Multiple solutions for elliptic equations with singularities

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Abstract. In this work we prove a multiplicity result for a class of quasilinear elliptic equation involving the subcritical Hardy-Sobolev exponent, and singularities both in the operator and in the nonlinearity. Precisely, we study the problem

$$\begin{cases} -\operatorname{div}\left[|x_N|^{-ap}|\nabla u|^{p-2}\nabla u\right] + \lambda |x_N|^{-(a+1-c)p}|u|^{p-2}u \\ \qquad = |x_N|^{-bq}|u|^{q-2}u + f \qquad \text{in } \mathbb{R}^N_+ \\ \quad u = 0 \qquad \text{on } \partial\mathbb{R}^N_+ \end{cases}$$

where we denote $x = (x_1, x_2, \ldots, x_N) = (x', x_N) \in \mathbb{R}^{N-1} \times \mathbb{R}$, $\mathbb{R}^N_+ = \left\{x \in \mathbb{R}^N : x_N > 0\right\}$, $\partial\mathbb{R}^N_+ = \left\{x \in \mathbb{R}^N : x_N = 0\right\}$, and we consider $1 < p < N$, $0 \leqslant a < (N-p)/p$, $a < b < a+1$, $c = 0$, $d \equiv a+1-b$, $q = q(a,b) \equiv Np/(N-pd)$ (the Hardy-Sobolev critical exponent), $\lambda \in \mathbb{R}$ is a parameter, and $f \in \left(L^q_b(\mathbb{R}^N_+)\right)^*$, the dual space of the weighted Lebesgue space. We prove an existence result for the case $f \equiv 0$ and a multiplicity result in the case $\lambda = 0$ for non autonomous perturbations $f \not\equiv 0$.