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2007 Chapter

Mini-course and List of Abstracts

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Contents

MINI-COURSE

by

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Lecture 1.
Bifurcation and continuous transitions of attractors in autonomous and nonautonomous systems

Nonautonomous bifurcation theory studies the change of attractors of nonautonomous systems which are introduced here with the process formalism as well as the skew product formalism.

We present a total stability theorem ensuring the existence of nearby attractors of perturbed systems. They depend continuously on a parameter if and only if the attraction is uniform w.r.t. parameter, i.e. the attractors are equi-attracting.

We apply these principles to explicit systems to clarify the meaning of continuous and abrupt transitions of attractors in contrast to bifurcations, i.e. splitting of minimal invariant subsets into others within the attractor. Several examples are treated, including a nonautonomous pitchfork bifurcation.

References

[1] P.E.Kloeden and S. Siegmund, Bifurcation and continuous transition of attractors in autonomous and nonautonomous systems, Inter. J. Bifurcation & Chaos. 15 (2005), 743-462.

Lecture 2.
Perturbation of attractors of skew-product difference systems witha shadowing driving system

The influence of the driving system on a skew-product flow generated by a triangular system of difference or differential equations can perturbed in two ways, directly by perturbing the vector field of the driving component itself or indirectly by perturbing its input variable in the vector field of the coupled component. The effect of such perturbations on a nonautonomous attractor of the driven component is discussed here. In particular, it is shown that a perturbed nonautonomous attractor with nearby components exists in the indirect case if the drievn sysetm has an inflated nonautonomous attractor and that the direct case can be reduced to this case if the driving system is shadowing.

References

[1] P.E. Kloeden and V.S. Kozyakin, The perturbation of attractors of skew-product flows with a shadowing driving system. Discrete & Continuous Dynamical Systems, 7(2001), 883-893.

Lecture 3.
The persistence of synchronization under environmental noise.

It is shown that the synchronization of dissipative systems persists when they are disturbed by additive noise no matter how large the intensity of the noise provided asymptotically stable stationary stochastic solutions are used instead of asymptotically stable equilibria

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[1] T. Caraballo and P.E. Kloeden, The persistence synchronization under environmental noise. Proc. Roy. Soc. London. A461 (2005), 2257–2267.

LIST OF ABSTRACTS

Existence of periodic solution for a class of system involving nonlinear wave equations

by

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In this conference, we show the existence of $2j\pi$-periodic solutions for the following class of system

$$(S)\qquad \begin{cases} u_{tt}-u_{xx}=v|v|^{p-2}, & x\in(0,\pi), t\in I\!R & (i)\\ v_{tt}-v_{xx}=u|u|^{q-2}, & x\in(0,\pi), t\in I\!R & (ii)\\ u(0,t)=u(\pi,t)=0, & t\in I\!R & (iii)\\ u(x,t+2j\pi)=u(x,t), & x\in[0,\pi] \text{ and } t\in I\!R & (iv)\\ v(0,t)=v(\pi,t)=0, & t\in I\!R & (v)\\ v(x,t+2j\pi)=v(x,t), & x\in[0,\pi] \text{ and } t\in I\!R & (vi) \end{cases}$$

where $2<p,q<+\infty$. The main tools used is a duality argument of Clarke & Ekeland in conjunction with the mountain pass theorem of Ambrosetti & Rabinowitz.

Stability of periodic travelling wave solutions for the modified Boussinesq equation

by

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This talk is concerned with nonlinear stability properties of periodic travelling wave solutions of the modified Boussinesq equation

$$u_{tt}=u_{xx}-(u^3+u_{xx})_{xx},\quad x,t\in\mathbb{R}.$$

It is shown that the special *dnoidal wave solutions* are nonlinearly stable in the space $H^1_{per}([0,L])\times L^2_{per}([0,L])$ for a range of their speeds of propagation.

