Federson

Universidade de São Paulo

The purpose of averaging methods is to determine conditions under which the solutions of an autonomous differential system can approximate the solutions of a more complicated time varying system. The averaging method is therefore a powerful tool in studying the perturbation theory of differential equations, since it allows one to replace a time-varying small perturbation, acting on a long time interval, by a time-invariant perturbation and, in this process, only a small error is introduced.

In this work, we use a known correspondence between a class of solutions of generalized ODEs and a solution of other types of equations (for instance, measure functional differential equations with impulses, impulsive functional dynamic equations on time scales and others) to prove a periodic and non-periodic averaging method for these types of equations.

## Global stability of Cohen-Grossberg neural network with both time-varying and continuous distributed

José J. M. Oliveira

Universidade do Minho

In this talk, a generalized neural network of Cohen-Grossberg type with both discrete time-varying and distributed unbounded delays is considered. Based on M-matrix theory, sufficient conditions are established to ensure the existence and global attractivity of an equilibrium point. The global exponential stability of the equilibrium is also addressed but for the model with bounded discrete time-varying delays. A comparison of results shows that these results generalize and improve some earlier publications.

## Cauchy-Stieltjes integral on time scales and application

Luciano Barbanti, B.C. Damasceno, Márcia Federson

UNESP

Along with the consideration of discontinuous functions on Calculus in time scales (that is, with the time being considered more generally as a closed subset of  $\mathbb{R}$ ) it was necessary to define an integral of the Stieltjes type for deal within dynamical equations. In this case the usual Riemann-Stieltjes integral is insufficient for handling important situations. We consider here the Cauchy-Stieltjes integral both of the left and of the right sides, and will show its adequacy on the field. After by using this integral we will be doing some applications extending for example results by Krejci and Laurent on variational inequalities for play type operators in hysteresis.

## On a fourth order boundary value problem with $\phi$ -Laplacian

Ma To Fu

Universidade de São Paulo

This talk is concerned with a new class of fourth order two-point boundary value problems with second order perturbation of  $\phi$ -Laplacian type. Such kind of problems correspond to the steady state solutions of certain wave equations of Kirchhoff-Boussinesq models. Existence of positive solutions is established through the Krasnosel'skii fixed point theorem for cones of ordered spaces. A uniqueness result is also presented.

## Nonlinear oscillations in a mathematical model of an electronic circuit with a Memristor

Marcelo Messias, Marluce da C. Scarabello

FCT/ UNESP / Presidente Prudente

The *Memristor* is supposed to be the *fourth fundamental electronic element* in addition to the well-known Resistor, Inductor and Capacitor. Named as a contraction for *memory resistor*, its theoretical existence was postulated in 1971 by L. O. Chua, based on symmetrical and logical properties observed in some electronic circuits. However its physical realization was announced only in 2008 in a paper published in *Nature* by a research team from Hewlett-Packard Company. Such discovery has attracted worldwide attention due to the potential applications of Memristors, especially in the construction of the next generation computers. In this work we present a bifurcation analysis of a mathematical model

of an electronic circuit formed by the four basic elements: one Resistor, one Inductor, one Capacitor and one Memristor. The model is given by a 3-dimensional 5-parameter piecewise-linear system of ordinary differential equations. We show that the system presents a line of equilibrium points, whose stability change depending on the position of the equilibrium in the line. Moreover, depending on the parameter values the system presents nonlinear oscillations determined by the existence of infinitely many periodic orbits, which arise from the change in the stability of the equilibrium points on the mentioned line of equilibria. This phenomenon has an interesting similarity with the creation of a limit cycle in the well-known Hopf bifurcation, which will be commented in the talk.

## **Functional differential equations with Perron integrable right-hand sides and impulses**

**Márcia Federson**, Eduard Toon

Universidade de São Paulo

We recover Ralph Henstock's result which says that the right-hand side of a certain non-autonomous ODE can be written as the sum of a function of state and time which satisfies the classic Carathéodory's conditions and a function of time which is Perron integrable. Henstock also establishes conditions under which this ODE admits a solution. In our presentation, we consider functional differential equations with impulses and prove similar results. We also employ the theory of generalized ODEs to obtain a result on the uniqueness of a solution.

