MULTIPLICITY OF SOLUTIONS FOR A CONVEX-CONCAVE PROBLEM WITH A NONLINEAR BOUNDARY CONDITION

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We study the existence of multiple positive solutions for a convex-concave problem with a nonlinear boundary condition involving two critical exponents and two positive parameters λ and μ of the type

$$\begin{cases} -\Delta u + u = \lambda u^{q_1} + u^{p_1} & \text{in} \quad \Omega, \\ \frac{\partial u}{\partial \nu} = \mu u^{q_2} + u^{p_2} & \text{on} \quad \partial\Omega, \\ u > 0 & \text{in} \quad \Omega, \end{cases} \qquad (P_{\lambda\mu})$$

where $0 < q_i < 1 < p_i < \infty$ $(i = 1, 2)$, $\Omega \subset I\!R^N$ $(N \geq 3)$ is a smooth bounded domain and $\frac{\partial u}{\partial \nu}$ is the outer unit normal derivative.

We obtain a continuous strictly decreasing function f such that $K_1 \equiv \{(f(\mu), \mu) : \mu \in [0, \infty)\}$ divides $[0, \infty) \times [0, \infty) \setminus \{(0, 0)\}$ in two connected sets K_0 and K_2 such that problem $(P_{\lambda\mu})$ has at least two solutions for $(\lambda, \mu) \in K_2$, at least one solution for $(\lambda, \mu) \in K_1$ and no solution for $(\lambda, \mu) \in K_0$. This work is related with following papers [2], [3], [4] and [6].

By sub and super solution method [1], we obtain a minimal positive solution of $(P_{\lambda\mu})$, and we employ a version of the Ambrosetti-Rabinowitz Mountain Pass Theorem due to Ghoussoub and Preiss [5] in order to get the second positive solution.

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