Subcritical perturbations of a singular quasilinear elliptic equation involving the critical Hardy-Sobolev exponent

by

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In this work we improve some known results for a class of singular quasilinear elliptic operators involving the critical Hardy-Sobolev exponent and also for a wide class of lower-order terms by proving a multiplicity result. Precisely, we consider the problem

$$\begin{cases} -\mathrm{div}\left[|x|^{-ap}|\nabla u|^{p-2}u\right] = |x|^{-bq}|u|^{q-2}u + g(x,u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geqslant 3)$ is a bounded domain containing the origin with smooth boundary $\partial\Omega$, $0 \leqslant a < (N-p)/p$, $a < b \leqslant a+1$, $d \equiv a+1-b$, and $q \equiv Np/[N-p(a+1-b)]$ is the critical Hardy-Sobolev exponent. The perturbation term $g(x,s)$ can change sign and has subcritical growth at infinity, that is, it holds

$$\lim_{|s|\to\infty} \frac{g(x,s)}{|x|^{-bq}|s|^{q-1}} = 0$$

uniformly with respect to $x \in \Omega$.

The proof of existence of nontrivial solutions for problem (1) under appropriate hypotheses is made by applying the generalized mountain-pass theorem due to Ambrosetti and Rabinowitz. To do this, we show that the minimax levels are in a convenient range by combining a special class of approximating functions, due to Gazzola and Ruf, with the concentrating functions of the best Sobolev constant.

Extremal Functions for Sharp Sobolev Inequalities: The Nonconstante Positive Scalar Curvature Case

by

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Sharp first-order L^2-Sobolev inequalities on compact Riemannian manifolds have been extensively studied in the literature and surprising results have been obtained by showing the influence of the geometry on such problems. For example, if the scalar curvature is either nonpositive or constant, then the exixtence of extremal functions for sharp first-order L^2-Sobolev inequalities always holds. However, if the scalar curvature is nonconstant and positive , the existence of extremal functions is an open question.

We present a method for to build examples of compact Riemannian manifolds with nonconstant positive scalar curvature such that first-order L^2-Sobolev inequalities possesses extremal functions. For instance, we have the following result:
let (M,g) be a compact Riemannian manifold of dimension $n \geq 4$ non-conformal to standard n-sphere (S^n, g_0) and $a \in C^\infty(M)$. Supose that $\mu_g > 0$ and

$$0 < \max_M a(x) < \frac{1}{K(n,2)^2 \mu_g} \min_M a(x),$$

where $\mu_g > 0$ is the Yamabe invariant. Then exists a metric h conformal to g such that $Scal_h = a$ and $(J^2_{h,opt})$ possesses an extremal function.

One easily may construct concrete examples of such manifolds. For example, take the product $S^1 \times S^{n-1}$ or the standard real projective space P^n, and a nonconstant positive function $\overline{a} \in C^\infty(M)$ such that

$$\overline{a}(x) < \frac{1}{K(n,2)^2} - \mu_g$$

for all $x \in M$. Clearly, the function $a = \overline{a} + \mu_g$ satisfies the condition of the result above. Our approach is a combination of prescribed scalar curvature's arguments and results on the Yamabe problem. For more details, see [1].

References

[1] E. R. Barbosa, M. Montenegro, Prescribed scalar curvature and extremal functions on conformal deformations. Preprint

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Bifurcation of Periodic Solutions for Delay Differential Equations on Compact Manifolds

by

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We study the nonlinear delay differential equation

$$\dot{x}(t) = \lambda f(t, x(t), x(t-1)), \quad \lambda \geq 0,$$

the following assumptions: given a smooth manifold (possibly with boundary) bedded in $\mathbb{R}^k$, $f : \mathbb{R} \times M \times M \to \mathbb{R}^k$ is a continuous map, T-periodic in the first variable and tangent to M in the second one; that is, $f(t+T, p, q) = f(t, p, q) \in T_pM$ r all $(t, p, q) \in \mathbb{R} \times M \times M$, where $T_pM \subseteq \mathbb{R}^k$ denotes the tangent space of M at p. Using a topological approach based of the fixed point index we obtain global furcation results for periodic solutions of the above problem.