## FLUID DYNAMICS

**Organizer:** Anne Caroline Bronzi

### **Convergence of statistical solutions of the 3D Navier-Stokes- $\alpha$ model as $\alpha \rightarrow 0$**

**Anne C. Bronzi**, Ricardo M. S. Rosa

Universidade Federal do Rio de Janeiro

In this talk we consider statistical solutions of the 3D Navier-Stokes- $\alpha$  model with periodic boundary condition. We prove that under certain conditions statistical solutions of the 3D Navier-Stokes- $\alpha$  model converge to statistical solutions of the exact 3D Navier-Stokes equations as  $\alpha \rightarrow 0$ . The statistical solutions that we consider here arise from measures in suitable trajectory spaces, in a sense akin to that considered by Vishik and Fursikov.

### **Multi-scale interactions in the atmospheric dynamics**

**Carlos Raupp**

Universidade de São Paulo

In the large-scale atmospheric dynamics are two basic kinds of wave motions, namely the low-frequency Rossby waves that are characterized by an approximate geostrophic balance and the high frequency inertio-gravity waves (also called Poincaré waves), which depart considerably from geostrophic equilibrium and are thus mostly related to deep convective cells. Due to the large time-scale separation between these two wave types, these modes have been treated of completely different ways in numerical weather forecast models. While the Rossby modes are responsible for the evolution of synoptic scale weather disturbances, the Poincaré modes are in principle irrelevant for synoptic-scale weather forecast, being essentially considered as a noise in atmospheric general circulation models. In the midlatitude dynamics where baroclinic instability is the main energy source for large-scale atmospheric disturbances, this approach has shown to be reasonable and justifies the good skill of weather forecasts in the extratropics.

Nevertheless, in the tropical region the meridional temperature gradient associated with the solar forcing is weak and, consequently, the main energy source for synoptic and planetary scale wave disturbances is associated with clouds and moist convection through scale interactions. As a consequence, the atmospheric general circulation numerical models generally poorly represent both the diurnal cycle of tropical convection and the large to planetary scale convectively coupled tropical disturbances. In this context, in the present talk I will discuss about the possibility of an inertio-gravity mode to excite a Rossby mode in view of the wave interaction mechanisms and present a linear wave interaction theory involving equatorially trapped Rossby and Poincaré modes through the diurnal cycle of the background moisture field in the context of deep convection parameterization. The difficulty of atmospheric general circulation models in representing planetary scale wave disturbances in the tropics will also be discussed in view of the present theory.

Financial Support: FAPESP

## **Modeling of internal waves propagation in a two-fluids system over a variable topography**

**Daniel G. A. Vigo**, Ailin R. de Zarate, Andre Nachbin

Universidade Federal do Rio de Janeiro

We present a higher-order strongly nonlinear model to describe the evolution of large amplitude internal waves over arbitrary bathymetric variations, in a two-layer system configuration where the upper layer is shallow while the lower layer is comparable to the characteristic wavelength. This new system of nonlinear evolution equations with variable coefficients is a generalization of the deep water configuration model proposed by Choi and Camassa (J. Fluid Mech., (1999)) that takes into account the effects of a variable bottom topography. We used this model to study the effective behaviour of weakly nonlinear long internal waves propagating over a rapidly-varying topography, using a multiscale averaging technique. It is shown that the system reduces to an effective Intermediate Long Wave (ILW) equation.

## **The vortex-wave system with a finite number of vortices as the limit of the Euler- $\alpha$ equations**

**Eleonora Pinto de Moura**, Helena J. N. Lopes, Milton C. L. Filho

Universidade Federal do Rio de Janeiro

In this talk I will discuss solutions of the two-dimensional Euler- $\alpha$  equations when the initial vorticity is the superposition of a finite number of point vortices and a bounded background vorticity. Finally I will show that these solutions converge, as  $\alpha \rightarrow 0$ , to a weak solution of the vortex-wave system, introduced by Marchioro and Pulvirenti (1991).

