# ON THE EXISTENCE OF PATTERNS IN A REACTION-DIFFUSION EQUATION WITH NONLINEAR NEUMANN BOUNDARY CONDITION

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In this work we prove existence and determine the asymptotic profile of a family of layered stable stationary solutions (patterns, for short) to the following reaction-diffusion equation

$$
\begin{cases} \dfrac{\partial u}{\partial t} = \varepsilon^2 \Delta u + f(u) \quad \text{in } \Omega \\ \varepsilon \dfrac{\partial u}{\partial \nu} = \delta_\varepsilon g(u) \quad \text{on } \partial\Omega \\ u(0,x) = u_0(x), \; x \in \overline{\Omega} \end{cases} \tag{1}
$$

where $\Omega \subset \mathbb{R}^3$ is a $C^2$ simply connected bounded domain and $\varepsilon$ a small positive parameter. It is assumed that:

- $\delta_\varepsilon \geq 1$ satisfies $\lim_{\varepsilon \to 0} \varepsilon \ln \delta_\varepsilon = \kappa$ with $0 \leq \kappa < \infty$.
- $\int_\alpha^\beta f = 0$
- $\int_{\alpha'}^{\beta'} g = 0$,

with $\alpha' \leq \alpha < \beta \leq \beta'$ and $f(\alpha) = f(\beta) = 0$, $g(\alpha') = g(\beta') = 0$.

Above relation holds, for example, when $\delta_\varepsilon = \varepsilon^{-n}$, $n \in \mathbb{N}$ as well as $\delta_\varepsilon = e^{\kappa/\varepsilon}$, $\kappa \geq 0$. In particular when $\delta_\varepsilon = \varepsilon^{-1}$ there holds that diffusibility in $\Omega$ and $\partial\Omega$ are the same.

The equal-area conditions for $f$ and $g$ are actually necessary for existence of such solutions (see [2]). The nonlinear Neumann boundary condition gives rise to an involved geometric profile of the patterns, namely, the trace of the function the family of solutions approach on $\Omega$, as $\varepsilon \to 0$, is not the function the solutions approach on $\partial\Omega$.

Main tools used are Gamma-convergence of functionals, variational techniques and results of dynamical systems in infinite dimension.

## References

[1] G. Alberti, E. Bouchitté, and P. Seppecher, "Phase Transition with the Line-Tension Effect," *Arch.Rational Mech. Anal.* **144**, 1-46 (1998).

[2] A. S. do Nascimento, R. J. de Moura, "The Role of Equal Area Condition in Internal and Superficial Layered Solutions to Some Nonlinear Boundary Value Elliptic Problem," *Progress In Nonlinear Differential Equations And Their Applications* **66** , 415-427 (2005).

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