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[2] P. Benevieri, A. Calamai, M. Furi, M.P. Pera, *On Forced Fast Oscillations for Forced lay Differential Equations on Compact Manifolds*, submitted.

Uniform Stabilization of the wave equation on compact surfaces and locally distributed damping

by

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This paper is concerned with the study of the wave equation on compact surfaces and locally distributed damping, described by

$$u_{tt} - \Delta_{\mathcal{M}} u + a(x)\, g(u_t) = 0 \quad \text{ on } \mathcal{M} \times]0, \infty[\,,$$

where $\mathcal{M} \subset \mathbb{R}^3$ is an oriented embedded compact surface without boundary, such that $\mathcal{M} = \mathcal{M}_0 \cup \mathcal{M}_1$, where

$$\mathcal{M}_1 := \{x \in \mathcal{M}; m(x) \cdot \nu(x) > 0\} \text{ and } \mathcal{M}_0 = \mathcal{M} \backslash \mathcal{M}_1.$$

Here, $m(x) := x - x^0$, ($x^0 \in \mathbb{R}^3$ fixed) and ν is the unit normal vector towards $\mathcal{M}$.

Assuming that the part $\mathcal{M}_0$ of $\mathcal{M}$ is *sectionally umbilical* or, more generally, that the principal curvatures k_1 and k_2 satisfy $|k_1(x) - k_2(x)| < \varepsilon_i$ (ε_i considered small enough) for all $x \in \mathcal{M}_{0i}$, $i = 1, \cdots, k$, where, $\cup_{i=1}^{k} \mathcal{M}_{0i} \supset \mathcal{M}_0$ and, moreover that the *mean curvature* H of $\mathcal{M}_0$ is *non-positive* (i.e. $H \leq 0$ on $\mathcal{M}_0$), then, supposing that $a(x) \geq a_0 > 0$ on a neighborhood $\mathcal{M}_*$ of $\mathcal{M}$ which contains $\overline{\mathcal{M}_1}$ strictly and, in addition, that g is a monotonic increasing function such that $k|s| \leq |g(s)| \leq K|s|$ for all $|s| \geq 1$, uniform decay rates of the energy are proved.

Wellposedness and optimal decay rates for wave equation with nonlinear boundary damping-source interaction

by

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We establish, subject to some natural additional assumptions imposed on the relation between the source and the damping, both wellposedness and effective optimal decay rates for the solutions of a semilinear model of the wave equation. The theory presented allows to consider both superlinear and sublinear behaviour of the dissipation in the presence of unstructured sources. In addition, finite time blow-up phenomenon is exhibited for finite energy solutions.

The solvability of Dirichlet problem for a class of degenerate elliptic equations with L^1-data

by

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Let L be a degenerate elliptic operator in divergence form

$$Lu(x) = -\sum_{i,j=1}^{n} D_j\big(a_{ij}(x) D_i u(x)\big), \quad \text{with} \quad D_j = \frac{\partial}{\partial x_j}, \tag{2}$$

where the coefficients a_{ij} are measurable, real-valued functions defined on a bounded open set $\Omega \subset \mathbb{R}^n$, and whose coefficient matrix $\mathbf{A}(x) = (a_{ij}(x))$ is symmetric and satisfies the degenerate ellipticity condition

$$\lambda |\xi|^2 \omega(x) \leq \sum_{i,j=1}^{n} a_{ij}(x) \xi_i \xi_j \leq \Lambda |\xi|^2 \omega(x) \tag{3}$$

for all $\xi \in \mathbb{R}^n$ and almost everywhere $x \in \Omega$, ω is a weight function (i.e., a locally integrable function on $\mathbb{R}^n$ such that $\omega(x) > 0$ for a.e. $x \in \mathbb{R}^n$), λ and Λ are positive constants.