## **A mathematical analysis of a model for phase transitions of thermoviscoelastic isochoric materials**

**José L. Boldrini**, Welington Vieira Assunção

Universidade Estadual de Campinas

We analyze a highly nonlinear system of partial differential equations used to model solid-liquid phase transitions in thermoviscoelastic isochoric materials with the possibility of motion during the process.

This system consists of an internal energy balance equation, governing the evolution of temperature, an evolution equation for the phase field, whose values determine the state of material, and a moment balance equation governing the material displacement. For this model, we prove existence and uniqueness of solutions.

## A Riemann solution for thermal multiphase flow in porous media

Julio D. Silva

IMPA

We consider a nonlinear system of conservation laws arising in petroleum engineering for a model that describes the injection of a mixture of gas and oil, in any proportion, into a porous medium filled with a similar mixture, at different temperatures.

We will focus on a particularly interesting and very strong degeneracy found in this model: the Riemann solution is given by a unique wave group in a full open set of Riemann data. This solution is constructed around an organizing structure: a singular point, intrinsically associated to most bifurcations in the Riemann solutions for this class of models.

## Solitons in in-situ combustion

Pablo Castañeda

IMPA

The *in-situ* combustion within a reservoir aids with the extraction of heavy oils. Small amounts of crude are burnt in the process which actually increases the recovery. However, how to maintain such behavior within the reservoir is not fully understood. A mathematical model was developed based upon a porous media chemical reactor with the intent to understand the characteristics required by the combustion process in order to sustain the reaction temperature. (The chemical reactor was presented in the ICMC 2010 Chapter.)

Our unconventional approach comes from a simple model where the reactor “moves” at constant speed. Nonetheless, velocities and reaction temperatures are found for any configuration of reservoir and injection temperatures. It is possible to find more than one solution for a single configuration; a stability analysis would reveal the behavior of such solutions.

A practical conclusion is the challenge to maintain the conditions for such combustion profiles to exist for waves expanding in two or three dimensions; keeping the initial combustion behavior requires that oxygen injection must increase with time!

## Mathematical analysis of a solidification problem for isochoric materials

Wellington V. Assunção

Universidade de Brasília

We consider a system of highly nonlinear partial differential equations modeling phase changes in isochoric materials with viscoelastic properties subject to thermal effects. This system features a balance equation for internal energy, governing the evolution of temperature, an evolution equation for the phase field parameter, whose values determine the state of material, and a moment balance equation governing the displacement.

In this talk, we establish results of existence, uniqueness and regularity of solutions for such system. Joint work with José Luiz Boldrini (IMECC-UNICAMP)

## COMPUTATIONAL DYNAMICS

**Organizer:** Marcio Gameiro

### **Sharp verified high-order enclosures of invariant manifolds of ODEs**

**Alexander Wittig**

IMPA (Brazil) and Michigan State University (USA)

A method to construct very accurate polynomial approximations of invariant manifolds of hyperbolic critical points in sufficiently smooth autonomous ordinary differential equations is presented. This construction is performed using differential algebra methods and also works in resonant cases as well as in arbitrary dimensions.

The constructed polynomial manifold approximation is then outfit with a tight remainder bound (typically of size  $10^{-12}$  or less), the correctness of which is verified using Taylor Model techniques. While the methods presented work in arbitrary dimension, special attention is given to the three dimensional case. The result is a very sharp, verified high-order enclosure of the local invariant manifolds. Using verified integration, these local manifold pieces can be integrated to obtain rigorous enclosures of large pieces of the invariant manifolds.

We show various examples of the use of the methods, including manifolds of the Lorenz ODE and other dynamical systems.

### **Time series analysis using topological computation method: a preliminary report**

**Hiroshi Kokubu**

Kyoto University (JST-CREST)

We propose a new approach to the analysis of time series generated by an unknown dynamical system. The method is based on a recently developed method of the Conley-Morse graphs [Z. Arai et al, SIADS 8 (2009)]. In the talk, we give an idea of the method along with a few test examples in order to show potential usefulness and limitation of the method. This is a joint work with Ippei Obayashi (Kyoto U).

