

RADIAL SYMMETRY OF MINIMIZERS FOR A CLASS OF NONLOCAL VARIATIONAL PROBLEMS

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We present some results about the radial symmetry of minimizers for some constrained variational problems for nonlocal functionals. For instance, consider the problem of minimizing

$$E(u, v) = \int_{R^N} \left(\frac{1}{2} |\text{grad } u|^2 + \frac{1}{2} |(-\Delta)^{1/4} u|^2 \right) dx + \int_{R^N} F(u, v) dx$$

subject to

$$Q(u, v) = \frac{1}{2} \int_{R^N} G(u, v) dx = c.$$

Under smoothness and growth assumptions on $F(u, v)$ and $G(u, v)$, and except for translation in the space variable, the minimizers are radially symmetric. There is no cooperativity assumption and we do not assume that the minimizer is positive. In that case, symmetrization cannot be used. Our approach is a combination of reflection presented in [1] with an identity involving the nonlocal term. Further examples are given.

REFERENCES

- [1] O. Lopes, Radial symmetry of minimizers for some translation and rotation invariant functionals. J. Differential Equations 124 (1996), no. 2, 378-388.

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