The main purpose of this paper is to establish the existence and uniqueness of entropy solutions for the Dirichlet problem

$$(P)\begin{cases} Lu(x) &= f(x), \text{ in } \Omega \\ u(x) &= 0, \text{ in } \partial\Omega \end{cases}$$

where $f \in L^1(\Omega)$. We propose to solve the problem (P) by approximation with variational solutions: we take $f_n \in C_0^\infty(\Omega)$ such that $f_n \to f$ in $L^1(\Omega)$, we find a solution $u_n \in W_0^{1,2}(\Omega, \omega)$ for the problem with second member f_n and we will try to pass the limit $n \to \infty$.

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Sobre Problemas Elípticos Singulares Envolvendo o p(x)-Laplaciano

by

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Mostraremos a existência de soluções positivas para um problema elíptico singular envolvendo o p(x)-Laplaciano. Usaremos um teorema devido a Rabinowitz o qual garante a existência de uma componente não-limitada de soluções de um problema não-linear de autovalor.

Periodic solutions for an extended Fisher-Kolmogorov and Swift-Hohenberg equations by truncature techniques

by

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Combining truncature techniques with variational approach we establish an existence result of nontrivial periodic solutions for a class of fourth-order ordinary differential equations of the form

$$(P) \qquad u^{iv} + qu'' + \alpha(x)u = f(x, u, u', u'', u''') \quad x \in \mathbb{R}.$$

The equation (P) when $q > 0$ is called Swift-Hohenberg equation and extended Fisher-Kolmogorov equation when $q \leq 0$.

In our work [1], we used similar arguments from de Figueiredo, Girardi and Matzeu (in [2]) to study the following problem with Navier boundary conditions

$$(P_1) \qquad \begin{cases} u^{iv} + qu'' + \alpha(x)u = f(x, u, u', u'', u''') \quad \text{in} \quad (0, L) \\ \\ u(0) = u(L) = u''(0) = u''(L) = 0, \end{cases}$$

where L is a positive real number.

By imposing some additional assumptions, we can get a periodic solution for problem (P).

References

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A transmission problem for elastic systems with a moving interface

by

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In this work one considers partial differential equations modeling an elastic system composed by two different elastic components, coupled through a moving interface. Assuming that one of its components is purely elastic (non-dissipative) while the other is viscoelastic. The model lies in the class of hyperbolic transmission problems. Results concerned with existence and regularity of solutions are presented using Galerkin methods. We also show exponential decay of the solution as time goes to infinity.

A nonlinear equation in Banach spaces and applications to well-posedness of Cauchy problems

by

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We analyze a nonlinear equation in Banach spaces, with nonlinearity composed by multiple terms of different degrees. We prove a theorem regarding the existence of solutions for such equations. Moreover, we show how this result may be applied to obtain well-posedness of various parabolic initial value problems.

Positive solutions for some quasilinear equations with critical and supercritical growth

by

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We establish results concerning the existence and multiplicity of positive solutions for the problem

$$-\mathrm{div}(a(\varepsilon x)|\nabla u|^{p-2}\nabla u) + u^{p-1} = f(u) + u^{p^*-1} \text{ in } \mathbb{R}^N,\ u \in W^{1,p}(\mathbb{R}^N),$$

where $\varepsilon > 0$ is a small paramenter, $2 \leq p < N$, $p^* = Np/(N-p)$, a is a positive potential and f is a superlinear function. We obtain existence of a ground state solution and relate the number of positive solutions with the topology of the set where a attains its minimum. We also prove a multiplicity result for a supercritical version of the above problem. In the proofs we use minimax theorems and Ljusternik-Schnirelmann theory

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On the number of solutions of some resonant elliptic systems

by

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We consider the problem of existence and multiplicity of solutions for the gradient system

$$\begin{cases} -\Delta u = F_u(x,u,v) & \text{in } \Omega, \\ -\Delta v = F_v(x,u,v) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain and $N \geq 3$. The function $F \in C^2(\Omega \times \mathbb{R}^2, \mathbb{R})$ is asymptotically linear and resonant cases are considered. The results are proved by applying minimax arguments and Morse Theory. The talk is based in a jointly work with Francisco O.V. de Paiva [1].