### **Computer-assisted verification method for invariant measure and decay rate of correlation functions**

**Ippei Obayashi**

Kyoto University

In this talk, I will show you a new computer-assisted verification method for invariant densities and decay rates of correlations for one-dimensional expanding maps.

The result is published in Computer-Assisted Verification Method for Invariant Densities and Rates of Decay of Correlations Obayashi Ippei, SIAM J. Appl. Dyn. Syst. **10**, 788 (2011), DOI:10.1137/09077864X



## Forcing chaos in the Swift-Hohenberg equation via topologically validated numerics

Jan B. van den Berg, Jean-Philippe Lessard

VU University Amsterdam

The Swift-Hohenberg equation is a fourth order parabolic PDE that models certain aspects of pattern formation, such as the finite wavelength instability in Rayleigh-Bénard convection. We focus on stationary solutions of the equation in one dimension, and interpret solutions of this ODE as braided strands. Using a variational principle this leads to topological forcing results, analogous to the famous “period-3 implies chaos” for interval maps. On the other hand, recent advances in rigorous numerics allow us to topologically validate the existence of a special periodic solution. Through the forcing result, this periodic orbit is the seed for chaotic dynamics in the Swift-Hohenberg ODE for a large range of parameter values.

## Numerical computation of one parameter branches of (un)stable manifolds with rigorous error bounds

Jason D. M. James, Konstantin Mischaikow

Rutgers University

I will discuss a method for computing high order polynomial approximations of a branch of chart maps for stable/unstable manifolds in a one parameter family of analytic dynamical systems (discrete or continuous time). The method is based on the so called Parameterization Method for Invariant Manifolds. An important aspect of the method is that we can provide a-posteriori theorems which give rigorous bounds on the truncation errors. The hypotheses of the a-posteriori theorems are checked by a computer assisted argument. Examples and applications will be discussed as time permits.

## Rigorous ODE integrations using Taylor models

TBA

Michigan State University

A Taylor model of a smooth function  $f$  over a domain  $D$  is a pair  $(P, e)$ , where  $P$  is the Taylor polynomial of  $f$  at a point  $d$  in  $D$ , and  $e$  is an error bound such that  $f$  differs from  $P$  by not more than  $e$  over  $D$ . A calculus including addition, multiplication, various intrinsic functions and differentiation/integration is developed to compute Taylor models for code lists, resulting in a method to compute rigorous sharp enclosures of arbitrary computer functions in terms of Taylor models. Compared to common interval-based rigorous methods, this approach substantially reduces the so-called dependency problem, the main limiting factor of interval methods.

Besides the common elementary operations, the Taylor model representation lends itself naturally to the introduction of integration and differentiation as further intrinsics. This fact leads the method of Taylor models to rich applications to efficient rigorous integrations of ODEs and flows.

We review the development of the Taylor-model based integrator COSY-VI. Particular emphasis is given to the suppression of the infamous wrapping effect to very high accuracy while representing large ranges of initial conditions. We will discuss the technique of error parameterization, which not only further suppresses the growth of remainder errors but also provides guidance for automatic domain decomposition. The result is a rigorous integrator for long term integration of large ranges of initial conditions. Some examples will be shown to demonstrate the methods.

## Rigorous computation of smooth branches of solutions of PDEs

**Marcio Gameiro**

ICMC - USP

In this talk we present a rigorous numerical method that combines classical predictor corrector algorithms, analytic estimates and the uniform contraction principle to prove existence of global smooth branches of solutions of nonlinear PDEs. We provide explicit examples for PDEs defined on two- and three-dimensional spatial domains.

