C^k-solvability near the characteristic set of a class of complex vector fields

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Abstract. Let

$$\mathcal{L} = \partial/\partial t + x^r(a+ib)(x,t)\partial/\partial x,$$

be a complex vector field defined on $\Omega_\epsilon = (-\epsilon, \epsilon) \times S^1$, $\epsilon > 0$, where a and b are C^∞ real-valued functions. Suppose that $\Sigma = \{0\} \times S^1$ is the characteristic set of $\mathcal{L}$. Let $f \in C^\infty(\Omega_\epsilon)$ satisfying

$$\int_0^{2\pi} \frac{\partial^{(j)} f}{\partial x^j}(0,t)dt = 0, \quad j = 0, \cdots, r-1.$$

In this lecture we are going to talk about existence of C^k solutions for the equation $\mathcal{L}u = f$ in a full neighborhood of Σ, when $\mathcal{L}$ satisfies some additional hypothesis:

• when $r = 1$ and $t \mapsto b(0,t) \neq 0$, $\forall t \in S^1$, we will show that the equation $\mathcal{L}u = f$ has a C^k solution defined in some neighborhood of Σ, for any $k \geq 1$;

• when $r = 2$ we will consider that $a + ib$ depends only on x-variable. We will show that the interplay between the order of vanishing of a and b has influence on the C^k-solvability.

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