References

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Comportamento na fronteira de soluções homogêneas de campos vetoriais

by

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Uma função holomorfa F definida no disco com crescimento temperado na fronteira têm um valor de bordo bF no sentido fraco. Se bF é uma função de potência p-integrável, as restrições de F aos círculos de raio $r < 1$ terão p-norma uniformemente limitada. Este comportamento global admite versões de natureza local. Considerando que as funções holormorfas são soluções homogêneas de um campo vetorial (o operador de Cauchy-Riemann), discutimos questões análogas para as soluções homogêneas de campos não necessariamente elípticos.

Exact Controllability of the 1D Nonlinear Damped Wave Equation

by

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In this paper we prove the exact controllability of the following one dimensional nonlinear damped wave equation

$$\begin{cases} w_{tt} + cw_t - dw_{xx} = u(t,x) + f(t,w,w_t,u(t,x)), & 0 < x < 1 \\ w(t,0) = w(t,1) = 0, & t \in I\!R \end{cases}$$

where $d > 0$, $c > 0$, the distributed control u belong to $L^2(0,t_1;L^2(0,1))$ and the nonlinear term $f(t,w,v,u)$ is a function $f : [0,t_1] \times I\!R^3 \to I\!R$ whose partial derivatives $\frac{\partial f}{\partial w}(t,w,v,u)$, $\frac{\partial f}{\partial v}(t,w,v,u)$

preserve sings and

$$\sup_{[0,t_1]\times I\!R^3}\left|\frac{\partial f}{\partial w}(t,w,v,u)\right| < d\pi^2, \qquad \sup_{[0,t_1]\times I\!R^3}\left|\frac{\partial f}{\partial v}(t,w,v,u)\right| < c.$$

Moreover, we compute the control steering the initial state $(w(0), w_t(0))$ to the final state $(w(t_1), w_t(t_1))$.

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Vanishing viscosity limit of incompressible flow around a sufficiently small obstacle

by

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Há alguns anos vimos estudando, junto com Milton Lopes Filho e Dragos Iftimie, o limite de escoamentos incompressíveis no exterior de um pequeno obstáculo quando o diâmetro do obstáculo tende a zero. Nesta palestra descreverei um resultado recente em que mostramos que o limite de viscosidade evanescente para escoamentos em domínio exterior a um pequeno obstaculo são soluções das equações de Euler, desde que o obstaculo seja suficientemente pequeno em relação à viscosidade.

Solitary waves for some nonlinear Schrodinger systems

by

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In this paper we study the existence of radially symmetric positive solutions in $H^1_{rad}(I\!R^N) \times H^1_{rad}(I\!R^N)$ of the elliptic system:

$$-\Delta u + u - (\alpha u^2 + \beta v^2)u = 0$$
$$-\Delta v + \omega^2 v - (\beta u^2 + \gamma v^2)v = 0,$$

$N = 1, 2, 3$, where α and γ are positive constants (β will be allowed to be negative). This system has trivial solutions of the form $(\phi, 0)$ and $(0, \psi)$ where ϕ and ψ are nontrivial solutions of scalar equations and we are interested in showing the existence of nontrivial solutions. The region of the parameters for which we prove existence is described in terms of a certain function $\lambda_1(\eta)$. For $N = 2, 3$ this function can be calculated only numerically but for $N = 1$ the region of existence can be described very explicitly in terms of the parameters α, β and γ. At least for $N = 1$, there is some numerical evidence that our results may be optimal for existence of such solutions. Our main results concern the case where the trivial solutions $(\phi, 0)$ and $(0, \psi)$ have Morse index equal to one. Our main tools are a mountain pass argument on the Nehari manifold $\mathcal{N}$ and some spectral analysis.