## Computer assisted proof of chaoticity of the Lorenz system for large ranges of parameters

**Martin Berz**

Michigan State University

The Lorenz system is one of the important showcases of dynamical systems. However, while the dynamics appears chaotic for large ranges of parameters, rigorous proofs of these conjectures have so far been possible only for isolated values of parameters. We study heuristically determined parameter dependent topological rectangles on a fixed surface that appear to lead to horseshoe crossings in the second return to the surface. Using the verified integrator COSY-VI, the parameter dependent boundaries of the rectangles are transported to the second return and projected to the Poincare surfaces, and used for the verification of the Markov crossing property. Utilizing the  $>$  continuity of the second return map, this allows to show chaoticity a range of  $\rho$  from about 25 to 95.

## Characteristic algorithms for the solution of the Boltzmann equation

**Nilson C. Roberty**, Marcelo L. S. Rainha

UFRJ

We introduce algorithms marching over an polygonal mesh with elements consistent with the propagation directions of the particle (radiation) flux. The decision for adopted this kind of mesh to solve the one-speed Boltzmann transport equation is due to characteristics of domain of the transport operator which controls derivatives only in the direction of propagation of the particles (radiation) flux in the absorbing and scattering media. This a-priori adaptivity has the advantages that it formulates an consistent scheme which makes appropriated the application of the Lax equivalence theorem framework to the problem. In this work we present the main functional spaces involved in the formalism and a description of the algorithms for the mesh generation and the transport equation solution. Some numerical examples related with the solution of an transmission problem in a high contrast model with absorption and scattering are presented. Also, a comparison with benchmarks problems for source and reactor criticality simulations shows the compatibility between calculations with the algorithms here proposed and theoretical results.

## Automatic computation of global dynamics for flows

Paweł Pilarczyk

Universidade do Minho, Braga, Portugal

A method for automatic computation of global dynamics for flows will be introduced, which also allows to classify this dynamics over a few varying parameters. The core idea behind this approach is to work with the time- $t$  map of the flow with a suitably chosen  $t > 0$ , and to apply the method for the classification of global dynamics for maps, introduced in [1]. This method is based on algorithmic decomposition of the phase space into chain recurrent components, such that in the remaining part of the space the dynamics is gradient-like, introduced in [2]. In this way, a Morse decomposition [3] of the space is constructed. This decomposition is computed using a uniform rectangular grid in a bounded rectangular box of the phase space, to which the analysis of the dynamics is restricted.

The correspondence between the isolating neighborhoods, the Conley indices, and the isolated invariant sets found for the time- $t$  map and the respective structures for the underlying flow is guaranteed by several theorems, including Theorem 1 and Corollary in [4] and Lemma 6 in [5].

### References

- [1] Z. Arai, W. Kalies, H. Kokubu, K. Mischaikow, H. Oka and P. Pilarczyk. *A database schema for the analysis of global dynamics of multiparameter systems*. SIAM J. Appl. Dyn. Syst. **8** (2009), 757–789.
- [2] H. Ban and W. Kalies. *A computational approach to Conley's decomposition theorem*. Journal of Computational and Nonlinear Dynamics **1** (2006), 312–319.
- [3] C. Conley. *Isolated invariant sets and the Morse index*, CBMS Regional Conference Series in Mathematics Vol. 38, American Mathematical Society, Providence, R.I., 1978.
- [4] M. Mrozek. *The Conley Index on Compact ANR's is of Finite Type*. Results in Mathematics **18** (1990), 306–313.
- [5] P. Pilarczyk. *Computer assisted method for proving existence of periodic orbits*. Topol. Methods Nonlinear Anal. **13** (1999), 365–377.

## Dynamics of the area preserving Henon map at 3:1 resonance

Piotr Zgliczynski, Carles Simo, Tomasz Kapela

Jagiellonian University, Kraków, Poland

We investigate the following area preserving Henon map

$$h(x, y) = R_\alpha(x, y - x^2), \quad \alpha = 2\pi/3 \quad (1)$$

where  $R_\alpha$  is a rotation by  $\alpha$ . We focus on  $\alpha = 2\pi/3$  case, the 3:1 resonance.

