# PATTERNS AND LOCAL MINIMIZERS FOR BOUNDARY REACTIONS

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The existence of nonconstant stable solutions (so called patterns) for diffusion equations with some kind of nonlinear boundary reactions is an important question often related with the geometry of the domain. Basically we are going to consider

$$\left\{ \begin{array}{rcll} u_t - \Delta u & = & 0, & \text{in } \Omega \\ u_\nu & = & \frac{1}{\varepsilon} f(u), & \text{on } \Gamma = \partial\Omega \end{array} \right. \tag{1}$$

for a model reaction term $f(u) = u - u^3 = u(1 - u^2)$.

In this talk we present numerical evidence of the existence of nonconstant stable equilibria for the unit square, computing families of equilibria branching off the unstable constant equilibria $u = 0$. We remark that it is the first approximation to the existence of patterns for convex domains.

The existence of equilibria con be treated as well as a variational problem and found the solutions minimizing the energy with a constraint as in the well known Ginzbrug-Landau vortices phenomena. Given any $\varepsilon > 0$ we may consider solutions of (1), $u_\varepsilon$, which are local minima of the associated energy

$$E_\varepsilon(u) = \frac{1}{2} \int_\Omega |\nabla u|^2 \, dx + \frac{1}{\varepsilon} \int_\Gamma G(u) \, d\ell \,, \tag{2}$$

where $G' = -f$, with a constraint, that is,

$$E_\varepsilon(u_\varepsilon) = \min_{u \in X_a} E_\varepsilon(u)$$

with $X_a = \{u \in H^1(\Omega) : \|\gamma u - \chi^{p_0,q_0}\|^2_{L(\Gamma)} \leq a^2\}$, and, where $\chi^{p_0,q_0}$ is the characteristic function with transition points $p_0$, $q_0 \in \Gamma$, alternating values $-1$ and $+1$.

In this way in the second part of the talk we show that we can find local minimizers as local minimizers of the renormalized energy associated to (2). As the reaction acts on the boundary these minimizers correspond to pairs of points of this boundary and the solution is obtained by harmonic extension to the interior of the domain. Theses patterns present a layer in the transiton points, or minimizing pair, where the solution goes from $-1$ to $+1$ in a very small region.

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