The limiting problem of a heat equation in a dumbbell domains

by

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In this lecture we present an *ill posed* heat equation. It consists of the heat equation in a domain consisting of an open, bounded and smooth domain $\Omega \subset \mathbb{R}^N$ to which we attach a curve R_0. In Ω the evolution is independent of the evolution in R_0 whereas in R_0 the evolution depends on the evolution in Ω, through its values in the points where the curve touches the boundary of Ω. We analyze in detail the linear elliptic and parabolic ill posed problem and give meaning to the existence of solutions (in some wide sense) to it. This together with some variation of constants formula is used to obtain solutions for the associated semilinear problems. The existence of attractors and some special solutions are also considered.

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Pseudodifferential operators with C*-algebra-valued symbols: abstract characterizations

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Given a separable unital C*-algebra C with norm $||\cdot||$, let E_n denote the Banach-space completion of the C-valued Schwartz space on $\mathbb{R}^n$ with norm $||f||_2 = ||\langle f, f\rangle||^{1/2}$, $\langle f, g\rangle = \int f(x)^* g(x)dx$. The assignment of the pseudodifferential operator $A = a(x, D)$ with C-valued symbol $a(x, \xi)$ to each smooth function with bounded derivatives $a \in \mathcal{B}^C(\mathbb{R}^{2n})$ defines an injective mapping O, from $\mathcal{B}^C(\mathbb{R}^{2n})$ to the set $\mathcal{H}$ of all operators with smooth orbit under the canonical action of the Heisenberg group on the algebra of all adjointable operators on the Hilbert module E_n. In [3], we construct a left-inverse S for O and prove that S is injective if C is commutative. This generalizes Cordes' description [1] of $\mathcal{H}$ in the scalar case. Combined with previous results [4] of the second-named author, our main theorem implies that, given a skew-symmetric $n \times n$ matrix J, and if C is commutative, then any $A \in \mathcal{H}$ which commutes with every pseudodifferential operator with symbol $F(x + J\xi)$, $F \in \mathcal{B}^C(\mathbb{R}^n)$, is a pseudodifferential operator with symbol $G(x - J\xi)$, for some $G \in \mathcal{B}^C(\mathbb{R}^n)$. That was conjectured by Rieffel [5].

The talk will start with a discussion of the scalar case [1, 2], when $C = \mathbb{C}$ and E_n coincides with $L^2(\mathbb{R}^n)$.

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On positive solution for a class of degenerate quasilinear elliptic semipositone and positone systems

by

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This work deals with existence and nonexistence of positive solutions for a class of degenerate quasilinear elliptic systems involving singularity, such as

$$\left\{\begin{array}{rcll} -div(|x|^{-ap}|\nabla u|^{p-2}\nabla u) & = & \lambda|x|^{-(a+1)p+c_1}f_1(x,u,v) & \text{in } \Omega, \\ -div(|x|^{-bq}|\nabla v|^{q-2}\nabla v) & = & \lambda|x|^{-(b+1)q+c_2}f_2(x,u,v) & \text{in } \Omega, \\ u=v & = & 0 & \text{on } \partial\Omega, \end{array}\right. \tag{4}$$

where Ω is a bounded smooth domain of $\mathbb{R}^N$ with $0 \in \Omega$, $1 < p, q < N$, $0 \leq a < (N-p)/p$, $0 \leq b < (N-q)/q$, $c_1, c_2 > 0$, λ a positive parameter and $f_1, f_2 : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are continuous and monotone functions. Actually, the nonlinearities involved have semipositone/positone structures. The existence results are obtained by applying the lower and upper solution method.

The extremal solution of a boundary reaction problem

by

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We consider the problem $\Delta u = 0$ in Ω with $\partial u/\partial \nu = \lambda e^u$ on Γ_1 and $u = 0$ on Γ_2 where $\lambda > 0$ and Γ_1, Γ_2 is a partition of $\partial\Omega$ and $\Omega \subset \mathbb{R}^N$. We determine sharp conditions on the dimension N such that the extremal solution is bounded, where the extremal solution refers to the one associated to the largest λ for which a solution exists. Optimal conditions on the dimension N are intimately related to Hardy inequalities.