I will discuss proofs of

- the existence of periodic islands
- the existence of the symbolic dynamics
  - and the existence of the “hyperbolic” set with some interesting properties:
    - some points have zero Liapunov exponent, some nonzero and for others points the limit defining Liapunov exponent oscilates between zero and some nonzero value.
- Gevrey character of the branches of unstable set for the resonant fixed point

## Computing homology via discrete Morse theory

Shaun Harker, Konstantin Mischaikow, Marian Mrozek

Rutgers University

Homology computation of a computer-represented chain complex can be expensive, primarily due to the cost of finding the Smith Normal Form (SNF) of a matrix. We show how one can significantly reduce the size of a chain complex by preprocessing it with Discrete Morse Theory, producing a reduced Morse complex. The particular discrete Morse functions we construct are derived from the central idea of Marian Mrozek's "Coreduction Homology Algorithm" paper, thus making this approach a direct generalization. The cost of SNF on the reduced complex can be dramatically smaller, easily justifying the cost of our preprocessing in a many practical settings. Additionally, we may compute homology generators by "lifting" the homology generators computed in the reduced complex back into the original complex. Consequently, this algorithm is suitable for use in the computation of the homology of maps.

*ICMC Summer Meeting on  
Differential Equations  
2012 Chapter*

Posters



## Limit solution for the Chern-Simons system

Adilson E. Presoto, Augusto C. Ponce  
UFSCar

In the paper [1], Lin, Ponce and Yang studied and developed some important results concerning the nonlinear elliptic system

$$\begin{cases} -\Delta u + e^v(e^u - 1) = \mu \\ -\Delta v + e^u(e^v - 1) = u \end{cases} \text{ in } \mathbb{R}^2,$$

where  $\mu$  and  $u$  are Radon measures. The existence of solutions for the system above can be obtained as the limit of solutions for bounded subsets  $\Omega \in \mathbb{R}^2$ ,

$$\begin{cases} -\Delta u + e^v(e^u - 1) = \mu \\ -\Delta v + e^u(e^v - 1) = u \\ u = v = 0 \end{cases} \text{ in } \Omega, \quad (2)$$

In their work, the authors investigated a condition on the measures  $\mu$  and  $u$  that ensures that (2) has a solution:

**Theorem:** *Let  $\mu$  and  $u$  be Radon measures. Then (2) has a solution if, and only if,  $\mu(x) + u(x) \leq 4\pi$ .*

We are particularly interested in better understanding the lack of solution when the condition above is not satisfied. For this purpose, we approach the measures by smooth functions in some weak sense. If the solutions of the approximate problems converge, we wonder what Chern-Simons system the limit solution will satisfy. In a crucial step, we use some results of the same approach method applied to equation

$$\begin{cases} -\Delta u + e^u - 1 = \mu \\ u = 0 \end{cases} \text{ in } \Omega, \text{ on } \partial\Omega,$$

which is addressed in the paper [3] and were presented in V ENAMA.

## References

- [1] Lin, Chang-Shou, Ponce, Augusto C, and Yang, Yisong - *A system of Elliptic Equations Arising in Chern-Simons Field Theory*. Journal of Functional Analysis., 289-350, 2007.
- [2] Brezis, H, Marcus, M, and Ponce, A. C. - *Nonlinear elliptic equations with measures revisited*. Annals of Mathematics Studies., 55-110, 2007.
- [3] Ponce, A. C., and Presoto, A. E. - *Limit Solutions for a Semilinear Elliptic Equation with Exponential Nonlinearity*, To appear..

## Singularly perturbed biharmonic problems with superlinear nonlinearities

Marcos T. de O. Pimenta, Sérgio H. M. Soares

Universidade Estadual de Londrina - UEL

We are interested in finding a family of solutions to a singularly perturbed biharmonic equation which has a concentration behavior. The proof is based on variational methods and it is used a weak version of the Ambrosetti-Rabinowitz condition.

## Asymptotic behavior of solutions to linear neutral delay

Patricia H. Tacuri, Miguel V. S. Frasson

ICMC-USP

We study the asymptotic behavior of the solutions of a class of linear neutral delay differential equations (NDDE) with discrete delay where the coefficients of the non neutral part are periodic functions which are rational multiples of all time delays. We show that this technique is applicable to a broader class where the coefficients of the neutral part are periodic functions as well.

