Second order Moser type inequalities: a borderline case

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Abstract. We study optimal embeddings for the space of functions whose Laplacian Δu belongs to $L^1(\Omega)$, where $\Omega \subset R^N$ is a bounded domain. This function space turns out to be strictly larger than the Sobolev space $W^{2,1}(\Omega)$ in which the whole set of second order derivatives is considered. In particular, in the limiting Sobolev case, when $N = 2$, we establish a sharp embedding inequality into the Zygmund space $L_{exp}(\Omega)$. On one hand, this result enables us to improve the regularity estimate of Brezis–Merle for the Dirichlet problem $\Delta u = f(x) \in L^1(\Omega)$, $u = 0$ on $\partial\Omega$; on the other hand, it represents a borderline case of D.R. Adams' generalization of Trudinger-Moser type inequalities to the case of higher order derivatives.