Diffusion equations and the Feynman-Kac formula

by

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Certain state functions are expressible as the mean value of a kernel function which is itself a random function of an underlying stochastic process. The pde satisfied by the state function is then deducible from a pde satisfied by the simpler kernel function. This idea is illustrated by examples from Brownian motion, option pricing theory, and quantum mechanics.

KdV Periodic Travelling Waves

by

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Travelling waves of evolution equations are obtained by searching for solutions $u(x,t)$ of the form $u(x,t) = \phi(x - ct)$ where c is the speed of propagation. The terminology used to classify the different types of travelling waves depends essentially on their shape. Those waves that are symmetric, single crested, and whose profile is like $\text{sech}^2(x)$ are known as Scott Russel's solitary waves or solitons; and the waves that are periodic in the x-variable, are known as Boussinesq's and Korteweg-de Vries' cnoidal waves. The theory of existence and stability of solitary waves has been improved in several ways since Benjamin in 1972. Comparatively, cnoidal waves have received little attention. The orbital stability of the Korteweg-de Vries (KdV) cnoidal waves was only recently studied by Angulo, Bona and Scialom. In this talk we will discuss mainly two questions: the convergence of cnoidal waves to the soliton, as the period tends to infinity; and, the conditions under which the family of linearized operators at the cnoidal waves is isoinertial.

A-priori bounds and positive solutions to a class of quasilinear elliptic equations

by

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We present some results about a-priori estimates for positive solutions u of the equation

$$-div(|\nabla u|^{p-2}\nabla u) = f(u)$$

on a bounded domain Ω with $u = 0$ on $\partial\Omega$ and $1 < p < 2$. Whenever $f(u) = u^q$ our results include the range $p-1 < q < p^* - 1$ and nonconvex domains. We obtain an existence result without assume any superlinear assumption. A result like this was obtained in [1] for the case $p = 2$.

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Transversal Periodic–to–Periodic Homoclinic Orbits

by

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We consider an autonomous system of ordinary differential equations with a hyperbolic periodic orbit. A transversal homoclinic orbit is an orbit along which the stable and unstable manifolds of the periodic orbit intersect transversally. Sil'nikov showed that the dynamics is chaotic in the neighbourhood of such a homoclinic orbit. In this talk we discuss the theory of these homoclinic orbits and also how systems possessing such orbits can be constructed using perturbation theory and rigorous numerical shadowing. Some of this is joint work with Flaviano Battelli, Brian Coomes and Huseyin Kocak.

On Attractors for Multivalued Semigroups Defined by Generalized Semiflows

by

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We extend results from Semigroup Theory on existence and characterization of attractors in order to include multivalued semigroups $T(t)$ defined by generalized semiflows $\mathcal{G}$. In particular we show that, if $\mathcal{G}$ is continuous, possesses a Lyapunov function, and $\mathcal{G}$ has a global attractor $\mathcal{A}$ which is maximal compact invariant, then $\mathcal{A} = W^u(Z(\mathcal{G}))$, where $Z(\mathcal{G})$ is the stationary solutions set and $W^u(Z(\mathcal{G}))$ is the unstable set of $Z(\mathcal{G})$. We introduce the φ-attractor concept which does not enjoy any uniformity on time of attraction and we prove, under suitable conditions, that the global φ-attractor $\widehat{N}$ is the set of asymptotic states described by $Z(\mathcal{G})$.

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Global solvability of the initial boundary value problem for Navier-Stokes equation

by

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We use a theory of Fractional Powers in order to find some conditions for existence of a global solution of Navie-Stokes equation in the 3-dimensional case.

"Averaging" e Órbitas Periódicas

by

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O objetivo da palestra e apresentar alguns resultados sobre a existência de ciclos limites para sistemas em dimensões 2,3 e 4 que são perturbacoes de centros através de um método já conhecido (Averaging) mas com uma nova roupagem.