## Klein-Gordon-Maxwell equations without Ambrosetti-Rabinowitz condition

Patrícia L. Cunha, Sérgio H. M. Soares

Universidade de São Paulo

We introduce a general nonlinearity in the nonlinear Klein-Gordon-Maxwell equations which does not require the usual Ambrosetti-Rabinowitz condition. Based on variational methods we prove the existence of positive and ground state solutions.

## Exponential trichotomies and continuity of invariant manifolds

Severino H. da Silva, Antônio L. Pereira

Universidade Federal de Campina Grande

In this work, we consider the invariant manifolds for the family of equations

$$\dot{x} = Ax + f(\varepsilon, x),$$

where  $A$  is the generator of a strongly continuous semigroup of linear operators in a Banach space  $X$  and  $f(\varepsilon, \cdot) : X \rightarrow X$  is continuous. The existence of stable (unstable) and center-stable (center-unstable) manifolds for a large class of these equations has been proved in [?]. We prove here that, if  $A$  admits an exponential trichotomy and  $f$  satisfies some suitable regularity hypotheses, then those manifolds are continuous with respect to the parameter  $\varepsilon$ .



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## **Boundedness of solutions of retarded FDEs with variable impulses via GODEs**

**Suzete M. Afonso**, Everaldo M. Bonotto, Márcia Federson, Luciana P. Gimenes

UNESP

In this work, we give sufficient conditions for the uniform boundedness and uniform ultimate boundedness of solutions of a class of retarded functional differential equations with impulse effects acting on variable times. We employ the theory of generalized ordinary differential equations to obtain our results.



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Programme



	SUNDAY 05	MONDAY 06	TUESDAY 07	WEDNESDAY 08
Anacã Hotel	<b>Registration</b> 17:00-18:00			
Reception lounge	08:00-08:20	<b>Registration</b>		
		<i>Chairman: Hildebrando M. Rodrigues</i>	<i>Chairman: Tibor Krisztin</i>	<i>Chairman: Jerome A. Goldstein</i>
Auditorium	08:20-09:00	<b>Opening</b>	Jerome A. Goldstein	Djairo G. de Figueiredo
Auditorium	09:00-09:40	Shui-Nee Chow	José M. Arrieta	Alfonso Castro
Auditorium	09:40-10:20	Martin Bohner	Tomás Caraballo	Boyan Sirakov
Library Ground Floor	10:20-10:50	Coffee break	Coffee break	Coffee break
		<i>Chairman: Tomás Caraballo</i>	<i>Chairman: Konstantin Mischaikow</i>	<i>Chairman: Bernold Fiedler</i>
Auditorium	10:50-11:30	Konstantin Mischaikow	Bernold Fiedler	Tibor Krisztin
Auditorium	11:30-12:10	Yingfei Yi	Carlos Tomei	Marco A. Teixeira
	12:10-14:00	Lunch	Lunch	Lunch
		<i>Chairman: Alfonso Castro</i>	<i>Chairman: José M. Arrieta</i>	<i>Chairman: Liliame A. Maia</i>
Auditorium	14:00-14:30	Jan Cholewa	José Valero	Gustavo A. Perla Menzala
Auditorium	14:30-15:00	Claudianor O. Alves	Andras Ronto	Benedetta Pellacci
Auditorium	15:00-15:30	Denis Bonheure	José Langa	João M. B. do Ó
Library ground floor	15:30-16:00	Coffee break	Coffee break	Coffee break
<b>Special Session on Elliptic Equations</b>				
		<i>Chairman: Claudianor Alves</i>	<i>Chairman: Marcelo Furtado</i>	<i>Chairman: Giovany Figueiredo</i>
Auditorium	16:00-16:30	Olimpio Miyagaki	Liliane Maia	Angelo R. Furtado de Holanda
Auditorium	16:30-17:00	Giovany Figueiredo	Francesca Dalbono	Jefferson Abrantes dos Santos
Auditorium	17:00-17:30	Marcelo Furtado	Humberto Quoirin	Everaldo Souto de Medeiros
Auditorium	17:30-18:00	Gustavo Ferron Madeira	Uberlandio Batista Severo	Edcarlos Domingos da Silva
Auditorium	18:00-18:30	Carlos Alberto Santos		
<b>Special Session on Ordinary and Functional Diff. Equations</b>				
		<i>Chairman: Marcelo Messias</i>	<i>Chairman: Luciano Barbanti</i>	<i>Chairman: Dana Frankova</i>
Room 5001	16:00-16:30	Luciano Barbanti	Giselle Antunes Monteiro	Benito Frazão Pires
Room 5001	16:30-17:00	Jaqueline Godoy Mesquita	<i>Dana Frankova</i>	Marcelo Messias
Room 5001	17:00-17:30	Andréa Cristina Prokopczyk Arita	Ma To Fu	Eduard Toon
Room 5001	17:30-18:00	José Joaquim Martins Oliveira	Márcia Federson	

**Special Session on Nonlinear Dynamical Systems**

		<i>Chairman:</i> <i>Pedro Marin-Rubio</i>	<i>Chairman:</i> <i>Gustavo A. Perla Menzala</i>	<i>Chairman:</i> <i>Sergio Oliva</i>
Room 5003	16:00-16:30	Mikolaj Sierzega	Julio C. Ruiz Claeysen	Pedro Marín-Rubio
Room 5003	16:30-17:00	Hugo Leiva	Sergio Oliva	Luis Felipe Rivero Garvía
Room 5003	17:00-17:30	German Lozada Cruz	Tomoyuki Miyaji	Jacson Simsen
Room 5003	17:30-18:00	Matheus Cheque Bortolan	Éder Ritis Aragão Costa	Alexandre Nolasco de Carvalho

**Special Session on Fluid Dynamics**

		<i>Chairman:</i> <i>Anne Caroline Bronzi</i>	<i>Chairman:</i> <i>Eleonora Pinto de Moura</i>
Room 5101	16:00-16:30	Carlos F. Mendonça Raupp	Jose Luiz Boldrini
Room 5101	16:30-17:00	Eleonora Pinto de Moura	Daniel Gregorio Alfaro Vigo
Room 5101	17:00-17:30	Julio Daniel Machado Silva	Anne Caroline Bronzi
Room 5101	17:30-18:00	Wellington Vieira Assunção	Pablo Castañeda

**Special Session on Computational Dynamics**

		<i>Chairman:</i> <i>Marcio Gameiro</i>	<i>Chairman:</i> <i>Hiroshi Kokubu</i>	<i>Chairman:</i> <i>Pawel Pilarczyk</i>
Room 5004	16:00-16:30	Hiroshi Kokubu	Piotr Zgliczynski	Nilson Costa Roberty
Room 5004	16:30-17:00	Pawel Pilarczyk	Jan Bouwe van den Berg	Kyoko Makino
Room 5004	17:00-17:30	Martin Berz	Jason D Mireles James	Alexander Wittig
Room 5004	17:30-18:00	Shaun Harker	Ippei Obayashi	Marcio Gameiro

**Special Session on Linear Equations**

		<i>Chairman:</i> <i>Paulo L. Dattori</i>
Room 5103	16:00-16:30	Rafael Fernando Barostichi
Room 5103	16:30-17:00	Evandro Raimundo Da Silva
Room 5103	17:00-17:30	Francisco Braun

**Poster Session**

<b>Library ground floor</b>	10:20 - 10:50 and 15:30 - 16:00	Adilson Eduardo Presoto Marcos T. O. Pimenta Patricia Hilario Tacuri Patrícia Leal da Cunha Severino Horácio da Silva Suzete Maria Silva Afonso
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**Social Events**

<b>Library ground floor</b>	18:30	Cocktail
<b>Anacã Hotel</b>	20:30	Conference Banquet

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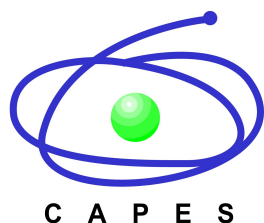
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